International Trade and Macroeconomic Dynamics with Heterogeneous Firms[*]

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Abstract

We develop a stochastic, general equilibrium, two-country model of trade and macroeconomic dynamics. Productivity differs across individual, monopolistically competitive firms in each country. Firms face a sunk entry cost in the domestic market and both fixed and per-unit export costs. Only relatively more productive firms export. Exogenous shocks to aggregate productivity and entry or trade costs induce firms to enter and exit both their domestic and export markets, thus altering the composition of consumption baskets across countries over time. In a world of flexible prices, our model generates endogenously persistent deviations from PPP that would not exist absent our microeconomic structure with heterogeneous firms. It provides an endogenous, microfounded explanation for a Harrod-Balassa-Samuelson effect in response to aggregate productivity differentials and deregulation. Finally, the model matches several moments of the U.S. and international business cycle quite well.

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1 Introduction

Formal models of international macroeconomic dynamics do not usually address or incorporate the determinants and evolution of trade patterns. The vast majority of such macroeconomic models take the pattern of international trade and the structure of markets for goods and factors of production as given.[1] The determinants of such trade patterns are, in turn, analyzed within methodologically distinct models that are generally limited to comparisons of long-run positions or growth dynamics after changes in some determinants of trade. These models do not consider short- to medium-run business cycle dynamics and their effect on the pattern of trade over time.[2] This separation between modern models of international macroeconomics and trade theory is somewhat unnatural. Modern international macroeconomics prides itself on its microfoundations. Yet, it neglects to analyze the effects of macro phenomena on the microeconomic underpinnings of its structure. Similarly, much of trade theory does not recognize the aggregate effects of micro dynamics that feed back into further micro adjustments over time.

This paper contributes to bridging the gap between international macroeconomics and trade theory by using Melitz's (2003) model of trade with monopolistic competition and heterogeneous firms as the microeconomic underpinning of a two-country, dynamic, stochastic, general equilibrium (DSGE) model of international trade and macroeconomics.[3] Although international macro models incorporate firm-level decisions, they do not usually address the firms' entry and exit behavior, and how the induced dynamics affect the transmission of shocks to the macroeconomy. Once we allow for the entry and exit of firms into and from markets over the economic cycle in an international macro model, the trade pattern becomes endogenous, and it is then no longer possible to separate macro and micro phenomena in a general equilibrium framework.

We adopt a simplified version of Melitz's (2003) setup, although one that is rich enough to deliver a variety of novel results. We assume that productivity differs across individual, monopolistically competitive firms in each country. Firms face some initial uncertainty concerning their future productivity when making an irreversible investment to enter the domestic market. In addition to the sunk entry cost, firms face both fixed and per-unit export costs. Forward-looking firms formulate entry and export decisions based on expectations of future market conditions. Only a

[1] See Lane (2001) for a survey of the recent literature. We discuss the relation between our work and some exceptions to this trend in international macroeconomics below.

[2] Baldwin and Krugman (1989) is an exception here. They analyze the effects of cycles in exchange rates on trade patterns.

[3] Melitz (2003) focuses on the analysis of the steady-state effects of trade on intra-industry reallocations between heterogeneous firms and their effect on aggregate productivity.

subset of relatively more productive firms export, while the remaining, less productive firms only serve their domestic market. This microeconomic structure endogenously determines the extent of the traded sector and the composition of consumption baskets in both countries. Exogenous shocks to aggregate productivity or entry and trade costs induce firms to enter and exit both their domestic and export markets, thus altering the composition of consumption baskets across countries over time.

We first introduce this microeconomic structure in a flexible-price model with no international trade in financial assets. We therefore initially focus on the role of goods market dynamics in a relatively simple setup. We show that the microeconomic features of our model have important consequences for macroeconomic variables. Macroeconomic dynamics, in turn, feed back into firm level decisions, further altering the pattern of trade over time. Our model generates deviations from purchasing power parity (PPP) that would not exist absent our microeconomic structure with heterogeneous firms. It provides an endogenous, microfounded explanation for a Harrod-Balassa-Samuelson (HBS) effect in response to aggregate productivity differentials and deregulation.

Textbook analysis of the HBS effect assumes the exogenous existence of a non-traded sector and a favorable productivity shock to the traded sector alone (Obstfeld and Rogoff, 1996, pp. 210-212). The shock causes the relative price of non-traded goods to increase, leading to a real exchange rate appreciation. An aggregate productivity increase (across all sectors) would have no effect on the real exchange rate. Balassa (1964), Harrod (1933), and Samuelson (1964) first pointed out the tendency for countries with higher productivity (assumed to predominantly affect the traded-goods sectors) to have higher prices.

In our model, all goods are *tradeable*; some are *non-traded* in equilibrium. More productive firms self-select into the traded sector in each country. In this respect, our model is consistent with the evidence in Bernard, Eaton, Jensen, and Kortum (2003), who document than only 21 percent of U.S. manufacturing plants actually export and with evidence in several studies on the self-selection of more productive plants into the traded sector.[4] Our model predicts that more productive economies, or less regulated ones (phenomena that effect *all* firms in the economy), exhibit higher average prices relative to their trading partners. This endogenous HBS effect is driven by two key new features in our setup: We show that effective labor units (adjusting for productivity) must relatively appreciate in the economy providing the more attractive environment for firms. This is a key consequence of endogenous firm entry – along with entry costs for the

[4] See references in Melitz (2003).

deregulation scenario. (This effect is very similar to the home market effect first described by Krugman, 1980.) Given the existence of a non-traded sector (implied by the fixed export costs), the relative increase in labor costs must induce an appreciation. In addition, changes in relative labor costs further induce changes in the composition of the traded sector in both countries. Import prices in the economy with higher labor costs (say, home) rise as lower productivity foreign firms now export to it. Conversely, import prices in the economy with lower labor costs decrease as the relatively less productive home exporters drop out of the export market. These effects necessarily reinforce the real exchange rate appreciation. Here, the endogenous determination of the traded sector plays a key role. An expenditure switching effect further reinforces the appreciation.

In addition to providing new foundations for the HBS effect (relying – we will argue – on an empirically relevant mechanism), we show that this mechanism generates deviations from PPP that display substantial endogenous persistence in response to transitory aggregate shocks (for very plausible calibrated parameters). Our model's micro-foundations therefore explain such persistent PPP deviations, even when prices are fully flexible. More generally, the introduction of micro dynamics motivated by heterogeneity and entry and trade costs significantly improves the ability of the model to generate endogenously persistent dynamics – a stumbling bloc for many well-known DSGE macro models. When we remove the assumption of financial autarky and allow for international trade in bonds, our model predicts that more productive economies, or less regulated ones, run persistent foreign debt positions to finance faster entry of new firms into a relatively more favorable environment. This combines with the endogenous HBS effect to deliver foreign debt and real exchange rate dynamics that are consistent with stylized facts for the United States in the 1990s.

A stochastic exercise shows that the model matches several important moments of the U.S. and international business cycle quite well for reasonable assumptions about parameters and productivity. In contrast to benchmark international real business cycle (RBC) models, our setup generates positive GDP correlation across countries, it does not automatically produce high correlation between relative consumption and the real exchange rate, and it improves substantially on the traditional framework as far as the "consumption-output anomaly" is concerned. These results confirm Obstfeld and Rogoff's (2001) finding that trade costs help explain international macroeconomic puzzles raised in the international RBC literature of the early 1990s (for instance, Backus, Kehoe, and Kydland, 1992, and Backus and Smith, 1993).

The assumption that firms pay fixed export costs that do not vary with export volume is clearly central to our results. Recent micro-level empirical studies have documented the importance and relevance of sunk market entry costs in explaining firm export behavior.[5] Interviews with managers making export decisions also confirm that firms in differentiated product markets face significant fixed costs associated with entry in export markets (Roberts and Tybout, 1997*a*): A firm must find and inform foreign buyers about its product and learn about the foreign market. It must research the foreign regulatory environment and adapt its product to ensure conformity to foreign standards (including testing, packaging, and labeling requirements). An exporting firm must also set up new distribution channels in the foreign country and conform to all the shipping rules specified by the foreign customs agency. Governments often manipulate some of these costs to erect non-tariff barriers to trade. We assume fixed, per-period export costs rather than sunk export market entry costs to keep our model simple. This implies that there is no hysteresis in entry and exit decisions in export markets – which would add persistence to our model's dynamics. However, we do model the sunk nature of entry costs for new firms, which induces persistence in the firm-level entry and exit decisions.

As far as bringing together trade theory and macroeconomics, Dornbusch, Fischer, and Samuelson (DFS, 1977) is probably the best known antecedent of our work, though our approach differs in several respects.[6] Baldwin and Krugman's (1989) model is closer to ours in spirit. They analyze how cycles in nominal exchange rates can have long-lasting effects through their impact on trade patterns when prices are sticky. Their model features entry and exit decisions as a key mechanism of transmission. However, theirs is a partial equilibrium, stylized framework. We highlight important trade and macro consequences of general equilibrium dynamics – and leave the introduction of price stickiness for future work.

Other contributions to the international macroeconomic literature have emphasized the role of trade costs and composition effects in the propagation of shocks. Already Backus, Kehoe, and Kydland (1992) showed that the inclusion of trade frictions improves the quantitative performance of an international RBC model. Obstfeld and Rogoff (2001) present simple models in which the addition of per-unit trade costs and the potentially endogenous nature of "tradedness" help explain a number of puzzles in international macroeconomics. Burstein, Eichenbaum, and Rebelo (2002)

[5] See Bernard and Jensen (2001) (for the U.S.), Bernard and Wagner (2001) (for Germany), Das, Roberts, and Tybout (2001) (for Colombia), and Roberts and Tybout (1997*b*) (for Colombia).

[6] See also Obstfeld and Rogoff (1996, pp. 235-257) on the DFS model. Kehoe and Ruhl (2002) develop a version of the model suitable for calibration to match observed post-trade-liberalization export growth on the intensive and extensive margins. Kraay and Ventura (2002) extend the DFS model to allow for international trade in a complete set of Arrow-Debreu securities and study the consequences of trade integration on the dynamics of the trade balance.

focus on the role of composition effects.[7]

Corsetti, Martin, and Pesenti (2003) develop a two-country, sticky-wage model with entry subject to a fixed cost. All goods are traded in their model, and they explore the implications of entry for the transmission of monetary shocks. Bergin and Glick (2003*a*, *b*) introduce heterogeneous, good-specific, "melting-iceberg" costs in perfect-foresight, two-period, small, open, endowment economy models. Bergin, Glick, and Taylor (2003) use a model with monopolistic competition, fixed export costs, and heterogeneous productivity (but no entry into domestic markets) in their analysis of the HBS effect. Betts and Kehoe (2001) introduce heterogeneous, per-unit trade costs in a multi-country, trade and macro model with complete asset markets and differentiated goods. As in our paper, endogenous non-tradedness is central to macro dynamics in Bergin and Glick's work and in Betts and Kehoe's. The focus on fixed costs, heterogeneous productivity, and entry and exit decisions of monopolistically competitive firms over the business cycle distinguishes our approach from those of these contributions. Alessandria and Choi (2003), Ruhl (2003), and Russ (2003) develop models that are closest to ours. Alessandria and Choi assume that firm-specific productivity displays no persistence, whereas we make the opposite assumption of extreme persistence. They focus on specific business cycle moments, while we draw attention also to long-run results and the way shocks propagate over time to explore issues such as the HBS effect. Ruhl uses a model with financial autarky that includes an exogenously non-traded good to reconcile estimates of the elasticity of substitution between home and foreign goods in the trade and macro literatures. Russ includes foreign direct investment (FDI) and nominal stickiness in her analysis and focuses on the relation between FDI and the nominal exchange rate.

The rest of the paper is organized as follows. Section 2 describes the benchmark model with financial autarky. Section 3 presents results on the determinants of the real exchange rate in our setup. These results guide our interpretation of the impulse responses in Section 4, which analyzes the dynamics of the model in response to shocks to aggregate productivity, sunk entry costs (interpreted as changes in domestic goods market regulation facing firms in each country), and trade costs (interpreted as changes in trade policy). Section 5 discusses how allowing for international bond trading affects dynamics and expounds on the implications of our model for current account dynamics. It also presents the results of a stochastic simulation of the model. Section 6 concludes.

[7] Cuñat and Maffezzoli (2002), Kraay and Ventura (2000 and 2001), and Ricci (1997) are other studies that combine trade theory and macroeconomics.

2 The Model

We begin by developing a version of our model in which there is no international trade in financial assets.

Household Preferences and Intratemporal Choices

The world consists of two countries, home and foreign. We denote foreign variables with a superscript star. Each country is populated by a unit mass of atomistic households. All contracts and prices in the world economy are written in nominal terms. Prices are flexible. Thus, we only solve for the real variables in the model. However, since the composition of consumption baskets in the two countries changes over time (affecting the definitions of the consumption-based price indexes), we introduce money as a convenient unit of account for contracts. Money plays no other role in the economy. For this reason, we do not model the demand for cash currency, and resort to a cashless economy as in Alessandria and Choi (2003) and Woodford (2003).[8]

The representative home household supplies L units of labor inelastically in each period at the nominal wage rate W_t, denominated in units of home currency. The household maximizes expected intertemporal utility from consumption (C): $E_t\left[\sum_{s=t}^{\infty}\beta^{s-t}C_s^{1-\gamma}/\left(1-\gamma\right)\right]$, where $\beta\in(0,1)$ is the subjective discount factor and $\gamma>0$ is the inverse of the intertemporal elasticity of substitution.

At time t, the household consumes the basket of goods C_t, defined over a continuum of goods Ω: $C_t=\left(\int_{\omega\in\Omega}c_t\left(\omega\right)^{\frac{\theta-1}{\theta}}d\omega\right)^{\frac{\theta}{\theta-1}}$, where $\theta>1$ is the symmetric elasticity of substitution across goods. At any given time t, only a subset of goods $\Omega_t\subset\Omega$ is available. Let $p_t\left(\omega\right)$ denote the home currency price of a good $\omega\in\Omega_t$. The consumption-based price index for the home economy is then $P_t=\left(\int_{\omega\in\Omega_t}p_t\left(\omega\right)^{1-\theta}d\omega\right)^{\frac{1}{1-\theta}}$, and the household's demand for each individual good ω is $c_t\left(\omega\right)=\left(p_t\left(\omega\right)/P_t\right)^{-\theta}C_t$.

The foreign household supplies L^* units of labor inelastically in each period in the foreign labor market at the nominal wage rate W_t^* per unit, denominated in units of foreign currency. It maximizes a similar utility function, with identical parameters and a similarly defined consumption basket. Crucially, the subset of goods available for consumption in the foreign economy during period t is $\Omega_t^*\subset\Omega$ and can differ from the subset of goods that are available in the home economy.

[8] Models of monetary economies in which policy is conducted through interest rate setting have de-emphasized the role of money demand even under sticky prices (Woodford, 2003).

Firms

There is a continuum of firms in each country, each producing a different variety $\omega \in \Omega$. Production requires only one factor, labor. Aggregate labor productivity is indexed by Z_t (Z_t^*), which represents the effectiveness of one unit of home (foreign) labor. Firms are heterogeneous as they produce with different technologies indexed by relative productivity z. A home firm with relative productivity z produces $Z_t z$ units of output per unit of labor employed. Productivity differences across firms therefore translate into differences in the unit cost of production. This cost, measured in units of the consumption good C_t, is $w_t / (Z_t z)$, where $w_t \equiv W_t / P_t$ is the real wage. Similarly, foreign firms are indexed by their productivity z and unit costs (measured in units of the foreign consumption good) $w_t^* / (Z_t^* z)$, where $w_t^* \equiv W_t^* / P_t^*$ is the real wage of foreign workers.[9]

Prior to entry, firms are identical and face a sunk entry cost of $f_{E,t}$ ($f_{E,t}^*$) effective labor units, equal to $w_t f_{E,t}/Z_t$ ($w_t^* f_{E,t}^*/Z_t^*$) units of the home (foreign) consumption good. Upon entry, home firms draw their productivity level z from a common distribution $G(z)$ with support on $[z_{\min}, \infty)$. Foreign firms draw their productivity level from an identical distribution. This relative productivity level remains fixed thereafter. Since there are no fixed production costs, all firms produce in every period, until they are hit with a "death" shock, which occurs with probability $\delta \in (0, 1)$ in every period. This exit inducing shock is independent of the firm's productivity level, so $G(z)$ also represents the productivity distribution of all producing firms. Home and foreign firms can serve both their domestic market as well as the export market. Exporting is costly, and involves both a melting-iceberg trade cost $\tau_t \geq 1$ ($\tau_t^* \geq 1$) as well as a fixed cost $f_{X,t}$ ($f_{X,t}^*$) (measured in units of effective labor). We assume that firms hire workers from their respective domestic labor markets to cover these fixed costs. These costs, in real terms, are then $w_t f_{X,t}/Z_t$ for home firms (in units of the home consumption good) and $w_t^* f_{X,t}^*/Z_t^*$ for foreign firms (in units of the foreign consumption good). The fixed export costs are paid on a period-by-period basis rather than sunk upon entry in the export market.[10]

All firms face a residual demand curve with constant elasticity θ in both markets, and they set fully flexible prices that reflect the same proportional markup $\theta / (\theta - 1)$ over marginal cost. Let $p_{D,t}(z)$ and $p_{X,t}(z)$ denote the nominal domestic and export prices of a home firm. We assume that

[9] We use the same index z for both home and foreign firms as this variable only captures firm productivity relative to the distribution of firms in that country.

[10] Even if there is substantial evidence of the importance of sunk export costs, introducing them in our model implies complications that we leave for future work. We could have also modeled an overhead fixed cost for selling in the domestic market – so long as this cost is low enough that firms with productivity $z_{\min}$ earn a non-negative profit from domestic sales. We ignore this cost for simplicity.

export prices are denominated in the currency of the export market. Prices, in real terms relative to the price index in the destination market, are then given by:

$$\rho_{D,t}(z) \equiv \frac{p_{D,t}(z)}{P_t} = \frac{\theta}{\theta - 1}\frac{w_t}{Z_t z}, \quad \rho_{X,t}(z) \equiv \frac{p_{X,t}(z)}{P_t^*} = Q_t^{-1}\tau_t \rho_{D,t}(z) \tag{1}$$

where $Q_t \equiv \varepsilon_t P_t^*/P_t$ is the consumption-based real exchange rate (units of home consumption per unit of foreign consumption; ε_t is the nominal exchange rate, units of home currency per unit of foreign).[11] However, due to the fixed export cost, firms with low productivity levels z may decide not to export in any given period. When making this decision, a firm decomposes its total profit $d_t(z)$ ($d_t^*(z)$) (returned to households as dividends) into portions earned from domestic sales $d_{D,t}(z)$ ($d_{D,t}^*(z)$) and from potential export sales $d_{X,t}(z)$ ($d_{X,t}^*(z)$). All these profit levels (dividends) are expressed in real terms in units of the consumption basket in the firm's location.[12] In the case of a home firm, they are given by $d_t(z) = d_{D,t}(z) + d_{X,t}(z)$, where

$$d_{D,t}(z) = \frac{1}{\theta}\left[\rho_{D,t}(z)\right]^{1-\theta} C_t, \quad d_{X,t}(z) = \begin{cases} \frac{Q_t}{\theta}\left[\rho_{X,t}(z)\right]^{1-\theta} C_t^* - \frac{w_t f_{X,t}}{Z_t} & \text{if firm } z \text{ exports,} \\ 0 & \text{otherwise.} \end{cases}$$

Similarly in the case of a foreign firm. (In this case, if the firm exports, its export profits are $d_{X,t}^*(z) = Q_t^{-1}\left[\rho_{X,t}^*(z)\right]^{1-\theta} C_t/\theta - w_t^* f_{X,t}^*/Z_t^*$.) As expected, a firm's total profit increases with its productivity level z (even though these firms set relatively lower prices – see (1)). A firm will export if and only if it would earn non-negative profit from doing so. For home firms, this will be the case so long as productivity z is above a cutoff level $z_{X,t} = \inf\{z : d_{X,t}(z) > 0\}$. A similar cutoff level $z_{X,t}^* = \inf\left\{z : d_{X,t}^*(z) > 0\right\}$ holds for foreign exporters. We assume that the lower bound productivity $z_{\min}$ is low enough relative to the export costs that $z_{X,t}$ and $z_{X,t}^*$ are both above $z_{\min}$. This ensures the existence of an endogenously determined non-traded sector: the set of firms who *could* export, but decide not to. These firms, with productivity levels between $z_{\min}$ and the export cutoff level, only produce for their domestic market.[13] This set of firms fluctuates over time with changes in the profitability of the export market, inducing changes in the cutoff levels $z_{X,t}$ and $z_{X,t}^*$. Since we do not have sunk export costs, firms "freely" enter and exit the export market as soon as

[11] Similar price equations hold for foreign firms. Note that $\rho_{X,t}^*(z) \equiv p_{X,t}^*(z)/P_t = Q_t \tau_t^* \rho_{D,t}^*(z)$.

[12] Note that an exporter's relative price $\rho_{X,t}(z)$ ($\rho_{X,t}^*(z)$) is expressed in units of C_t^* (C_t) (the consumption good at the location of sales) but the profits from export sales $d_{X,t}(z)$ ($d_{X,t}^*(z)$) are expressed in units of C_t (C_t^*) (the consumption basket in the firm's location).

[13] All firms that pay the sunk entry cost will produce for their domestic market as they all earn positive profits from doing so (including the firm with the lowest productivity level $z = z_{\min}$).

export conditions change.

Firm Averages

In every period, there is a mass $N_{D,t}$ ($N^*_{D,t}$) of firms producing in the home (foreign) country. These firms have a distribution of productivity levels over $[z_{\min}, \infty)$ given by $G(z)$. Among these firms, there are $N_{X,t} = [1 - G(z_{X,t})] N_{D,t}$ and $N^*_{X,t} = \left[1 - G(z^*_{X,t})\right] N^*_{D,t}$ exporters. These exporters have a distribution of productivity levels over $[z_{X,t}, \infty)$ and $[z^*_{X,t}, \infty)$ given by the conditional distribution of $G(z)$ on these intervals. Following Melitz (2003), we define two special "average" productivity levels – an average $\tilde{z}_D$ for all producing firms (in each country), and an average $\tilde{z}_{X,t}$ for all home exporters – as:

$$\tilde{z}_D \equiv \left[\int_{z_{\min}}^{\infty} z^{\theta-1} dG(z)\right]^{\frac{1}{\theta-1}}, \quad \tilde{z}_{X,t} \equiv \left[\frac{1}{1 - G(z_{X,t})} \int_{z_{X,t}}^{\infty} z^{\theta-1} dG(z)\right]^{\frac{1}{\theta-1}}.$$

(The definition of $\tilde{z}^*_{X,t}$ is analogous to that of $\tilde{z}_{X,t}$.) As shown in Melitz (2003), these productivity averages – based on weights that are proportional to relative output shares of firms – summarize all the information on the productivity distributions relevant for all macroeconomic variables. In essence, our model is isomorphic to one where $N_{D,t}$ ($N^*_{D,t}$) firms with productivity level $\tilde{z}_D$ produce in the home (foreign) country and $N_{X,t}$ ($N^*_{X,t}$) firms with productivity level $\tilde{z}_{X,t}$ ($\tilde{z}^*_{X,t}$) export to the foreign (home) market.

In particular, $p_{D,t}(\tilde{z}_D)$ ($p^*_{D,t}(\tilde{z}_D)$) represents the average nominal price of home (foreign) firms in their domestic market, and $p_{X,t}(\tilde{z}_{X,t})$ ($p^*_{X,t}(\tilde{z}^*_{X,t})$) represents the average nominal price of home (foreign) exporters in the export market. The price index at home therefore reflects the prices of the $N_{D,t}$ home firms (with average price $p_{D,t}(\tilde{z}_D)$) and the $N^*_{X,t}$ foreign exporters to the home market (with average price $p^*_{X,t}(\tilde{z}^*_{X,t})$). As shown in Melitz (2003), the home price index can thus be written:

$$P_t = \left\{N_{D,t} \left[p_{D,t}(\tilde{z}_D)\right]^{1-\theta} + N^*_{X,t} \left[p^*_{X,t}(\tilde{z}^*_{X,t})\right]^{1-\theta}\right\}^{\frac{1}{1-\theta}}, \quad \text{or} \quad 1 = N_{D,t} \left(\tilde{\rho}_{D,t}\right)^{1-\theta} + N^*_{X,t} \left(\tilde{\rho}^*_{X,t}\right)^{1-\theta},$$

where $\tilde{\rho}_{D,t} \equiv \rho_{D,t}(\tilde{z}_D)$ and $\tilde{\rho}^*_{X,t} \equiv \rho^*_{X,t}(\tilde{z}^*_{X,t})$ represent the average relative prices of home producers and foreign exporters in the home market. A similar equation holds abroad.

The productivity averages $\tilde{z}_D$, $\tilde{z}_{X,t}$, and $\tilde{z}^*_{X,t}$ are constructed in such a way that $\tilde{d}_{D,t} \equiv d_{D,t}(\tilde{z}_D)$ ($\tilde{d}^*_{D,t} \equiv d^*_{D,t}(\tilde{z}_D)$) represents the average firm profit earned from domestic sales for all home (foreign) producers; and $\tilde{d}_{X,t} \equiv d_{X,t}(\tilde{z}_{X,t})$ ($\tilde{d}^*_{X,t} \equiv d^*_{X,t}(\tilde{z}^*_{X,t})$) represents the average firm export profits for all home (foreign) exporters.[14] Thus, $\tilde{d}_t = \tilde{d}_{D,t} + [1 - G(z_{X,t})]\, \tilde{d}_{X,t}$ and $\tilde{d}^*_t = \tilde{d}^*_{D,t} + \left[1 - G(z^*_{X,t})\right] \tilde{d}^*_{X,t}$ represent the average *total* profits of home and foreign firms, since $1 - G(z_{X,t})$ and $1 - G(z^*_{X,t})$ represent the proportion of home and foreign firms that export and earn export profits.[15]

Firm Entry and Exit

In every period, there is an unbounded mass of prospective entrants in both countries. These entrants are forward looking, and correctly anticipate their future expected profits $\tilde{d}_t$ ($\tilde{d}^*_t$) in every period (the pre-entry expected profit is equal to post-entry average profit) as well as the probability δ (in every period) of being hit with the exit-inducing shock. We assume that entrants at time t only start producing at time $t+1$, which introduces a one-period time-to-build lag in the model.[16] The exogenous exit shock occurs at the very end of the time period (after production and entry). Some entrants may therefore never produce as they can incur the exit shock immediately upon entry. Prospective home entrants in period t therefore compute their expected post-entry value given by the present discounted value of their expected stream of profits $\{\tilde{d}_s\}_{s=t+1}^{\infty}$:

$$\tilde{v}_t = E_t \sum_{s=t+1}^{\infty} \left[\beta \left(1 - \delta\right)\right]^{s-t} \left(\frac{C_{t+s}}{C_t}\right)^{-\gamma} \tilde{d}_s. \tag{2}$$

This will also represent the average value of incumbent firms *after* production has occurred (since both the new entrants and the incumbents then face the same probability $1-\delta$ of survival and production in the subsequent period). The firm discounts future profit with the household's stochastic discount factor, adjusted for the probability of firm survival $1-\delta$. Entry will incur until the average firm value is equalized with the entry cost, leading to the free entry condition $\tilde{v}_t = w_t f_{E,t}/Z_t$. This condition will hold so long as there is a positive mass $N_{E,t}$ of entrants. We assume that the macroeconomic shocks are small enough that this holds in every period. Finally, the timing of entry and production we have assumed implies that the number of home producing firms during period t ($N_{D,t}$) is such that $N_{D,t} = (1-\delta)\left(N_{D,t-1} + N_{E,t-1}\right)$. Similar free entry condition, requirement on the size of shocks, and law of motion for the number of producing firms hold in the foreign country.

[14] This implies $d_{D,t}(\tilde{z}_D) = \int_{z_{\min}}^{\infty} d_{D,t}(z) dG(z)$ and $d_{X,t}(\tilde{z}_{X,t}) = \{1/\left[1 - G(z_{X,t})\right]\} \int_{z_{X,t}}^{\infty} d_{X,t}(z) dG(z)$. Similar results hold for the average profits of foreign firms. See Melitz (2003) for proofs.

[15] Again, $\tilde{d}_t$ and $\tilde{d}^*_t$ represent the average firm profit levels in the sense that $\tilde{d}_t = \int_{z_{\min}}^{\infty} d_t(z) dG(z)$ and $\tilde{d}^*_t = \int_{z_{\min}}^{\infty} d^*_t(z) dG(z)$.

[16] Backus, Kehoe, and Kydland's (1992) international RBC model features time-to-build lags as in Kydland and Prescott (1982).

Parametrization of Productivity Draws

In order to solve our model, we parametrize the distribution of firm productivity draws $G(z)$. We assume that productivity z is distributed Pareto with lower bound $z_{\min}$ and shape parameter $k > \theta - 1$. The assumption of Pareto distribution induces a size distribution of firms that is also Pareto, which fits firm-level data quite well. k indexes the dispersion of productivity draws: Dispersion decreases as k increases and the firm productivity levels are increasingly concentrated toward their lower bound $z_{\min}$.[17] These assumptions imply that $G(z) = 1 - (z_{\min}/z)^k$. Letting $\varphi \equiv \{k/[k-(\theta-1)]\}^{1/(\theta-1)}$, the average productivities $\tilde{z}_D$ and $\tilde{z}_{X,t}$ are given by $\tilde{z}_D = \varphi z_{\min}$ and $\tilde{z}_{X,t} = \varphi z_{X,t}$. The share of home exporting firms is then $N_{X,t}/N_{D,t} = 1 - G(z_{X,t}) = \varphi^k (z_{\min}/\tilde{z}_{X,t})^k$, and the zero export profit condition for the cutoff firm with productivity $z_{X,t}$ implies that average export profits must satisfy $\tilde{d}_{X,t} = (\theta-1)\left(\varphi^{\theta-1}/k\right) w_t f_{X,t}/Z_t$. Analogous results hold for $\tilde{z}^*_{X,t}$, $N^*_{X,t}/N^*_{D,t}$, and $\tilde{d}^*_{X,t}$.

Household Budget Constraint and Intertemporal Choices

Households in each country hold two types of assets: shares in a mutual fund of domestic firms and domestic, risk-free bonds. (We assume that bonds pay risk-free, consumption-based real returns.)

Focus on the home economy. Let x_t be the share in the mutual fund of home firms held by the representative home household *entering* period t. The mutual fund pays a total profit in each period (in units of home currency) that is equal to the average total profit of all home firms that produce in that period, $\tilde{D}_t N_{D,t}$, where $\tilde{D}_t \equiv P_t \tilde{d}_t$. During period t, the representative home household buys x_{t+1} shares in a mutual fund of $N_{H,t} \equiv N_{D,t} + N_{E,t}$ home firms (those already operating at time t and the new entrants). Only $N_{D,t+1} = (1-\delta) N_{H,t}$ firms will produce and pay dividends at time $t+1$. Since the household does not know which firms will be hit by the exogenous exit shock δ at the *very end* of period t, it finances continuing operation of all pre-existing home firms and of all new entrants during period t. The date t price (in units of home currency) of a claim to the future profit stream of the mutual fund of $N_{H,t}$ firms is equal to the average nominal price of claims to future profits of home firms, $\tilde{V}_t \equiv P_t \tilde{v}_t$.

The household enters period t with bond holdings B_t in units of consumption and mutual fund share holdings x_t. It receives gross interest income on bond holdings, dividend income on mutual fund share holdings and the value of selling its initial share position, and labor income.

[17] The standard deviation of log productivity is equal to $1/k$. The condition that $k > \theta - 1$ ensures that the variance of firm size is finite.

The household allocates these resources between purchases of bonds and shares to be carried into next period and consumption. The period budget constraint (in units of consumption) is:

$$B_{t+1}+\tilde{v}_t N_{H,t}x_{t+1}+C_t=(1+r_t)B_t+\left(\tilde{d}_t+\tilde{v}_t\right)N_{D,t}x_t+w_tL, \quad (3)$$

where r_t is the consumption-based interest rate on holdings of bonds between $t-1$ and t (known with certainty as of $t-1$). The home household maximizes its expected intertemporal utility subject to (3).

The Euler equations for bond and share holdings are:

$$(C_t)^{-\gamma}=\beta\left(1+r_{t+1}\right)E_t\left[(C_{t+1})^{-\gamma}\right], \quad \text{and} \quad \tilde{v}_t=\beta\left(1-\delta\right)E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\left(\tilde{v}_{t+1}+\tilde{d}_{t+1}\right)\right],$$

respectively. As it should be expected, forward iteration of the equation for share holdings and absence of speculative bubbles yield the asset price solution in equation (2).[18]

Aggregate Accounting and Balanced Trade

Aggregating the budget constraint (3) across (symmetric) home households and imposing the equilibrium conditions under financial autarky ($B_{t+1}=B_t=0$ and $x_{t+1}=x_t=1$) yields the aggregate accounting equation $C_t=w_tL+N_{D,t}\tilde{d}_t-N_{E,t}\tilde{v}_t$. A similar equation holds abroad. Consumption in each period must equal labor income plus investment income net of the cost of investing in new firms. Net investment income is the sum of profits from domestic sales and export profits, minus entry costs. Since $N_{E,t}\tilde{v}_t$ is the value of home investment in new firms, aggregate accounting also states the familiar equality of spending (consumption plus investment) and income (labor plus dividend) that must hold in a financially closed economy.

To close the model, observe that financial autarky implies balanced trade: The value of home exports must equal the value of foreign exports: $Q_tN_{X,t}\left(\tilde{\rho}_{X,t}\right)^{1-\theta}C_t^*=N_{X,t}^*\left(\tilde{\rho}_{X,t}^*\right)^{1-\theta}C_t$.

Summary

Table 1 summarizes the main equilibrium conditions of the model. The equations in the table constitute a system of 19 equations in 19 endogenous variables: w_t, w_t^*, $\tilde{d}_t$, $\tilde{d}_t^*$, $N_{E,t}$, $N_{E,t}^*$, $\tilde{z}_{X,t}$,

[18] We omit the transversality conditions for bonds and shares that must be satisfied to ensure optimality. The foreign household maximizes its utility function subject to a similar budget constraint, resulting in analogous Euler equations and transversality conditions.

$\tilde{z}^*_{X,t}$, $N_{D,t}$, $N^*_{D,t}$, $N_{X,t}$, $N^*_{X,t}$, r_t, r^*_t, $\tilde{v}_t$, $\tilde{v}^*_t$, C_t, C^*_t, Q_t.[19] Of these endogenous variables, 4 are predetermined as of time t: the total numbers of firms at home and abroad, $N_{D,t}$ and $N^*_{D,t}$, and the risk-free interest rates, r_t and r^*_t. Finally, the model features 8 exogenous variables: the aggregate productivities Z_t and Z^*_t, and the policy variables $f_{E,t}$, $f^*_{E,t}$, $f_{X,t}$, $f^*_{X,t}$, τ_t, τ^*_t. We interpret changes in $f_{E,t}$ and $f^*_{E,t}$ as changes in market regulation facing a country's firms in the respective domestic markets and changes in $f_{X,t}$, $f^*_{X,t}$, τ_t, and τ^*_t as changes in trade policy. Since $f_{X,t}$ and τ_t are trade costs facing home firms, they are best interpreted as the foreign government's trade policy instruments.

3 Entry, Endogenous Non-Tradedness, and the Real Exchange Rate

Manipulating the price index equations and using the definition of the real exchange rate Q_t yields:

$$Q_t^{1-\theta} = \frac{N^*_{D,t}\left(TOL_t\right)^{1-\theta} + N_{X,t}\left(\tau_t \frac{\tilde{z}_D}{\tilde{z}_{X,t}}\right)^{1-\theta}}{N_{D,t} + N^*_{X,t}\left(TOL_t \tau^*_t \frac{\tilde{z}_D}{\tilde{z}^*_{X,t}}\right)^{1-\theta}}, \tag{4}$$

where we defined the terms of labor, TOL_t, as $TOL_t \equiv \varepsilon_t \left(W^*_t/Z^*_t\right)/\left(W_t/Z_t\right)$. TOL_t measures the relative cost of effective labor in the two countries. Higher TOL_t indicates a depreciation of home effective labor relative to foreign. Equation (4) holds regardless of whether there is financial autarky or not. Suppose $\tau_t = \tau^*_t = 1$ and $f_{X,t} = f^*_{X,t} = 0$. Because there is no fixed export cost, all firms export and there are no non-traded goods: $\tilde{z}_{X,t} = \tilde{z}^*_{X,t} = \tilde{z}_D$, $N_{X,t} = N_{D,t}$, $N^*_{X,t} = N^*_{D,t}$. Then, equation (4) immediately implies that $Q_t = 1$ in all periods. Absent trade costs, our model is isomorphic to Obstfeld and Rogoff's (1995) as far as the real exchange rate is concerned: PPP holds since all goods are traded, the law of one price holds, and preferences for consumption are identical across countries.

The definition of the real exchange rate $Q_t \equiv \varepsilon_t P^*_t/P_t$ uses the consumption-based price indexes in the two economies, P_t and P^*_t. These price indexes change over time for the variety effect implied by entry of new firms and availability of new goods in the economy. Arguably, this is not how the Bureau of Labor Statistics calculates the CPI for the U.S. economy: CPI data are based on average prices rather than constantly adjusted for availability of new varieties. Denote the consumer price level based on average prices with $\tilde{P}_t$. The average price level $\tilde{P}_t$ is such that $(P_t)^{1-\theta} =$

[19] To reduce the dimensionality of the system in Table 1, we apply the previously introduced definitions of average real prices and average profits/dividends from domestic and export sales as functions of variables in this set and exogenous variables. (See bottom of table for a summary.)

$\left(N_{D,t} + N^*_{X,t}\right)\left(\tilde{P}_t\right)^{1-\theta}$. A similar relation holds abroad: $\left(P^*_t\right)^{1-\theta} = \left(N^*_{D,t} + N_{X,t}\right)\left(\tilde{P}^*_t\right)^{1-\theta}$. Let us now define a measure of the real exchange rate based on average consumer price levels: $\tilde{Q}_t \equiv \varepsilon_t \tilde{P}^*_t / \tilde{P}_t$. It is immediate to verify that

$$\tilde{Q}_t = \left(\frac{N^*_{D,t} + N_{X,t}}{N_{D,t} + N^*_{X,t}}\right)^{\frac{1}{\theta-1}} Q_t. \tag{5}$$

PPP holds for both measures of the real exchange rate in the absence of trade costs. The "average" real exchange rate $\tilde{Q}_t$ provides a measure of the real exchange rate that is empirically more appealing than Q_t, as it is more in line with what we would obtain from using consumer price data constructed by the Bureau of Labor Statistics.

Denote steady-state levels of variables by dropping the time subscript. We assume $f_E = f^*_E$, $f_X = f^*_X$, $\tau = \tau^*$, $L = L^*$, and $Z = Z^* = 1$. The model has a unique, symmetric steady state with $\tilde{Q} = Q = TOL = 1$ under these assumptions. We log-linearize the system around the steady state under assumptions of log-normality and homoskedasticity of exogenous stochastic shocks, and we solve for the dynamics in response to exogenous shocks numerically with the method of undetermined coefficients. We denote percentage deviations from the steady state with sans serif fonts.

It is possible to show that the following equation holds for the real exchange rate $\tilde{Q}_t$ regardless of whether agents can trade bonds across countries or not:

$$\begin{aligned} \tilde{\mathsf{Q}}_t &= (2s_D - 1)\,\mathsf{TOL}_t - (1 - s_D)\left[\left(\tilde{\mathsf{z}}_{X,t} - \tilde{\mathsf{z}}^*_{X,t}\right) - \left(\mathsf{t}_t - \mathsf{t}^*_t\right)\right] \\ &\quad - \frac{1}{\theta - 1}\left(s_D - \frac{N_D}{N_D + N_X}\right)\left[\left(\mathsf{N}^*_{D,t} - \mathsf{N}_{X,t}\right) - \left(\mathsf{N}_{D,t} - \mathsf{N}^*_{X,t}\right)\right], \end{aligned} \tag{6}$$

where s_D is the steady-state share of spending on domestic goods ($s_{D,t} \equiv N_{D,t}\left(\tilde{\rho}_{D,t}\right)^{1-\theta}$) and t_t (t^*_t) denotes the percentage deviation of τ_t (τ^*_t) from the steady state. When there are trade costs, $s_D > 1/2$ – it is $1/2$ in the absence of trade costs. Hence, *ceteris paribus*, when $\mathsf{TOL}_t < 0$ (when home labor in effective units becomes more expensive relative to foreign), the real exchange rate $\tilde{Q}_t$ appreciates. The intuition is familiar. When home effective labor becomes relatively more expensive, this increases the price of non-traded goods at home relative to foreign and leads to appreciation. This is a channel for real exchange rate appreciation that depends on existence of a non-traded goods sector, but not on the fact that the latter is determined endogenously and can change over time. Abstracting from the fact that a non-traded sector exists in our model if and only if there are strictly positive fixed export costs (which ensure the endogenous determination of what is traded and what is not), this is the traditional channel for real appreciation in the textbook analysis of the HBS effect. However, the operation of this traditional channel is triggered by a novel mechanism in our model, which differs from the familiar assumption of a sector-specific productivity increase in the traded sector and is centered on the dynamics of firm entry in response to country-wide shocks.

Suppose there is a permanent increase in aggregate productivity, Z_t, in the home economy. Home becomes a relatively more attractive business environment for potential entrants. In the presence of trade costs, these will want to locate in the home economy – the relatively "bigger" market – in the same fashion as in Krugman's (1980) seminal paper. Absent a change in the relative cost of effective labor at home and abroad, eventually, no firm would be left in the foreign economy to keep foreign households employed (since a fraction δ of foreign firms incur the "death" shock in each period).[20] It follows that home effective labor must become more expensive relative to foreign in the long run for some firms to enter the foreign economy and keep foreign labor employed. It is entry in the home economy that eventually bids the home real wage up by enough that $\mathsf{TOL}_t < 0$ and the real exchange rate appreciates.

Interestingly, under financial autarky, the impact response to the same shock must be such that home effective labor depreciates ($\mathsf{TOL}_t > 0$) and $\tilde{Q}_t$ depreciates. The intuition is simple. In response to the shock, home households want to spread their increase in demand evenly over all available varieties. Home firms are more productive, foreign firms are not. Therefore, an even increase in demand across home and foreign varieties results in excess demand for foreign labor, and the terms of labor must rise to ensure equilibrium. This is where our model differs crucially from standard international macro models. Depreciation would be the effect of the shock on relative labor costs in the short *and* in the long run in a model without entry. In our model, sizably larger entry in the more attractive home economy reverses the movement of the terms of labor over time by generating increased demand for home labor, so that TOL_t falls below the steady state and converges to its new, appreciated long-run level. As we show in the numerical exercise below, the inversion in the movement of the terms of labor generated by entry yields appreciation of $\tilde{Q}_t$ not only in the long-run, but also during most of the transition for plausible parameter values.[21] Most importantly, this mechanism provides the key link between short-run and long-run dynamics as

[20] We assume that labor is immobile across countries.

[21] When we allow for international trade in bonds, faster entry into the more productive economy financed by borrowing causes TOL_t to appreciate also in the short run.

well as between macro- and micro-driven results in our setup.[22]

How does endogenous non-tradedness enter the picture? The direct contribution of endogenous non-tradedness to real exchange rate dynamics plays a key role through the term that depends on the export cutoff differential $\tilde{\mathsf{z}}_{X,t} - \tilde{\mathsf{z}}^*_{X,t}$ in equation (6) and the term that reflects variation in the relative numbers of home exporters into the foreign economy and foreign exporters into home – the latter capturing an expenditure-switching effect. Focus on the export cutoffs first. Changes in relative labor costs further induce changes in the composition of the traded sector in both countries. Import prices in the economy with higher labor costs (home, in the example above, after entry has driven TOL_t below the steady state) rise as lower productivity foreign firms can now export to it. Conversely, import prices in the economy with lower labor costs decrease as lower productivity home exporters are forced to drop out of the export market. (Recall that firms must hire domestic labor to cover fixed export costs.) These effects necessarily reinforce the real exchange rate appreciation caused by an increase in the effective cost of home labor.[23] As for the last term in equation (6), $s_D - N_D/(N_D + N_X)$ is the difference between the market (expenditure) share of home firms competing in the home market and the number share of these firms. It reflects the average productivity difference between home firms ($\tilde{z}_D$) and foreign exporters ($\tau\tilde{z}_X$, adjusted for the iceberg export cost). In steady state, this difference is negative for plausible parameter values, as the average productivity advantage of exporters is larger than the transport cost. Since exporters have lower prices relative to domestic firms, an increase in the relative number of home exporters into the foreign economy ($\mathsf{N}^*_{D,t} - \mathsf{N}_{X,t} < 0$) leads, *ceteris paribus*, to a decrease in the average price of consumption abroad and appreciation of $\tilde{Q}_t$. Similarly, a decrease in the relative number of foreign exporters into home ($\mathsf{N}_{D,t} - \mathsf{N}^*_{X,t} > 0$) causes appreciation. Higher aggregate productivity at home results in more entry of firms into home than it increases the number of foreign exporters, so that the expenditure share on domestic goods increases at home ($\mathsf{N}_{D,t} - \mathsf{N}^*_{X,t} > 0$) in conjunction with higher relative labor costs. Conversely, the expenditure share on domestic goods in the other economy decreases ($\mathsf{N}^*_{D,t} - \mathsf{N}_{X,t} < 0$) as less firms enter foreign, but there are

[22] Deregulation of the home market, which we model as a decrease in the size of the sunk entry cost $f_{E,t}$ along the lines of Blanchard and Giavazzi (2003), has similar consequences for TOL_t and $\tilde{Q}_t$. The more attractive economy has more expensive labor and higher average prices. The main difference is that there is no short-run depreciation of the terms of labor since there is no productivity difference across home and foreign producers when home households allocate their initial increase in demand.

[23] In the long run, the number of home exporters is higher than in the initial steady state following a permanent increase in aggregate home productivity, because *all* home firms are more productive than before. However, the long-run increase is smaller than the short-run increase in the number of home exporters due to the dynamics of the terms of labor: Over time, the relatively less productive home exporters drop out of the foreign market as home labor becomes relatively more expensive.

more home exporters. But a higher (lower) share of spending on domestic goods (which include products of domestic exporters *and* non-exporters) implies a shift in the composition of spending toward (away from) domestic, non-traded goods. As non-traded goods have higher prices (relative to traded goods), the shift in expenditures further increases the real exchange rate appreciation.

The operation of the traditional channel triggered by the novel inclusion of entry and the additional channel generated by endogenous non-tradedness combine to deliver a fully microfounded, endogenous HBS effect in response to aggregate productivity increases (or deregulation).

Given the dynamics of $\tilde{Q}_t$, we can use equation (5) to solve for Q_t. It is:

$$\begin{aligned}\mathsf{Q}_t &= (2s_D - 1)\,\mathsf{TOL}_t - (1 - s_D)\left[\left(\tilde{\mathsf{z}}_{X,t} - \tilde{\mathsf{z}}^*_{X,t}\right) - (\tau_t - \tau^*_t)\right] \\ &\quad -\frac{s_D}{\theta - 1}\left(\mathsf{N}^*_{D,t} - \mathsf{N}_{D,t}\right) + \frac{1 - s_D}{\theta - 1}\left(\mathsf{N}^*_{X,t} - \mathsf{N}_{X,t}\right). \qquad (7)\end{aligned}$$

Just like the price indexes P_t and P^*_t can change either because average prices change or because product variety does, Q_t is affected both by changes in average relative prices – $\tilde{Q}_t$ – and changes in the relative number of varieties consumed – $\left(N^*_{D,t} + N_{X,t}\right) / \left(N_{D,t} + N^*_{X,t}\right)$. The terms of labor, export cutoffs, and iceberg trade costs affect Q_t directly through their effects on average prices and $\tilde{Q}_t$. Hence, the corresponding terms in the log-linear equation for Q_t are unchanged. The effects of changes in the number of firms in the domestic and foreign market are dominated by the effect of product variety. *Ceteris paribus*, increases in the number of varieties available for consumption push the price indexes down.[24] An increase in the number of foreign exporters relative to home ($\mathsf{N}^*_{X,t} - \mathsf{N}_{X,t} > 0$) leads to depreciation of Q_t because, *ceteris paribus*, it pushes the home price index down relative to the foreign one. An increase in the total number of home firms relative to foreign ($\mathsf{N}^*_{D,t} - \mathsf{N}_{D,t} < 0$) generates depreciation for the same reason. In particular, the sign of the effect of $\mathsf{N}^*_{D,t} - \mathsf{N}_{D,t}$ on Q_t changes relative to the effect on $\tilde{Q}_t$, exactly because product variety becomes dominant over average price effects in the determination of Q_t. Thus, for plausible parameter values, a shock that causes home households to have access to more varieties than foreign households generates depreciation of Q_t even when $\tilde{Q}_t$ moves in the opposite direction.

[24] This is true both in nominal and real terms. Take for example the home price index. It is $(P_t)^{1-\theta} = \left(N_{D,t} + N^*_{X,t}\right)\left(\tilde{P}_t\right)^{1-\theta}$. Multipling both sides by $(W_t)^{1-\theta}$ shows that the consumption-based price index in units of labor (P_t/W_t) falls – the real wage rises – if product variety increases even if there is no change in the average price of consumption in units of labor $(\tilde{P}_t/W_t)$.

Consumption Smoothing and the Real Exchange Rate under Financial Autarky

The results on the real exchange rate that we discussed above hold regardless of whether there is international trade in financial assets or not. We can develop additional intuition on the determination of the consumption-based real exchange rate under financial autarky by observing that the balanced trade condition implies: $(Q_t)^{\theta} = \left(N^*_{X,t}/N_{X,t}\right) \tilde{T}_t^{\theta-1} (C_t/C^*_t)$, where we defined the average terms of trade $\tilde{T}_t \equiv \varepsilon_t \tilde{p}_{X,t}/\tilde{p}^*_{X,t}$, so that an increase in $\tilde{T}_t$ is an improvement in home's average terms of trade. *Ceteris paribus*, increases in C_t/C^*_t, $N^*_{X,t}/N_{X,t}$, and $\tilde{T}_t$ must be accompanied by depreciation of the consumption-based real exchange rate to preserve balanced trade.

Assume temporarily that $f_{X,t} = f^*_{X,t} = 0$ and $\tau_t = \tau^*_t = 1$. Suppose further that we reduce our model to a version of Obstfeld and Rogoff's (1995) setup by assuming constant numbers of firms at home and abroad, so that the ratio $N^*_{X,t}/N_{X,t} = N^*_{D,t}/N_{D,t}$ is constant. In this case, C_t/C^*_t is proportional to $\tilde{T}_t^{1-\theta}$. Adjusting for the fact that the definition of the terms of trade in Obstfeld and Rogoff's model is the reciprocal of ours, this is the relation between the consumption differential and the terms of trade in that model if one removes international borrowing or lending. In other words, holding the ratio $N^*_{X,t}/N_{X,t}$ constant, the standard model with PPP implies that the terms of trade adjust proportionally to offset movements in the consumption differential to preserve balanced trade.

The consumption differential is not tied to the average terms of trade in our setup. The consumption-based real exchange rate depreciates in response to a shock that causes home consumption to rise above foreign, and the terms of trade deterioration that would make imports more expensive for home consumers is correspondingly dampened relative to Obstfeld and Rogoff's scenario.

4 International Trade and Macroeconomic Dynamics

This section substantiates the results and intuitions of the previous section by means of a numerical exercise.

Calibration

We calibrate parameters as follows. We interpret periods as quarters and set $\beta = .99$ and $\gamma = 2$ – both standard choices for quarterly business cycle models. We normalize the endowment of labor of each household to 1 and set the size of the exogenous firm exit shock $\delta = .025$ to mimic the evidence of destruction of one in ten jobs in the course of a year. We borrow the value of θ from Bernard, Eaton, Jensen, and Kortum (BEJK, 2003) and set $\theta = 3.8$, which was calibrated to fit U.S. plant and macro trade data. BEJK also reports that the standard deviation of log U.S. plant sales is 1.67. In our theoretical model, this standard deviation is equal to $1/(k - \theta + 1)$. Given $\theta = 3.8$, we deduce $k = 3.4$, which satisfies the requirement $k > \theta - 1$. We postulate $\tau = 1.3$, roughly in line with Obstfeld and Rogoff (2001), and set the steady-state fixed export cost f_X such that the proportion of exporting plants matches the number reported in BEJK (21 percent). This leads to a fixed export cost f_X equal to 23.5 percent of the per-period, amortized flow value of the entry cost, $[1 - \beta(1 - \delta)] / [\beta(1 - \delta)] f_E$.[25] Changing the entry cost f_E while maintaining the same ratio f_X/f_E does not affect any of the impulse responses. The total number of firms in steady state is inversely proportional to f_E – and the size and value of all firms are similarly proportional to f_E. Basically, changing f_E for given ratio f_X/f_E amounts to changing the unit of measure for output and number of firms. But it does not matter whether output is measured in units, tens of units, etc., and the same is true for the number of firms. Hence, we set f_E to 1 for simplicity. For the same reason, we normalize $z_{\min}$ to 1 without loss of generality. Our calibration implies that exporters are on average 58.2 percent more productive than non-exporters. The steady-state share of expenditure on domestic goods is .733, and the share of expenditure on non-traded domestic goods is .176. The relative size differential of exporters relative to non-exporters in the domestic market is 3.61.

It may be argued that the value of θ results in a steady-state markup that is too high relative to the evidence. A standard choice in the macro literature is $\theta = 6$ to deliver a 20 percent markup of price over marginal cost (Rotemberg and Woodford, 1992). However, it is important to observe that, in models without any fixed cost, $\theta/(\theta - 1)$ is a measure of both markup over marginal cost and average cost. In our model with entry costs, free entry ensures that, *on average*, firms earn zero profits *net* of the entry cost. This means that, on average, firms price at average cost (inclusive of the entry cost). More productive firms price above their average cost (since the latter is lower as they spread the entry cost over larger amounts of output produced), and less productive firms price below their average cost. The firm with productivity $z_{\min}$ must always price below its average cost – which means that the net present value of its profits never end up covering the entry cost. Thus, although $\theta = 3.8$ implies a fairly high markup over marginal cost, our model delivers reasonable

[25] We tried using different values of τ (1.1, 1.2, 1.25) and recalculated f_X relative to f_E to match the 21 percent of exporting plants. The impulse responses were very similar in all cases.

markups over average costs.

Productivity Shocks

Following Backus, Kehoe, and Kydland (1992) and Baxter (1995), we assume the following bivariate process for the percentage deviations of home and foreign productivities Z_t and Z_t^* from the steady state:

$$\begin{bmatrix} \mathsf{Z}_t \\ \mathsf{Z}_t^* \end{bmatrix} = \begin{bmatrix} \phi_Z & \phi_{ZZ^*} \\ \phi_{Z^*Z} & \phi_{Z^*} \end{bmatrix} \begin{bmatrix} \mathsf{Z}_{t-1} \\ \mathsf{Z}_{t-1}^* \end{bmatrix} + \begin{bmatrix} \xi_t^Z \\ \xi_t^{Z^*} \end{bmatrix}, \tag{8}$$

where the persistence parameters ϕ_Z and ϕ_{Z^*} are in the interval $[0,1]$, the spillover parameters ϕ_{ZZ^*} and ϕ_{Z^*Z} are non-negative, and ξ_t^Z and $\xi_t^{Z^*}$ are zero-mean innovations. In Section 5, we calibrate the matrix of persistence and spillover parameters and the variance-covariance matrix of the innovations to match empirical evidence. Here, we assume $\phi_{ZZ^*} = \phi_{Z^*Z} = 0$ and focus on the consequence of a 1 percent increase in Z_t above the steady state with persistence $\phi_Z = 1$ or $\phi_Z = .9$.

A Permanent Increase in Home Productivity

Figure 1 presents the responses to a permanent 1 percent increase in home productivity. Consider first the long-run effects, in the new steady state. (A period corresponds to a quarter in our exercise, but the numbers 10 and 20 below the horizontal axis in each panel of the figures denote 10 and 20 years after the shock, respectively.)

Home is now endowed with more units of effective labor. At equal relative cost per unit of effective labor (holding the terms of labor – TOL – constant), whenever there are positive transport costs, entering firms will want to locate in the bigger market. Here, home is the bigger market (even when TOL does not change), as it has more units of effective labor. Thus, labor costs in foreign must relatively decrease (the terms of labor must fall) in order to keep firms in its market, and its labor force employed. This effect is described by Krugman (1980, pp. 954-955) in his seminal, first new trade theory paper. (If all prospective entrants decided to locate into home, firms would stop entering the foreign economy altogether, and pre-existing foreign firms would be eventually wiped out by the "death" shock δ.) Note that the free entry condition for firms in home and foreign plays a key role in determining this comparative static.

If the new effective labor units at home were all used by existing firms, then the increase in production would induce positive profits for all firms in the long run. Instead, the free entry condition implies entry of new home firms, and N_D increases.

Home consumption (C) increases owing to higher labor and dividend income (w and $\tilde{d}$, respectively). The increase in C (and in w and $\tilde{d}$) reflects also the availability of more varieties. For instance, since the number of available varieties increases, C would increase even if consumption of each individual variety remained constant. We can separate the variety effect from the other mechanisms at work in our model by decomposing changes in demand facing an individual producer among three channels: aggregate changes in demand (or changes in total nominal expenditure deflated by the average price of all varieties), changes in a firm's price relative to the average price, and changes in the number of competing firms. Consider variety z. Recall the relation between the consumption-based price index P and the average price level $\tilde{P}$: $(P_t)^{1-\theta} = \left(N_{D,t} + N^*_{X,t}\right)\left(\tilde{P}_t\right)^{1-\theta}$. Home demand for variety z can then be written as $\left(p_t(z)/\tilde{P}_t\right)^{-\theta}\left(N_{D,t} + N^*_{X,t}\right)^{-1}\left(P_t C_t/\tilde{P}_t\right)$. The term $P_t C_t/\tilde{P}_t$ captures the first channel, the term $\left(p_t(z)/\tilde{P}_t\right)^{-\theta}$ captures the second channel, and the term $\left(N_{D,t} + N^*_{X,t}\right)^{-1}$ captures the third channel. The latter disappears when the number of firms is fixed. In our model, holding overall demand and relative prices constant, increases in the number of firms competing in the home market decrease the demand level for each firm. (Alternatively, one can look at this effect through the consumption-based price index by recalling that, holding the average price level constant, an increase in the number of firms reduces the overall price index.)

In Figure 1, import demand at home for all foreign firms (the output of foreign exporters that reaches its destination, denoted $y^*_X(z)$) increases. This is induced by the overall increase in home demand ($PC/\tilde{P}$) and the decrease in the prices of foreign exports relative to domestic goods ($p^*_X(z)/\tilde{p}_D = \rho^*_X(z)/\tilde{\rho}_D$ falls) caused by lower TOL.[26] These two forces dominate the opposite effect of the increase in the number of firms selling at home. Increased demand for imports induces entry of new foreign exporters into the home market. Consequently, the foreign export productivity cutoff (z^*_X) falls, as less productive foreign firms are now exporting.

Export demand for home firms (the output of home exporters at destination, denoted $y_X(z)$) decreases, induced mainly by the increase in relative export prices: $p_X(z)/\tilde{p}^*_D = \rho_X(z)/\tilde{\rho}^*_D$ rises since TOL falls. A higher number of home exporters (N_X) than in the initial steady state contributes to this effect. Changes in foreign demand and in the overall number of foreign firms (N^*_D)

[26] It is $P_t C_t/\tilde{P}_t = \left(N_{D,t} + N^*_{X,t}\right)^{-1/(\theta-1)} C_t$. Log-linearizing this expression and using $\theta = 3.8$ and $N_X/N_D = .21$ in the initial steady state, it is easy to verify that $PC/\tilde{P}$ increases in Figure 1. Note also that the response of $\rho_D(z)$ to a shock is identical to the response of the average $\tilde{\rho}_D \equiv \rho_D(\tilde{z}_D)$ because $\tilde{z}_D$ is constant.

are small compared to this relative price change. (Foreign income and consumption increase by little.) Hence, the relative price effect dominates, and $y_X(z)$ is lower than in the initial steady state. The number of home exporters is higher than in the initial steady state since all home firms are permanently more productive. However, *ceteris paribus*, the decrease in export demand and the increase in the effective cost of home labor (w/Z) induce exit of lower productivity home exporters: The cutoff productivity for home exporters (z_X) rises, forcing relatively less productive home firms to abandon the foreign market.[27]

Now consider changes along the transition.

Absent sunk entry costs, there would be an immediate adjustment to the new steady state number of firms. However, the sunk costs – along with the timing of entry and production – induce a slow response in the number of producing firms. The N_D variable behaves very much like a capital stock, and the N_E variable very much like investment in a more standard macro model.

Abstract temporarily from the increase in N_X^* on impact. The immediate increase in home overall demand for given number of firms supplying goods in the home market induces higher initial demand for existing goods sold in the domestic market. Absent a change in TOL, the demand increase is spread evenly over home produced and imported varieties. This would represent an excess demand for foreign effective labor relative to the new increased level of effective labor available at home. This results in an initial increase in TOL. The increase in N_X^* on impact does not reverse this immediate effect. From that point on, the number of home firms steadily increases, but by less than $PC/\tilde{P}$, which shifts home demand toward home produced goods and reverses the initial excess demand for foreign effective labor. The steady increase in the number of varieties available in the home economy over time contributes to the increasing paths of home consumption (C) and the real wage (w).

Import demand at home $(y_X^*(z))$ increases monotonically, as the overall increase in demand at home outweighs the initial reversal in the direction of the relative price of foreign exports ($p_X^*(z)/\tilde{p}_D$ initially rises). Thus, there is no change in the direction of the foreign export productivity cutoff z_X^* during the transition.

Export demand for home firms initially increases, as the relative price is the dominant effect. Initially, $y_X(z)$ increases as $p_X(z)/\tilde{p}_D^*$ falls in response to the immediate upward movement of TOL.

[27] Recall that the fixed export cost in units of consumption is wf_X/Z. Since f_X is unchanged, *ceteris paribus*, increases in w/Z increase the burden of the fixed cost on the profitability of exporting. Among other sources of real wage variation, upward pressure on w originates in the variety effect and in increased labor demand to cover the sunk entry cost for new home entrants.

Thus the home export cutoff productivity level z_X initially falls, as temporarily stronger demand allows relatively less productive firms to export.

The dynamics of TOL in Figure 1 illustrate the intuitive arguments in the previous section. The average real exchange rate $\tilde{Q}$ depreciates in the short run, owing to the initial increase in TOL, but it subsequently appreciates, as the micro-macro mechanism of our model reverses the movement of TOL. Given the existence of a non-traded sector, the relative increase in labor costs must induce an appreciation by increasing the price of non-traded goods at home relative to foreign.[28] The path of $\tilde{Q}$ resembles that of TOL but, as we observed above, the effect of the terms of labor is reinforced by the dynamics of the export cutoffs and the implied adjustment in the composition of the traded sector.

The consumption-based real exchange rate Q depreciates in the short and in the long run, as its dynamics are dominated by the variety effect. Its qualitative dynamics track those of the consumption differential C/C^*. The average terms of trade deteriorate, since, *ceteris paribus*, higher productivity makes home exports cheaper relative to foreign exports as in the standard model with PPP. Absent changes in Q and in the relative number of exporters in the two countries, terms of trade deterioration would be proportional to the consumption differential to preserve balanced trade.[29] Terms of trade deterioration is dampened by a relative reallocation of resources away from the export sector in the home economy (the total number of firms eventually increases by more than the number of exporters) and into the export sector abroad. This leaves room for consumption-based real depreciation to perform part of the adjustment necessary to preserve balanced trade.

A Transitory Increase in Home Productivity

Figure 2 presents the responses to a 1 percent increase in home productivity with persistence .9 – a value of the persistence parameter at the lower end of the range usually considered in the RBC literature. The shock has no permanent effect, as all endogenous variables we focus on are stationary in response to stationary exogenous shocks. Our interest here is also in the persistence properties of the model's endogenous variables relative to the persistence of a transitory shock.

Home consumption rises on impact, in response to higher income. As in the case of the short-run response to a permanent shock, absent a change in TOL, this would result in excess demand for effective foreign labor. Therefore, TOL rises temporarily, which pushes the relative price of

[28] To see this, recall that the price $\rho_D(z)$ ($\rho_D^*(z)$) is the relative price of home (foreign) non-traded goods for all varieties such that $z \in [z_{\min}, z_{X,t})$, and observe that $\rho_D(z)$ increases by more than $\rho_D^*(z)$ in Figure 1.

[29] We report the response of $\tilde{T}_t^{\theta-1}$ in the figures to facilitate comparison with the consumption differential.

home exports ($p_X(z)/\tilde{p}_D^*$) down and the relative price of foreign exports ($p_X^*(z)/\tilde{p}_D$) up. The effect of higher home demand ($PC/\tilde{P}$) prevails on the higher relative price of foreign exports and the increase in the number of firms selling at home, and $y_X^*(z)$ increases. As a consequence, the number of foreign exporters rises, and the cutoff productivity level z_X^* falls as less productive foreign firms now find it profitable to export.

Financial autarky implies that foreign consumers cannot share the favorable consequences of the transitory increase in home productivity through international asset trading motivated by the desire of home households to smooth the dynamics of C. Foreign consumption increases by little in response to increased income from exporting to home.

The initial decline in the relative price of home exports $p_X(z)/\tilde{p}_D^*$ temporarily boosts output of exporters, so that $y_X(z)$ increases initially. In the short run, the number of home exporters increases, both because all firms are temporarily more productive for given fixed export cost and because the expansion in demand makes it profitable to export for relatively less productive firms. *Ceteris paribus*, the increase in N_X contributes to higher C^* through availability of more varieties. But this effect is short lived. Households use investment in new firms to smooth the consequences of the shock on consumption. The number of entrants in the home economy increases and so does the total number of home firms. In turn, as in the case of a permanent shock (and in a shorter amount of time), entry and the increase in the total number of home firms push the relative cost of home effective labor up (TOL falls), which reverses the movement in the relative price of home exports and quickly brings $y_X(z)$ below the steady state. The number of home exporters decreases as productivity returns to the steady state and output for each exporter has fallen. The decrease in $y_X(z)$ is mirrored by an increase in the export productivity cutoff for home firms (z_X), which causes relatively less productive home firms to leave the export market.

The path of the average real exchange rate $\tilde{Q}$ is qualitatively similar to that of TOL. The favorable productivity shock results in real appreciation throughout most of the transition for the same reasons we discussed above. Similarly, the consumption-based real exchange rate Q depreciates tracking the consumption differential C/C^*, and the terms of trade deteriorate.

The dynamics of key endogenous variables in Figure 2 display substantial persistence, well beyond the exogenous .9 persistence of the productivity shock. Approximately 84 percent of the initial increase in productivity has been reabsorbed 10 years after the shock. At that point in time, the average real exchange rate $\tilde{Q}$ still needs to cover roughly half the distance between the peak appreciation (which happens approximately 4 years after the shock) and the steady state.[30]

A large literature has developed in the past few years trying to explain real exchange rate movements in terms of nominal rigidity and local currency pricing. (See Chari, Kehoe, and McGrattan, 2002, and references therein.) The success has been, at best, mixed. Plausible degrees of nominal rigidity and local currency pricing (supported by the assumption of market segmentation) succeed in generating volatile real exchange rates, but only special assumptions deliver persistence in line with the data. Benigno (2004) highlights inertia in endogenous interest rate setting by central banks and differences in nominal rigidity as sources of real exchange rate persistence. Burstein, Eichenbaum, and Rebelo (2002) and Corsetti and Dedola (2002) recently put forth models that allow for the presence of distribution sectors and are a promising extension of the sticky-price literature in which structural features of the economy beyond nominal rigidity matter for real exchange rate dynamics. We propose a different mechanism that delivers substantial real exchange rate persistence in response to transitory shocks: firm entry and reallocation in and out of markets in a world of flexible prices.[31]

Deregulation

We model deregulation in the domestic market as a permanent decrease in the sunk entry cost f_E relative to its initial steady-state level. This is consistent with Blanchard and Giavazzi (2003).[32] Figure 3 presents the responses to a permanent 1 percent decrease in the home sunk entry cost.

As for the case of a permanent change in productivity, consider long-run effects first.

Absent changes in TOL, all entering firms will want to locate in home to take advantage of the lower entry costs. In equilibrium, home labor costs must rise relative to foreign to keep foreign labor employed, *i.e.*, TOL must fall. The lower entry costs must also induce entry of new firms, so that N_D rises. As in Blanchard and Giavazzi (2003), lower entry costs result in a larger number of firms and a higher real wage (w).

[30] This result does not depend on the presence of a steady-state iceberg cost $\tau = \tau^* \neq 1$. A similarly persistent deviation from PPP happens if $\tau = \tau^* = 1$.

[31] Imbs, Mumtaz, Ravn, and Rey (2002) argue that the "consensus" half-life of PPP deviations estimated by Rogoff (1996) (three to five years) is the outcome of aggregation. They show that the half life of deviations from PPP for disaggregated prices is much shorter (little longer than a year). Our model reconciles these results by generating persistent deviations of aggregate price indexes from PPP (as in Rogoff) in a world in which disaggregated prices converge to the law of one price quickly (as in Imbs, Mumtaz, Ravn, and Rey). (Caballero and Engel, 2003, argue that using disaggregated data biases estimates of the speed of macroeconomic adjustment upward. Our model fits their argument with respect to PPP at the aggregate versus disaggregate level. However, see also Chen and Engel, 2004, on the issue of aggregation and PPP.)

[32] See also Alesina, Ardagna, Nicoletti, and Schiantarelli (2003).

Along the transition, the major difference with the case of a permanent productivity increase is that home no longer also enjoys an increase in the number of effective labor units relative to foreign. There is therefore no need for an initial increase in TOL to keep the new units of labor at home employed. In fact, there is an initial increase in demand for home labor relative to foreign, driven by the labor demand of new entrants in the home market. Another major difference is that home consumers no longer enjoy an immediate boost in demand/consumption linked to higher productivity. In fact, consumption decreases on impact as consumers increase their savings to finance the entry of new firms.

There are two competing effects on import demand at home ($y_X^*(z)$). The decrease in consumption demand has a negative effect, while the relative decrease in foreign labor costs (lower TOL) has a positive effect through its impact on relative prices ($p_X^*(z)/\tilde{p}_D$ falls). An initial decrease in N_X^* reinforces the effect of TOL, since it implies that the number of firms competing in the home market falls on impact. Over time, the entry of new home firms and an increasing number of foreign exporters have a negative effect on $y_X^*(z)$. Initially, the decrease in consumption demand dominates, and $y_X^*(z)$ falls. Over time, changes in both consumption and relative prices induce an increase in $y_X^*(z)$. Changes in the foreign export cutoff level z_X^* mirror the change in export demand: z_X^* increases initially as $y_X^*(z)$ falls, and z_X^* falls thereafter as $y_X^*(z)$ increases.

Changes in home export demand ($y_X(z)$) are dominated by the change in the relative price $p_X(z)/\tilde{p}_D^*$ – both the change in foreign consumption and the effect of changes in the number of firms selling in the foreign market are small relative to the changes in this variable. $p_X(z)/\tilde{p}_D^*$ rises as TOL falls. Hence, $y_X(z)$ falls over the entire transition. As usual, the change in the export cutoff z_X mirrors this change: z_X increases over the entire transition.

The Endogenous Harrod-Balassa-Samuelson Effect

We pause to comment on the real exchange rate, the HBS effect, and some important implications of our results before analyzing the consequences of changes in trade policy.

Consistent with the HBS evidence, our model predicts that more productive economies, or less regulated ones, will exhibit higher average prices relative to their trading partners. This real exchange rate appreciation (based on average prices) is driven by two key new features of our model:

1. *Entry:* Effective labor units must relatively appreciate in the economy providing the more attractive environment for firms. This is a key consequence of introducing firm entry (along

with entry costs for the deregulation scenario). Given the existence of a non-traded sector, the relative increase in labor costs must induce an appreciation.

2. *Endogenous Non-Tradedness:* Changes in relative labor costs further induce changes in the composition of the traded sector in both countries:

   - Import prices in the economy with higher labor costs (home) rise as lower productivity firms can now export to it. Conversely, import prices in the economy with lower labor costs decrease as home exporters with relatively lower productivity are forced to drop out of the export market. These effects necessarily reinforce the real exchange rate appreciation.
   - The number of foreign exporters relative to home producers in the home market decreases in response to a permanent home productivity advantage. This shifts the composition of home spending toward non-traded goods and pushes the average price of consumption at home upward. Instead, the number of home exporters relative to foreign firms in the foreign market increases, shifting the composition of foreign spending toward traded goods and pushing the average price of consumption abroad downward. Both changes strengthen the appreciation of $\tilde{Q}$. (In the case of deregulation at home, $\mathsf{N}_D - \mathsf{N}_X^* > \mathsf{N}_D^* - \mathsf{N}_X$ contributes to the real appreciation.)

Endogenous determination of the traded sector over time plays a key role in our microfounded, endogenous HBS effect. All goods are tradeable in our model; some are non-traded in equilibrium, and the margin moves in response to shocks. Thus, the "non-tradedness" margin *within* tradeable sectors is central in our theory. How relevant is this margin on empirical grounds? There is evidence that endogenous changes in the composition of the traded sector take place over the normal length of a business cycle. Bernard and Jensen (2001) study a panel of U.S. manufacturing plants between 1987 and 1997. They find that, on average, 13.9 percent of non-exporters begin to export in any given year during the sample, and 12.6 percent of exporters stop. Consistent with the natural interpretation of our model, such entry and exit do not occur *across* sectors but *within* sectors: We do not observe some sectors switching between traded and non-traded over time. We observe firms/plants moving in and out of the export sectors within tradeable sectors. A model based on a traded/non-traded classification across sectors will therefore not capture the relevant margin for this process. As mentioned above, only 21 percent of U.S. manufacturing plants export. There is thus a very substantial non-traded margin of firms within the tradeable manufacturing sector. Entry and exit into the export market is also important in explaining aggregate trade flows (and hence other macro variables): Roberts and Tybout (1997*a*) report that the large export increases associated with devaluations in Mexico and Colombia were in the most part driven by the entry of new plants into the export market. These new exporters account for over 50 percent of the export growth in both countries. For the U.S., Bernard and Jensen (2003) report that 40 percent of the export growth between 1987-1992 was driven by new exporters. The changing non-tradedness margin within tradeable sectors is therefore empirically important for aggregate changes.[33]

Entry of firms into relatively more attractive business environments is central to our HBS result, because it induces a relative increase in the effective labor costs in this market. Another important implication is the divergence between the real exchange rate based on average price levels, $\tilde{Q}_t$, and its consumption-based counterpart, Q_t. Whereas the former appreciates, the latter depreciates, driven by an increase in product variety. Adjustments to PPP in international databases are made on the basis of consumer price data that are certainly closer to average price measures than to the variety-adjusted indexes. In other words, adjustments to PPP are based on $\tilde{Q}_t$ rather than Q_t. Yet, as PPP adjustments are meant to reflect welfare differences, users of such data should correct for such differences across countries. (For instance, Broda and Weinstein's, 2003, empirical results prove that the omission of variety – and quality – adjustments has quantitatively significant implications for the evaluation of the U.S. CPI.) Our results suggest that welfare-based PPP adjustments reflecting productivity (or deregulation) differences are opposite to those used in practice. The policy relevance of this implication is apparent, since international agencies use PPP-adjusted data to calculate levels of income per capita that determine, for instance, the allocation of international aid.

Trade Policy

We focus on the consequences of trade liberalization: a symmetric, worldwide decrease in the iceberg costs τ and τ^* or a symmetric, worldwide decrease in the fixed export costs f_X and f_X^*.

[33] Klein, Schuh, and Triest (2003) find evidence that real exchange rate movements lead to significant reallocations of labor in U.S. manufacturing. Our model provides a theoretical foundation for the effect of firm reallocation across sectors on the aggregate economy.

Lower Iceberg Costs

Figure 4 illustrates the response of the economy to a worldwide, permanent, 1 percent decrease in the iceberg trade costs τ and τ^*. Since the shock is symmetric, there is no movement in relative, cross-country variables such as the terms of labor, the terms of trade, and the real exchange rate. We report only changes in home variables as they are identical to those in foreign variables. Lower iceberg costs result in lower export prices and higher output for each individual exporter ($y_X(z)$). There is an immediate, substantial increase in the number of exporting firms, mirrored by a decrease in the export productivity cutoff. The increase in export output demand and the fact that more exporters are demanding labor to cover fixed export costs contribute to upward pressure on the real wage. Consumption rises in response to higher household wage and dividend income. However, *ceteris paribus*, a higher real wage increases the burden of the sunk entry cost in the domestic market. For this reason, the number of entrants drops, and the total number of firms that produce in each economy falls gradually. It is this gradual fall in N_D that causes the long-run increase in w and C to be smaller than the short-run movement through the variety effect. A higher real wage also drives the price of output for domestic sale up, so that domestic sales actually fall, though by less than the increase in output of each individual exporter, and aggregate GDP ($y_t \equiv w_t L + N_{D,t}\tilde{d}_t$) rises.

Lower Fixed Costs

Figure 5 shows the responses to a worldwide, permanent, 1 percent decrease in the fixed export costs f_X and f_X^*. Qualitatively, most effects are similar to those of a reduction in the size of the iceberg costs, although the size of the responses is significantly smaller in most cases. The main qualitative difference relative to Figure 4 is that export prices rise and export output falls. As in Figure 4, trade liberalization results in a sizable increase in the number of exporters. *Ceteris paribus*, this puts upward pressure on the real wage. Since there is no change in iceberg costs and the real exchange rate, both prices for domestic and export sales must rise. Higher export prices, a small change in overall demand, and lower demand for each individual exporter due to the larger number of exporters combine to generate a decrease in output for each exporter.

The dynamics of consumption in figures 4 and 5 imply that the welfare gains from trade liberalization are much larger when liberalization is accomplished by lowering per-unit trade costs than by reducing the fixed export cost. The changes in individual exporter output and the number of exporters in each country following trade liberalization are consistent with Kehoe and Ruhl's (2002) evidence that a substantial portion of the increase in trade after liberalization takes place on the extensive margin.

5 International Trade in Bonds

We now extend the model of Section 2 to allow for international trade in bonds. This allows us to study both how international bond trading affects the results we obtained so far and how the microeconomic dynamics in our model affect current account movements relative to more standard setups without the features of ours. Since the extension to international borrowing and lending does not contain especially innovative features relative to the financial autarky setup, we limit ourselves to describing its main ingredients in words here and present the relevant model equations in an appendix.

We assume that agents can trade bonds domestically and internationally. Home bonds, issued by home households, are denominated in home currency. Foreign bonds, issued by foreign households, are denominated in foreign currency. We keep the assumption that nominal returns are indexed to inflation in each country, so that the bonds issued by each country provide a risk-free, real return in units of that country's consumption basket.

International asset markets are incomplete, as only risk-free bonds are traded across countries. In the absence of any other change in our model, this would imply indeterminacy of steady-state net foreign assets and non-stationarity of the model. The choice of the initial position of the economy would be a matter of convenience, and all shocks would have permanent consequences via wealth reallocation across countries regardless of the nature of the disturbances, thus undermining the reliability of log-linear approximation and the validity of stochastic analysis. (Ghironi, 2000, discusses the issue in detail.) To solve this problem, we assume that agents must pay fees to domestic financial intermediaries when adjusting their bond holdings. We specify these fees as quadratic in the stock of bonds. This convenient specification is sufficient to pin down the steady state uniquely and deliver stationary model dynamics in response to temporary shocks.[34] Realistic choices of parameter values imply that the cost of adjusting bond holdings has a very small impact

[34] Turnovsky (1985) first used this approach, later adopted by Benigno (2001) and Laxton and Pesenti (2003), among others. Schmitt-Grohé and Uribe (2003) demonstrate that different approaches to the issue of steady-state determinacy and model stationarity under incomplete markets generate similar dynamics when the models are calibrated to match a given economy. We choose the cost of adjusting bond holdings over the alternatives for its analytical convenience and ease of interpretation. (One can think of the fees that households must pay as capturing a form of limited participation in financial markets as in Cooley and Quadrini, 1999.)

on model dynamics, other than pinning down the steady state and ensuring mean reversion in the long run if shocks are not permanent.[35]

We assume that financial intermediaries rebate the revenues from bond-adjustment fees to domestic households. In equilibrium, the markets for home and foreign bonds clear, and each country's net foreign assets entering period $t+1$ depend on interest income from asset holdings entering period t, labor income, net investment income, and consumption during period t. The change in asset holdings between t and $t+1$ is the country's current account. Home and foreign current accounts add to zero when expressed in units of the same consumption basket. There are now three Euler equations in each country: the Euler equation for share holdings, which is unchanged, and Euler equations for holdings of domestic and foreign bonds. The fees for adjusting bond holdings imply that the Euler equations for bond holdings feature a term that depends on the stock of bonds, and which is central to steady-state determinacy and model stationarity. Euler equations for bond holdings in each country imply a no-arbitrage condition between bonds. In the log-linear model, this no-arbitrage condition relates the real interest rate differential across countries to expected real exchange rate depreciation in a standard fashion.

The balanced trade condition closed the model in the case of financial autarky. We show that balanced trade implies labor market clearing in each country in Appendix B. Since the balanced trade condition no longer holds once we allow for international bond trading, we must impose labor market clearing conditions at home and abroad to close the model. These conditions state that the amount of labor used in production and to cover sunk entry costs and fixed export costs in each country must equal labor supply in that country in each period.

Since the costs of adjusting bond holdings pin down zero holdings of both domestic and foreign bonds in both countries as the unique steady state, the model with international bond trading of Appendix C has exactly the same steady state as the model under financial autarky (under the same assumptions about exogenous variables). As before, we log-linearize the system and solve it using the method of undetermined coefficients.[36]

[35] We could have introduced costs of adjusting bond holdings in the financial autarky case, but the costs would have had no role in equilibrium, as holdings of bonds are always zero under autarky, leaving the system of equations to be solved unchanged.

[36] Since steady-state holdings of bonds are zero, the percentage deviations of bond stocks from the steady state are normalized by the steady-state level of consumption when linearizing the model (for instance, $\mathsf{B}_{t+1} \equiv dB_{t+1}/C$). Similarly for the current account and the trade balance.

Impulse Responses

We consider the same productivity and deregulation shocks as in the case of autarky. The responses to trade liberalization are identical to those under financial autarky. Since the trade liberalization scenarios involve identical changes in trade policy instruments in the two countries and the initial steady state is symmetric, there is no incentive for agents to trade assets across countries in equilibrium even if they are allowed to do so.

We set the scale parameter for the cost of adjusting bond holdings to .0025 – sufficient to generate stationarity in response to transitory shocks but small enough to ensure that we are not attributing too large a role to this friction in the dynamics of our model.

A Permanent Increase in Home Productivity

Figure 6 shows impulse responses to a permanent 1 percent increase in home productivity. The response of several key variables to the shock is qualitatively similar to that under financial autarky. The permanent nature of the shock implies that home households do not have an incentive to accumulate or decumulate net foreign assets to smooth the effect of a *transitory* fluctuation in income on consumption. The path of C is very similar to that in Figure 1.

As in the case of financial autarky, TOL must decrease in the long run (home effective labor must relatively appreciate) to prevent all entering firms from locating into the home economy. The free entry condition implies entry of new firms, and N_D increases.

The home economy runs a current account deficit in response to the shock and accumulates net foreign debt.[37] Home households borrow from abroad to finance more investment in new, more productive home firms than under financial autarky in the first years after the shock, as one can see by comparing the responses of N_E in figures 1 and 6. The ability to borrow from abroad allows home agents to front-load the increase in the number of home firms that is triggered by the shock, so that the profile of N_D is steeper than in Figure 1. The home household's incentive to front-load entry of more productive firms is mirrored by the foreign household's desire to invest savings in the more attractive economy. Although foreign households cannot hold shares in the mutual portfolio of home firms (since we allow only bonds to be traded across countries), the return on bond holdings

[37] In Figure 6, ca is the current account and tb is the trade balance. The impulse response of the asset stock a cumulates holdings of domestic and foreign bonds. It shows the level of a at the *end* of each period. (The same is true for a^*.) The result that home runs a current account deficit does not change if one looks at the response of $P_t CA_t/\tilde{P}_t$ to adjust for the effect of changes in the number of available varieties on the current account in units of the consumption basket.

is tied to the return on holdings of shares in home firms by no-arbitrage between bonds and shares within the home economy. Therefore, foreign households can share the benefits of higher home productivity by lending to home.

Faster entry of new home firms puts upward pressure on the home real wage, so that TOL no longer depreciates on impact and home exporters no longer enjoy an initial boost in demand: $y_X(z)$ is below the steady state throughout the transition to the new, lower long-run position. In turn, this is mirrored by an increase in the export productivity cutoff z_X.

As under financial autarky, more overall demand for consumption at home results in more demand for imports: $y_X^*(z)$ rises and relatively less productive foreign firms enter the export market (z_X^* falls). Foreign consumption increases by less than under financial autarky on impact, and it takes a few years to achieve the new steady-state level, as foreign agents save in the form of lending to the home economy in the initial periods. In fact, the decision of foreign households to save in the form of foreign lending to the more productive country implies that less resources are available to fund entry of new firms in the foreign economy, so that N_E^* falls substantially in the first few years and there is a more pronounced decrease in the total number of foreign firms (N_D^*) than in Figure 1, resulting in a downward movement in foreign GDP at the time of the peak negative response in N_D^*.

The average real exchange rate $\tilde{Q}$ appreciates in response to the appreciation of the terms of labor and the endogenous adjustment of the traded sectors in both countries. As the results of Section 3 suggested, opening the economy to international asset trading does not change the functioning of the endogenous HBS mechanism at work in our model.

A Transitory Increase in Home Productivity

Figure 7 shows impulse responses to an increase in home productivity with persistence .9. Dynamics are qualitatively similar to those under financial autarky. As in the case of a permanent shock, an important difference is the absence of an initial depreciation of the terms of labor, again motivated by faster entry of new firms into the home economy than under autarky. Home households borrow initially to finance faster firm entry. However, borrowing is quickly reversed, and home runs current account surpluses for approximately seven years after the shock. The path of the current account is such that home's net foreign assets are actually above the steady state throughout the transition, except in the initial periods. When the shock is not permanent, lending abroad to smooth the consequences of a temporary, favorable shock on consumption becomes the main determinant of net foreign asset dynamics.[38]

Deregulation

Figure 8 shows impulse responses to deregulation of the home market. The comparison with Figure 3 can be understood along the same lines as that between figures 1 and 6. Home households borrow from abroad to front-load entry of new firms in the more favorable home market. The home country runs a current account (and trade) deficit and accumulates foreign debt. Home consumption initially declines and is permanently higher in the long run. Foreign consumption moves by more than in Figure 3 as foreign households initially save in the form of foreign lending and then receive income from their positive asset position. The terms of labor and the average real exchange rate $\tilde{Q}$ appreciate.

Once we allow for international borrowing and lending, our model predicts that (permanently) more productive, or less regulated, economies will have a stronger real exchange rate (the HBS effect) and will run persistent (though not permanent) foreign debt positions to finance faster entry of new firms in the economy. These predictions appear in line with the experience of the United States in the 1990s. As under financial autarky, however, the availability of more varieties causes the consumption-based real exchange rate Q to depreciate.

International Business Cycles

Backus, Kehoe, and Kydland (BKK, 1992) show that introducing trade costs in an international RBC model improves its ability to replicate second moments of U.S. and international data. They specify a resource cost of trade as a quadratic function of net exports. But even the introduction of these trade costs is not sufficient to remove the result that consumption is more strongly correlated across countries than aggregate output in their model, contrary to the data (the consumption-output anomaly). Backus and Smith (1993) show that international RBC models with internationally complete asset markets tie the cross-country consumption differential to the real exchange rate through international risk-sharing. Contrary to the prediction of perfect positive correlation between relative consumption and the real exchange rate, they document evidence of no clear pattern in the data (the consumption-real exchange rate anomaly). Chari, Kehoe, and McGrattan (CKM, 2002) report evidence of negative correlation between relative consumption and the real exchange

[38] Foreign households initially invest in the more productive home economy and then borrow to share the consumption benefits of temporarily higher home productivity.

rate for the U.S. relative to Europe. Their sticky-price model does better than BKK as far as the consumption-output anomaly is concerned, but it too fails when it comes to relative consumption and the real exchange rate – and to generating sufficient persistence in the latter.

Obstfeld and Rogoff (2001) argue that introducing iceberg trade costs helps explain a variety of puzzles in international comovements, including the BKK consumption-output anomaly. They observe that trade costs and incomplete asset markets can explain the consumption-real exchange rate anomaly.[39] Our model features trade costs and incomplete asset markets. Here, we investigate its ability to mimic key features of international business cycles and its performance in relation to the puzzles highlighted above.

The model includes only one source of fluctuations at business cycle frequencies, the shocks to aggregate productivities Z_t and Z_t^*. In this section, we calibrate the bivariate productivity process (8) based on U.S. and international data, holding other exogenous variables at their steady-state levels. For purposes of comparison, we use the symmetrized estimate of the bivariate productivity process for the U.S. and an aggregate of European economies in BKK and set

$$\begin{bmatrix} \phi_Z & \phi_{ZZ^*} \\ \phi_{Z^*Z} & \phi_{Z^*} \end{bmatrix} = \begin{bmatrix} .906 & .088 \\ .088 & .906 \end{bmatrix}.$$

This matrix implies a small, positive productivity spillover across countries, such that, if home productivity rises during period t, foreign productivity will also increase at $t+1$. We set the standard deviation of the productivity innovations to .00852 (a .73 percent variance) and the correlation to .258 (corresponding to a .19 percent covariance), as estimated by BKK. We calculate the implied second moments of endogenous variables using the frequency domain technique described in Uhlig (1999) and compare the model-generated moments with those of U.S. and international data computed in BKK and reported in Table 2 for the reader's convenience.[40] (Benigno, 2004, and CKM are the sources on the empirical properties of the real exchange rate. Benigno reports averages of data in Bergin and Feenstra – BF –, 2001, and CKM.) For consistency with BKK and CKM, who focus on the high-frequency properties of business cycles in the U.S. and abroad, we report second moments of Hodrick-Prescott (HP)-filtered variables. The productivity process has eigenvalues .994 and .818. A stationary process for productivity and model stationarity imply that all endogenous

[39] Benigno and Thoenissen (2003) show that international asset market incompleteness plays a central role in dealing with the Backus-Smith puzzle in a model in which intermediate goods are traded and households consume only non-traded goods.

[40] Results based on model simulation are similar.

variables of interest are stationary. However, productivity and key endogenous state variables – such as the number of firms and asset stocks – are persistent enough that model-generated moments calculated without HP filtering pick up low frequency fluctuations that are not featured in the HP-filtered data in BKK.

Households invest in the entry of new firms in the economy in our model. The number of new entrants behaves very much like investment in capital, with the latter proxied by the number of firms that produce in each period. For comparison with BKK's results on investment and capital, we compute second moments for $N_{E,t}$ and $N_{D,t}$ as well as for their overall values in terms of average firm valuation ($\tilde{v}_t N_{E,t}$ and $\tilde{v}_t N_{D,t}$, respectively). Consistent with BKK, we define saving as the difference between GDP and consumption ($y_t - C_t$) and investigate the correlation between the saving rate ($1 - C_t/y_t$) and the investment rate ($\tilde{v}_t N_{E,t}/y_t$) relative to the data. To control for the role of the variety effect on measurement, we consider two measures of real variables, in units of consumption (for instance, y_t) and deflated by the average price level $\tilde{P}_t$ ($P_t y_t/\tilde{P}_t$). In most cases, results are very similar across measures.

Table 3 reports our results for the benchmark calibration. The model underpredicts the standard deviation of aggregate output (measured by GNP in BKK) and overpredicts that of consumption, although it is successful at generating less volatile consumption than GDP. The ratio of the standard deviation of $P_t C_t/\tilde{P}_t$ to $P_t y_t/\tilde{P}_t$ is roughly .59, ten percent higher than the consumption-output volatility ratio in BKK's data. Like fixed investment in the data, investment in new firms is substantially more volatile than GDP. The trade balance/GDP ratio is much more volatile than the net exports/output ratio in the data. The trade costs in our model do not prevent the trade balance/GDP ratio from being quite volatile as the BKK specification does by penalizing trade balance movements directly.[41] Instead, the real exchange rate is clearly less volatile than in the data.

The model is quite successful at reproducing the autocorrelation function of key U.S. aggregate variables with output: The autocorrelation functions for output itself, consumption, investment in new firms, the stock of productive firms, and the trade balance in Table 3 all reproduce the qualitative pattern in Table 2. In the cases of output, consumption, and investment success is also reasonable, if not striking, on quantitative grounds.

The BKK model fails to deliver positive correlation between home and foreign output. A puzzling negative cross-country correlation of aggregate outputs is a standard result of the international

[41] Note, however, that the volatility of the trade balance itself is much smaller.

RBC literature. Our model successfully generates positive correlation between foreign and domestic GDPs. However, as the BKK setup, ours fails to generate cross-country consumption correlation that is smaller than the correlation across GDPs for the same parametrization of productivity. The contemporaneous correlation between saving and investment rates is positive, but stronger than in the data. The autocorrelation function for the real exchange rate $\tilde{Q}_t$ displays substantial persistence, with a first-order coefficient equal to .89 roughly in line with the BF and CKM evidence. The correlation between relative consumption spending (the ratio of home to foreign nominal consumption spending deflated by the respective average price levels) and the real exchange rate $\tilde{Q}_t$ is negative – as in the CKM data – but too large in absolute value. The correlation between relative consumption and the consumption based interest rate – which is 1 in Backus and Smith's (1993) complete markets world – is .71.

To verify the robustness of our results, we consider an alternative parametrization of the productivity process (8). In fact, Reynolds (1993) and Baxter and Crucini (1995) find that estimated spillover coefficients in (8) are insignificantly different from zero. Baxter and Crucini (1995) argue in favor of a near-unit-root process for productivity. Accordingly, Baxter (1995) and Baxter and Farr (2001) set the coefficients ϕ_{ZZ^*} and ϕ_{Z^*Z} to zero and consider values of $\phi_{ZZ} = \phi_{Z^*Z^*}$ ranging between .995 and .999 in their exercises. Table 4 reports results for $\phi_{ZZ^*} = \phi_{Z^*Z} = 0$ and $\phi_{ZZ} = \phi_{Z^*Z^*} = .999$. (As Baxter, 1995, we keep the same variance-covariance matrix of the productivity innovations as in BKK.) The majority of qualitative patterns are unchanged. As before, the model overpredicts the standard deviation of consumption and the trade balance-GDP ratio. It underpredicts the standard deviation of GDP and the real exchange rate. It does quite well in terms of the standard deviations of investment in new firms and the total stock of firms.

The model generates substantial GDP persistence in Table 4. The autocorrelation function for investment in new firms is very close to the empirical autocorrelation of fixed investment. The contemporaneous correlation of home consumption to home GDP is too strong relative to the BKK data. Most importantly, however, the consumption-output anomaly of BKK weakens substantially: The correlation of consumption across countries (C and C^*) is smaller than that of GDP (y and y^*). When productivity spillovers are present, the response of foreign consumption to a home shock is larger, as foreign households anticipate that foreign productivity will rise too. When we remove the spillovers, the increase in foreign consumption following a home shock is muted, resulting in lower consumption correlation than GDP correlation across countries. The consumption-output anomaly persists if we consider variables deflated by the respective average prices, but the correlation between $PC/\tilde{P}$ and $P^*C^*/\tilde{P}^*$ is only slightly larger than that between $Py/\tilde{P}$ and $P^*y^*/\tilde{P}^*$. The correlation between saving and investment rates is now in the same range as observed in the data. As in the case of the BKK productivity process, the correlation between relative consumption spending deflated by average prices and the real exchange rate is negative, but too large in absolute value. Importantly, the correlation between C/C^* and Q now drops to .13.[42]

Overall, we interpret the results of the stochastic exercise of this section as supportive of the novel features of our model as a mechanism for the propagation of business cycles across countries and over time. Even after HP filtering (and thus removing low-frequency fluctuations that are arguably important for medium- to long-run transmission), the model is very successful at generating persistent dynamics of endogenous variables and matching several key moments of the data for reasonable parameter values and a standard productivity process. Consistent with Obstfeld and Rogoff's (2001) arguments, the introduction of trade costs and market incompleteness pushes results in the right direction with respect to important puzzles in international macroeconomics, although assumptions about the exogenous shock process also play an important role.

6 Conclusions

We developed a two-country, stochastic, general equilibrium model of international trade and macroeconomic dynamics. Relative to existing international macro models, ours has the advantage of matching several features of empirical evidence in the micro, trade literature. It does so while preserving substantial tractability and the ability to provide intuitions for the main results.

We assumed that firms face some uncertainty about their future productivity when they make the decision whether or not to sink the resources necessary to enter the domestic market. Consistent with overwhelming empirical evidence, we assumed that firms face fixed costs as well as per-unit costs when they export. As a consequence of the fixed export cost, only the relatively more productive firms self-select into the export market. Aggregate productivity shocks, changes in domestic market regulation, and changes in trade policy cause firms to enter and exit markets and generate deviations from PPP that would not exist absent our microeconomic structure.

Our model provides a fully microfounded, endogenous explanation for the HBS effect, according to which more productive economies have relatively stronger real exchange rates. All goods are tradeable in our setup, some are non-traded in equilibrium, and the non-tradedness margin changes

[42] Setting the scale parameter for the costs of adjusting bond holdings to .01 left most results unaffected.

over time. In contrast to the textbook treatment of the HBS effect, shocks are aggregate rather than sector-specific. Our model predicts that more productive economies, or less regulated ones, will exhibit higher average prices relative to their trading partners. This real exchange rate appreciation is driven by entry and endogenous non-tradedness, the two key new features of our setup.

The same new features result in substantial persistence of key endogenous variables in response to transitory exogenous shocks. In particular, our model results in persistent PPP deviations in a world of flexible prices.

When we allow for international borrowing and lending, the model predicts that more productive, or less regulated, economies will experience HBS real appreciation and run persistent foreign debt positions to finance faster entry of new firms in the economy. Thus, our framework provides a novel perspective on recent stylized facts for the U.S. economy that are broadly in line with these predictions. In addition, the model matches several important moments of the U.S. and international business cycle quite well for reasonable assumptions about parameters and productivity. In particular, it generates positive GDP correlation across countries, it does not constrain the correlation between relative consumption and the real exchange rate to being perfect, and it improves on the standard international RBC setup as far as cross-country correlations of consumption and GDP are concerned, confirming Obstfeld and Rogoff's (2001) argument that trade costs help explain international macroeconomic puzzles.

Baldwin and Krugman (1989) used a mechanism similar to ours in a sticky-price, partial equilibrium model to explain persistent effects of nominal exchange rate movements. Although it does well on persistence grounds, the flexible-price setup of this paper underpredicts the volatility of GDP and the real exchange rate. We conjecture that including nominal rigidity and deviations from the law of one price along the lines of Benigno (2004) and Devereux and Engel (2003) will amplify the response of the real exchange rate to shocks and subject it to the consequences of monetary shocks, complementing our persistence-inducing mechanism with increased real volatility.[43] We will extend our model in this direction in future work.

Importantly, our model suggests a dichotomy between the behavior of relative CPIs as currently measured by the data and a welfare-based measure of the real exchange rate that accounts for availability of new varieties. Rather than appreciating, the latter depreciates as a consequence of a productivity advantage or deregulation. This raises the issue of PPP adjustments in international

[43] Goldberg and Verboven (2001) document strong evidence of deviations from the law of one price in the European car market. The results in Engel (1999) and Engel and Rogers (2001) suggest that deviations from the law of one price play a central role in short-run real exchange rate fluctuations.

statistics, which may contain substantial biases due to the omission of variety effects. We will delve deeper into this issue in our future research.

Appendix

A. The Steady State

We denote constant, steady-state levels of variables by dropping the time subscript and assume: $f_E = f_E^*$, $f_X = f_X^*$, $\tau = \tau^*$, $L = L^*$, and $Z = Z^* = 1$. Under these assumption, the steady state of the model is symmetric: $\tilde{Q} = Q = \mu = 1$ and the levels of all other endogenous variables are equal across countries.

Solving for $\tilde{z}_X$

Given the solution for the average export productivity $\tilde{z}_X$, we can obtain the cutoff level z_X from $\tilde{z}_X = \varphi z_X$, where $\varphi \equiv \{k/[k-(\theta-1)]\}^{1/(\theta-1)}$. We can solve for $\tilde{z}_X$ as follows. The Euler equation for share holdings yields:

$$\tilde{v} = \frac{\beta(1-\delta)}{1-\beta(1-\delta)}\left(\tilde{d}_D + \frac{N_X}{N_D}\tilde{d}_X\right). \tag{9}$$

Combining this equation with the free entry condition $\tilde{v} = f_E w$ implies:

$$\tilde{d}_D + \frac{N_X}{N_D}\tilde{d}_X = \frac{[1-(1-\delta)\beta]}{(1-\delta)\beta} f_E w. \tag{10}$$

The steady-state zero profit export cutoff equation is:

$$\tilde{d}_X = w f_X \frac{\theta-1}{k-(\theta-1)}. \tag{11}$$

Also, steady-state profits from selling at home and abroad are $\tilde{d}_D = (\tilde{\rho}_D)^{1-\theta} C/\theta$ and $\tilde{d}_X = (\tilde{\rho}_X)^{1-\theta} C/\theta - w f_X$, respectively. These two equations imply:

$$\tilde{d}_D = \left(\frac{\tilde{\rho}_X}{\tilde{\rho}_D}\right)^{\theta-1}\left(\tilde{d}_X + w f_X\right). \tag{12}$$

Optimal pricing yields $\tilde{\rho}_D = [\theta/(\theta-1)]\tilde{z}_D^{-1} w$ and $\tilde{\rho}_X = [\theta/(\theta-1)]\tau\tilde{z}_X^{-1} w$. Hence, $\tilde{\rho}_X/\tilde{\rho}_D =$

$\tau\tilde{z}_D/\tilde{z}_X$, and substituting this into (12), we have:

$$\tilde{d}_D = \left(\frac{\tau\tilde{z}_D}{\tilde{z}_X}\right)^{\theta-1}\left(\tilde{d}_X + wf_X\right), \tag{13}$$

or, taking (11) into account,

$$\tilde{d}_D = \left(\frac{\tau\tilde{z}_D}{\tilde{z}_X}\right)^{\theta-1}\left(wf_X\frac{\theta-1}{k-(\theta-1)} + wf_X\right). \tag{14}$$

The steady-state share of exporting firms in the total number of domestic firms is:

$$\frac{N_X}{N_D} = (z_{\min})^k(\tilde{z}_X)^{-k}\left[\frac{k}{k-(\theta-1)}\right]^{\frac{k}{\theta-1}}. \tag{15}$$

Substituting equations (11), (14), and (15) into (10), using $\tilde{z}_D = \{k/[k-(\theta-1)]\}^{\frac{1}{\theta-1}} z_{\min}$, and rearranging yields:

$$(\tilde{z}_X)^{1-\theta}(\tau z_{\min})^{\theta-1}\left[\frac{k}{k-(\theta-1)}\right]^2 + (\tilde{z}_X)^{-k}(z_{\min})^k\left[\frac{k}{k-(\theta-1)}\right]^{\frac{k}{\theta-1}}\frac{\theta-1}{k-(\theta-1)} = \frac{[1-(1-\delta)\beta]}{(1-\delta)\beta}\frac{f_E}{f_X}.$$

This equation can be rewritten as:

$$\xi_1(\tilde{z}_X)^{1-\theta} + \xi_2(\tilde{z}_X)^{-k} = \xi_3, \tag{16}$$

where

$$\begin{aligned}
\xi_1 &\equiv (\tau z_{\min})^{\theta-1}\left[\frac{k}{k-(\theta-1)}\right]^2 > 0,\\
\xi_2 &\equiv (z_{\min})^k\left[\frac{k}{k-(\theta-1)}\right]^{\frac{k}{\theta-1}}\frac{\theta-1}{k-(\theta-1)} > 0,\\
\xi_3 &\equiv \frac{[1-(1-\delta)\beta]}{(1-\delta)\beta}\frac{f_E}{f_X} > 0.
\end{aligned}$$

The left-hand side of equation (16) is a hyperbola. This guarantees existence and uniqueness of $\tilde{z}_X > 0$, the exact value of which we obtain numerically.

Solving for $\tilde{\rho}_X$

The law of motion for the total number of domestic firms implies:

$$N_E = \frac{\delta}{1-\delta} N_D. \tag{17}$$

Steady-state aggregate accounting yields $C = wL + N_D \tilde{d}_D + N_X \tilde{d}_X - N_E w f_E$. Using (10) and (17), this can be rewritten as:

$$\frac{C}{w} = L + N_D f_E \frac{1-\beta}{(1-\delta)\,\beta}. \tag{18}$$

Equation (11) and the expression for average export profits, $\tilde{d}_X = (\tilde{\rho}_X)^{1-\theta} C/\theta - w f_X$, imply:

$$\frac{C}{w} = \tilde{\rho}_X^{\theta-1} \frac{\theta k}{k-(\theta-1)} f_X. \tag{19}$$

The price index equation $N_D \tilde{\rho}_D^{1-\theta} + N_X \tilde{\rho}_X^{1-\theta} = 1$ yields:

$$\frac{\tilde{\rho}_X^{\theta-1}}{N_D} = \left(\frac{\tilde{\rho}_X}{\tilde{\rho}_D}\right)^{\theta-1} + \frac{N_X}{N_D},$$

or, using $\tilde{\rho}_X / \tilde{\rho}_D = \tau \tilde{z}_D / \tilde{z}_X$ and equation (15),

$$\frac{\tilde{\rho}_X^{\theta-1}}{N_D} = \left(\frac{\tau \tilde{z}_D}{\tilde{z}_X}\right)^{\theta-1} + \left(\frac{z_{\min}}{\tilde{z}_X}\right)^{k} \left[\frac{k}{k-(\theta-1)}\right]^{\frac{k}{\theta-1}}. \tag{20}$$

Together, equations (18), (19), and (20) yield the following equation for $\tilde{\rho}_X$:

$$\tilde{\rho}_X^{1-\theta} = \left[\frac{\theta k}{k-(\theta-1)} f_X - K^{-1} f_E \frac{1-\beta}{(1-\delta)\,\beta}\right] L^{-1}.$$

where K is the right-hand side of equation (20).

Special Case: All Firms Export In this case, equation (19) no longer holds since the zero cutoff profit condition (11) no longer holds. Using $\tilde{d}_D = (\tilde{\rho}_D)^{1-\theta} C/\theta$ and $\tilde{d}_X = (\tilde{\rho}_X)^{1-\theta} C/\theta - w f_X$, equation (10) can be written as:

$$\tilde{\rho}_X^{1-\theta} \frac{C}{\theta} \left(\tau^{\theta-1} + 1\right) - w f_X = \frac{[1-(1-\delta)\,\beta]}{(1-\delta)\,\beta} f_E w,$$

which implies:

$$\frac{C}{w} = \tilde{\rho}_X^{\theta-1} \frac{\theta}{\tau^{\theta-1}+1} \left\{ f_X + \frac{[1-(1-\delta)\,\beta]}{(1-\delta)\,\beta} f_E \right\}. \tag{21}$$

Equation (21) now replaces equation (19) when solving for $\tilde{\rho}_X$. This yields the following expression for $\tilde{\rho}_X$:

$$\tilde{\rho}_X^{1-\theta} = \left[\frac{\theta}{\tau^{\theta-1}+1} \left\{ f_X + \frac{[1-(1-\delta)\beta]}{(1-\delta)\beta} f_E \right\} - K^{-1} f_E \frac{1-\beta}{(1-\delta)\beta} \right] L^{-1}.$$

Solving for the Remaining Variables

The solutions for other endogenous variables are straightforward

- $N_D = K^{-1} \tilde{\rho}_X^{\theta-1}$;
- $\tilde{\rho}_D = \frac{\tilde{z}_X}{\tau \tilde{z}_D} \tilde{\rho}_X$ using $\tilde{\rho}_X / \tilde{\rho}_D = \tau \tilde{z}_D / \tilde{z}_X$;
- $w = \tilde{\rho}_X \frac{\theta-1}{\theta\tau} \tilde{z}_X$ using $\tilde{\rho}_X = [\theta / (\theta-1)] \tau \tilde{z}_X^{-1} w$;
- $C = w \left[L + N_D f_E \frac{1-\beta}{(1-\delta)\beta} \right]$ using (18);
- $N_E = \frac{\delta}{1-\delta} N_D$;
- $N_X = N_D (z_{\min})^k (\tilde{z}_X)^{-k} \left[\frac{k}{k-(\theta-1)} \right]^{\frac{k}{\theta-1}}$ using (15);
- $\tilde{d}_D = \frac{1}{\theta} (\tilde{\rho}_D)^{1-\theta} C$;
- $\tilde{d}_X = \frac{1}{\theta} (\tilde{\rho}_X)^{1-\theta} C - w f_X$;
- $\tilde{v} = w f_E$ (using the free entry condition);
- $1 + r = 1/\beta$ (using the Euler equation for bond holdings).

Symmetry of the steady state ensures $\tilde{z}_X^* = \tilde{z}_X$, $\tilde{\rho}_X^* = \tilde{\rho}_X$, $N_D^* = N_D$, $N_E^* = N_E$, $N_X^* = N_X$, $\tilde{d}_D^* = \tilde{d}_D$, $\tilde{d}_X^* = \tilde{d}_X$, $\tilde{v}^* = \tilde{v}$, in addition to $C^* = C$, $w^* = w$, and $r^* = r$.

B. Labor Market Clearing

Recall that a firm with productivity z produces $Z_t z$ units of output per unit of labor employed. Consider separately the labor used to produce goods for the domestic and export markets: let $l_{D,t}(z)$ and $l_{X,t}(z)$ represent the amount of labor hired to produce goods for each market. These only represent labor used in production; in addition, each new entrant hires $f_{E,t}/Z_t$ units of labor to cover the entry cost, and each exporter hires $f_{X,t}/Z_t$ units of labor to cover the fixed export cost in every period. The profits earned from domestic sales for a firm with productivity z are then given by:

$$d_{D,t}(z) = \rho_{D,t}(z) Z_t z l_{D,t}(z) - w_t l_{D,t}(z) = \frac{1}{\theta - 1} w_t l_{D,t}(z), \tag{22}$$

using $\rho_{D,t}(z) = \frac{\theta}{\theta-1}\frac{w_t}{Z_t z}$ from (1). This relationship holds for a firm with average productivity $\tilde{z}_D$, and also for averages across all domestic firms. This implies that the average amount of production labor hired to cover domestic sales is $(\theta - 1)\,\tilde{d}_{D,t}/w_t$. The total amount of such labor hired at home is thus $N_{D,t}\,(\theta - 1)\,\tilde{d}_{D,t}/w_t$.

The profits earned from export sales for an exporting firm with productivity z are given by:

$$d_{X,t}(z) = Q_t \rho_{X,t}(z) \frac{Z_t z l_{X,t}(z)}{\tau_t} - w_t \left[l_{X,t}(z) + \frac{f_{X,t}}{Z_t} \right] = \frac{1}{\theta - 1} w_t l_{X,t}(z) - w_t \frac{f_{X,t}}{Z_t}, \tag{23}$$

using $\rho_{X,t}(z) = Q_t^{-1}\tau_t \frac{\theta}{\theta-1}\frac{w_t}{Z_t z}$ from (1). Note that only $Z_t z l_{X,t}(z)/\tau_t$ export units are sold, although $Z_t z l_{X,t}(z)$ are produced (the remaining fraction having "melted" away in an iceberg fashion while crossing the border). Again, this relationship holds for a firm with average export productivity $\tilde{z}_{X,t}$, and also for averages across all exporters. The average amount of production labor hired to cover export sales is thus $(\theta - 1)\,\tilde{d}_{X,t}/w_t + (\theta - 1)\,f_{X,t}/Z_t$. Multiplying by $N_{X,t}$ yields the total amount of such labor for the home economy.

The total amount of production labor hired in the home economy is then

$$\frac{\theta - 1}{w_t} N_{D,t} d_{D,t}(z) + \frac{\theta - 1}{w_t} N_{X,t} \tilde{d}_{X,t} + \frac{\theta - 1}{Z_t} N_{X,t} f_{X,t}.$$

Adding the total amount of labor hired by new entrants, $N_{E,t} f_{E,t}/Z_t$, and that hired by exporters to cover the fixed costs, $N_{X,t} f_{X,t}/Z_t$, yields the aggregate labor demand for the home economy:

$$L_t = \frac{\theta - 1}{w_t} N_{D,t} \tilde{d}_{D,t} + \frac{\theta - 1}{w_t} N_{X,t} \tilde{d}_{X,t} + \frac{\theta}{Z_t} N_{X,t} f_{X,t} + \frac{1}{Z_t} N_{E,t} f_{E,t}. \tag{24}$$

Equating L_t to labor supply (L) yields the equilibrium condition (39) below. The derivation of (40) is analogous.

Balanced Trade Implies Labor Market Clearing

We now demonstrate that balanced trade under financial autarky implies labor market clearing.

Using the home price index equation $1 = N_{D,t}\left(\tilde{\rho}_{D,t}\right)^{1-\theta} + N_{X,t}^{*}\left(\tilde{\rho}_{X,t}^{*}\right)^{1-\theta}$, the balanced trade condition $Q_t N_{X,t} \left(\tilde{\rho}_{X,t}\right)^{1-\theta} C_t^* = N_{X,t}^* \left(\tilde{\rho}_{X,t}^*\right)^{1-\theta} C_t$ can be written:

$$Q_t N_{X,t} \left(\tilde{\rho}_{X,t}\right)^{1-\theta} C_t^* = \left[1 - N_{D,t} \left(\tilde{\rho}_{D,t}\right)^{1-\theta}\right] C_t.$$

This condition can be re-written as

$$C_t = \theta N_{X,t} \left(\tilde{d}_{X,t} + w_t \frac{f_{X,t}}{Z_t}\right) + \theta N_{D,t} \tilde{d}_{D,t},$$

since $\tilde{d}_{D,t} = \left(\tilde{\rho}_{D,t}\right)^{1-\theta} C_t/\theta$ and $\tilde{d}_{X,t} = Q_t \left(\tilde{\rho}_{X,t}\right)^{1-\theta} C_t^*/\theta - w_t f_{X,t}/Z_t$. Combining this with aggregate accounting ($C_t = w_t L + N_{D,t}\tilde{d}_{D,t} + N_{X,t}\tilde{d}_{X,t} - N_{E,t} w_t f_{E,t}/Z_t$) yields the labor market clearing condition for the home economy:

$$L = \frac{\theta - 1}{w_t} N_{D,t} \tilde{d}_{D,t} + \frac{\theta - 1}{w_t} N_{X,t} \tilde{d}_{X,t} + \frac{\theta}{Z_t} N_{X,t} f_{X,t} + \frac{1}{Z_t} N_{E,t} f_{E,t}.$$

The proof for the foreign economy follows the same steps.

C. International Bond Trading

The budget constraint of the representative home household, in units of the home consumption basket, is now:

$$\begin{aligned} & B_{t+1} + Q_t B_{*,t+1} + \frac{\eta}{2}\left(B_{t+1}\right)^2 + \frac{\eta}{2} Q_t \left(B_{*,t+1}\right)^2 + \tilde{v}_t N_{H,t} x_{t+1} + \frac{M_t}{P_t} + C_t + T_t \\ = \; & (1 + r_t) B_t + Q_t \left(1 + r_t^*\right) B_{*,t} + \left(\tilde{d}_t + \tilde{v}_t\right) N_{D,t} x_t + T_t^{fi} + \frac{M_{t-1}}{P_t} + w_t L, \end{aligned} \tag{25}$$

where B_{t+1} denotes holdings of home bonds, $B_{*,t+1}$ denotes holdings of foreign bonds, $(\eta/2)\left(B_{t+1}\right)^2$ is the cost of adjusting holdings of home bonds, $(\eta/2)\left(B_{*,t+1}\right)^2$ is the cost of adjusting holdings of foreign bonds (in units of foreign consumption), T_t^{fi} is the fee rebate, taken as given by the household, and equal to $(\eta/2)\left[\left(B_{t+1}\right)^2 + Q_t \left(B_{*,t+1}\right)^2\right]$ in equilibrium. For simplicity, we assume that the scaling parameter $\eta > 0$ is identical across costs of adjusting holdings of home and foreign bonds. Also, there is no cost of adjusting equity holdings. The justification for this is that, in equilibrium, bond holdings differ from zero only because of transactions with a foreign counterpart. As a consequence, in equilibrium, bond-adjustment fees actually capture fees on international transactions, which we assume absent from domestic transactions such as those involving equity to avoid adding unnecessary complication.[44]

The representative foreign household faces a similar constraint, in units of foreign consumption:

$$\begin{aligned} & \frac{B^*_{t+1}}{Q_t} + B^*_{*,t+1} + \frac{\eta}{2}\frac{\left(B^*_{t+1}\right)^2}{Q_t} + \frac{\eta}{2}\left(B^*_{*,t+1}\right)^2 + \tilde{v}^*_t N^*_{F,t} x^*_{t+1} + \frac{M^*_t}{P^*_t} + C^*_t + T^*_t \\ = \; & \frac{(1+r_t)}{Q_t} B^*_t + (1+r^*_t) B^*_{*,t} + \left(\tilde{d}^*_t + \tilde{v}^*_t\right) N^*_{D,t} x^*_t + T^{fi*}_t + \frac{M^*_{t-1}}{P^*_t} + w^*_t L^*, \end{aligned} \tag{26}$$

where B^*_{t+1} denotes holdings of the home bond, $B^*_{*,t+1}$ denotes holdings of the foreign bond, and $T^{fi*}_t = (\eta/2)\left[\left(B^*_{t+1}\right)^2/Q_t + \left(B^*_{*,t+1}\right)^2\right]$ in equilibrium.

Home and foreign households maximize the respective intertemporal utility functions subject to the respective constraints. The first-order conditions for the choices of share holdings in mutual portfolios of domestic firms at home and abroad are unchanged relative to the case of financial autarky. Instead, we have the following Euler equations for bond holdings. At home:

$$(C_t)^{-\gamma}(1+\eta B_{t+1}) = \beta(1+r_{t+1}) E_t\left[(C_{t+1})^{-\gamma}\right], \tag{27}$$

$$(C_t)^{-\gamma}(1+\eta B_{*,t+1}) = \beta\left(1+r^*_{t+1}\right) E_t\left[\frac{Q_{t+1}}{Q_t}(C_{t+1})^{-\gamma}\right]. \tag{28}$$

Abroad:

$$(C^*_t)^{-\gamma}\left(1+\eta B^*_{t+1}\right) = \beta(1+r_{t+1}) E_t\left[\frac{Q_t}{Q_{t+1}}\left(C^*_{t+1}\right)^{-\gamma}\right], \tag{29}$$

$$(C^*_t)^{-\gamma}\left(1+\eta B^*_{*,t+1}\right) = \beta\left(1+r^*_{t+1}\right) E_t\left[\left(C^*_{t+1}\right)^{-\gamma}\right]. \tag{30}$$

The presence of the terms that depend on the stock of bonds at the left-hand side of these equations is crucial for steady-state determinacy and model stationarity. It ensures that zero holdings of bonds is the *unique* steady state in which the product of β times the gross interest rate equals 1 in each country, so that economies return to this initial position after temporary shocks. If we had perfect foresight and $\eta = 0$, Euler equations for bond holdings at home and abroad would imply the no-arbitrage condition $(1+r_{t+1})/\left(1+r^*_{t+1}\right) = Q_{t+1}/Q_t$. This is the familiar condition that says that the real interest rate differential must be equal to expected real depreciation for agents

[44] We also experimented with a specification of the cost of adjusting bond holdings as a function of overall assets: $(\eta'/2)(A_{t+1})^2$, where $A_{t+1} \equiv B_{t+1} + Q_t B_{*,t+1}$. The specification we use has the advantage of pinning down uniquely the steady-state levels and dynamics of B_{t+1} and $B_{*,t+1}$ as well as of their aggregate. The alternative specification only pins down the latter. It is possible to verify that the two specifications yield identical log-linear dynamics under the assumptions that the steady-state levels of B_{t+1} and $B_{*,t+1}$ are zero when the cost $(\eta'/2)(A_{t+1})^2$ is used and $\eta' = (1/2)\eta$.

to be indifferent between home and foreign bonds. With perfect foresight and $\eta > 0$, no-arbitrage conditions for home and foreign households imply:

$$\frac{1+r_{t+1}}{1+r^*_{t+1}} = \frac{Q_{t+1}}{Q_t}\frac{1+\eta B_{t+1}}{1+\eta B_{*,t+1}} = \frac{Q_{t+1}}{Q_t}\frac{1+\eta B^*_{t+1}}{1+\eta B^*_{*,t+1}}. \tag{31}$$

Equilibrium requires that home and foreign bonds be in zero net supply worldwide:

$$B_{t+1} + B^*_{t+1} = 0, \tag{32}$$

$$B_{*,t+1} + B^*_{*,t+1} = 0. \tag{33}$$

Home and foreign holdings of each individual bond must add up to zero because each country is populated by a unitary mass of identical households that make identical equilibrium choices and only the home (foreign) country issues home (foreign) currency bonds. Using (32) and (33) in conjunction with the second equality in (31) makes it possible to show that $B_{t+1} = B_{*,t+1}$ and $B^*_{t+1} = B^*_{*,t+1}$ at an optimum under perfect foresight. Since households face quadratic costs of adjusting bond holdings with identical scale parameters across bonds, it is optimal to adjust holdings of different bonds equally so as to spread the cost evenly. The same result holds in the log-linear version of the stochastic model.

Aggregate accounting implies the following laws of motion for net foreign assets at home and abroad:

$$B_{t+1} + Q_t B_{*,t+1} = (1+r_t)B_t + Q_t\left(1+r^*_t\right)B_{*,t} + w_t L + N_{D,t}\tilde{d}_t - N_{E,t}\tilde{v}_t - C_t, \tag{34}$$

$$\frac{B^*_{t+1}}{Q_t} + B^*_{*,t+1} = \frac{(1+r_t)}{Q_t}B^*_t + \left(1+r^*_t\right)B^*_{*,t} + w^*_t L^* + N^*_{D,t}\tilde{d}^*_t - N^*_{E,t}\tilde{v}^*_t - C^*_t, \tag{35}$$

where holdings of individual bonds across countries are tied by the equilibrium conditions (32) and (33). Given these conditions, multiplying (35) times Q_t and subtracting the resulting equation from (34) yields an expression for home net foreign asset accumulation as a function of interest income and of the cross-country differentials between labor income, net investment income, and consumption:

$$\begin{aligned} B_{t+1} + Q_t B_{*,t+1} = {} & (1+r_t)B_t + Q_t\left(1+r^*_t\right)B_{*,t} + \frac{1}{2}\left(w_t L - Q_t w^*_t L^*\right) \\ & + \frac{1}{2}\left(N_{D,t}\tilde{d}_t - N^*_{D,t}Q_t\tilde{d}^*_t\right) - \frac{1}{2}\left(N_{E,t}\tilde{v}_t - N^*_{E,t}Q_t\tilde{v}^*_t\right) - \frac{1}{2}\left(C_t - Q_t C^*_t\right). \end{aligned} \tag{36}$$

Current accounts are by definition equal to the changes in aggregate bond holdings in the two countries:

$$CA_t \equiv B_{t+1} - B_t + Q_t \left(B_{*,t+1} - B_{*,t}\right), \tag{37}$$

$$CA_t^* \equiv \frac{B_{t+1}^* - B_t^*}{Q_t} + B_{*,t+1}^* - B_{*,t}^*. \tag{38}$$

It is straightforward to verify that the bond-market clearing conditions (32) and (33) imply $CA_t + Q_t CA_t^* = 0$ (a country's borrowing must equal the other country's lending) and $C_t + Q_t C_t^* = w_t L + Q_t w_t^* L^* + N_{D,t}\tilde{d}_t + N_{D,t}^* Q_t \tilde{d}_t^* - \left(N_{E,t}\tilde{v}_t + N_{E,t}^* Q_t \tilde{v}_t^*\right)$ (since the world as a whole is a closed economy, world consumption must equal world labor income plus world net investment income).

Labor market clearing conditions at home and abroad require:

$$L = \frac{\theta - 1}{w_t}\left(N_{D,t}\tilde{d}_{D,t} + N_{X,t}\tilde{d}_{X,t}\right) + \frac{1}{Z_t}\left(\theta N_{X,t} f_{X,t} + N_{E,t} f_{E,t}\right), \tag{39}$$

$$L^* = \frac{\theta - 1}{w_t^*}\left(N_{D,t}^*\tilde{d}_{D,t}^* + N_{X,t}^*\tilde{d}_{X,t}^*\right) + \frac{1}{Z_t^*}\left(\theta N_{X,t}^* f_{X,t}^* + N_{E,t}^* f_{E,t}^*\right). \tag{40}$$

We thus have 23 endogenous variables: w_t, w_t^*, $\tilde{d}_t$, $\tilde{d}_t^*$, $N_{E,t}$, $N_{E,t}^*$, $\tilde{z}_{X,t}$, $\tilde{z}_{X,t}^*$, $N_{D,t}$, $N_{D,t}^*$, $N_{X,t}$, $N_{X,t}^*$, r_t, r_t^*, $\tilde{v}_t$, $\tilde{v}_t^*$, C_t, C_t^*, Q_t, B_t, $B_{*,t}$, B_t^*, $B_{*,t}^*$. Of these, 8 are predetermined as of time t: the total numbers of firms at home and abroad, $N_{D,t}$ and $N_{D,t}^*$, the risk-free interest rates, r_t and r_t^*, and the stocks of bonds, B_t, $B_{*,t}$, B_t^*, and $B_{*,t}^*$. The model features the same 8 exogenous variables as in the financial autarky case. The 23 endogenous variables above are determined by a system of 23 equations that is identical to the system in Table 1 in the following blocs: Price Indexes, Profits, Free entry, Zero-profit export cutoffs, Share of exporting firms, Number of firms, Euler equation (shares). The 5 equations in Euler equation (bonds), Aggregate accounting, and Balanced trade are replaced by the 9 equations in Table C.1.[45]

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Table 1. Model summary, financial autarky

Price indexes	$N_{D,t}\left(\tilde{\rho}_{D,t}\right)^{1-\theta}+N_{X,t}^{*}\left(\tilde{\rho}_{X,t}^{*}\right)^{1-\theta}=1$ $N_{D,t}^{*}\left(\tilde{\rho}_{D,t}^{*}\right)^{1-\theta}+N_{X,t}\left(\tilde{\rho}_{X,t}\right)^{1-\theta}=1$
Profits	$\tilde{d}_t=\tilde{d}_{D,t}+\frac{N_{X,t}}{N_{D,t}}\tilde{d}_{X,t}$ $\tilde{d}_t^{*}=\tilde{d}_{D,t}^{*}+\frac{N_{X,t}^{*}}{N_{D,t}^{*}}\tilde{d}_{X,t}^{*}$
Free entry	$\tilde{v}_t=w_t\frac{f_{E,t}}{Z_t}$ $\tilde{v}_t^{*}=w_t^{*}\frac{f_{E,t}^{*}}{Z_t^{*}}$
Zero-profit export cutoffs[46]	$\tilde{d}_{X,t}=w_t\frac{f_{X,t}}{Z_t}\frac{\theta-1}{k-(\theta-1)}$ $\tilde{d}_{X,t}^{*}=w_t^{*}\frac{f_{X,t}^{*}}{Z_t^{*}}\frac{\theta-1}{k-(\theta-1)}$
Share of exporting firms	$\frac{N_{X,t}}{N_{D,t}}=\left(z_{\min}\right)^{k}\left(\tilde{z}_{X,t}\right)^{-k}\left[\frac{k}{k-(\theta-1)}\right]^{\frac{k}{\theta-1}}$ $\frac{N_{X,t}^{*}}{N_{D,t}^{*}}=\left(z_{\min}\right)^{k}\left(\tilde{z}_{X,t}^{*}\right)^{-k}\left[\frac{k}{k-(\theta-1)}\right]^{\frac{k}{\theta-1}}$
Number of firms	$N_{D,t}=(1-\delta)\left(N_{D,t-1}+N_{E,t-1}\right)$ $N_{D,t}^{*}=(1-\delta)\left(N_{D,t-1}^{*}+N_{E,t-1}^{*}\right)$
Euler equation (bonds)	$(C_t)^{-\gamma}=\beta\left(1+r_{t+1}\right)E_t\left[(C_{t+1})^{-\gamma}\right]$ $(C_t^{*})^{-\gamma}=\beta\left(1+r_{t+1}^{*}\right)E_t\left[(C_{t+1}^{*})^{-\gamma}\right]$
Euler equation (shares)	$\tilde{v}_t=\beta(1-\delta)E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-\gamma}\left(\tilde{v}_{t+1}+\tilde{d}_{t+1}\right)\right]$ $\tilde{v}_t^{*}=\beta(1-\delta)E_t\left[\left(\frac{C_{t+1}^{*}}{C_t^{*}}\right)^{-\gamma}\left(\tilde{v}_{t+1}^{*}+\tilde{d}_{t+1}^{*}\right)\right]$
Aggregate accounting	$C_t=w_tL+N_{D,t}\tilde{d}_t-N_{E,t}\tilde{v}_t$ $C_t^{*}=w_t^{*}L^{*}+N_{D,t}^{*}\tilde{d}_t^{*}-N_{E,t}^{*}\tilde{v}_t^{*}$
Balanced trade	$Q_tN_{X,t}\left(\tilde{\rho}_{X,t}\right)^{1-\theta}C_t^{*}=N_{X,t}^{*}\left(\tilde{\rho}_{X,t}^{*}\right)^{1-\theta}C_t$

In the equations above, it must be understood that the following, previously defined functions for average real prices and average profits/dividends from domestic and export sales are applied:

$$\begin{aligned}
\tilde{\rho}_{D,t}&\equiv\rho_{D,t}(\tilde{z}_D)=\tfrac{\theta}{\theta-1}\left(\tilde{z}_DZ_t\right)^{-1}w_t,\\
\tilde{\rho}_{D,t}^{*}&\equiv\rho_{D,t}^{*}(\tilde{z}_D)=\tfrac{\theta}{\theta-1}\left(\tilde{z}_DZ_t^{*}\right)^{-1}w_t^{*},\\
\tilde{\rho}_{X,t}&\equiv\rho_{X,t}(\tilde{z}_{X,t})=\tfrac{\theta}{\theta-1}\tau_t\left(\tilde{z}_{X,t}Z_t\right)^{-1}Q_t^{-1}w_t,\\
\tilde{\rho}_{X,t}^{*}&\equiv\rho_{X,t}^{*}(\tilde{z}_{X,t}^{*})=\tfrac{\theta}{\theta-1}\tau_t^{*}\left(\tilde{z}_{X,t}^{*}Z_t^{*}\right)^{-1}Q_tw_t^{*},
\end{aligned}$$

and

$$\begin{aligned}
\tilde{d}_{D,t}&\equiv d_{D,t}(\tilde{z}_D)=\tfrac{1}{\theta}\left[\rho_{D,t}(\tilde{z}_D)\right]^{1-\theta}C_t,\\
\tilde{d}_{D,t}^{*}&\equiv d_{D,t}^{*}(\tilde{z}_D)=\tfrac{1}{\theta}\left[\rho_{D,t}^{*}(\tilde{z}_D)\right]^{1-\theta}C_t^{*},\\
\tilde{d}_{X,t}&\equiv d_{X,t}(\tilde{z}_{X,t})=\tfrac{Q_t}{\theta}\left[\rho_{X,t}(\tilde{z}_{X,t})\right]^{1-\theta}C_t^{*}-w_t\tfrac{f_{X,t}}{Z_t},\\
\tilde{d}_{X,t}^{*}&\equiv d_{X,t}^{*}(\tilde{z}_{X,t}^{*})=\tfrac{Q_t^{-1}}{\theta}\left[\rho_{X,t}^{*}(\tilde{z}_{X,t}^{*})\right]^{1-\theta}C_t-w_t^{*}\tfrac{f_{X,t}^{*}}{Z_t^{*}}.
\end{aligned}$$

[46] These conditions hold only when $f_{X,t}$ and $f_{X,t}^{*}$ are strictly positive. If all firms export (*i.e.*, if $f_{X,t}=f_{X,t}^{*}=0$), then these conditions must be replaced with $\tilde{z}_{X,t}=\tilde{z}_{X,t}^{*}=\tilde{z}_D$. The same is true in the bond trading case.

Table 2. Business cycle data[47]

U.S.	Standard deviation	
Variable	Percentage	Relative to output
Output	1.71	1.00
Consumption	.84	.49
Investment	5.38	3.15
Capital stock	.63	.37
Net exports/output	.45	

$Corr(\text{Variable}_{t+j},\text{Output}_t)$, $j = -5, ..., 5$

Variable ↓	−5	−4	−3	−2	−1	0	1	2	3	4	5
Output	−.03	.15	.38	.63	.85	1.00	.85	.63	.38	.15	−.03
Consumption	.20	.38	.53	.67	.77	.76	.63	.46	.27	.06	−.12
Investment	.09	.25	.44	.64	.83	.90	.81	.60	.35	.08	−.14
Capital stock	−.60	−.60	−.54	−.43	−.24	.01	.24	.46	.62	.71	.72
Net exports/output	−.51	−.51	−.48	−.43	−.37	−.28	−.17	.00	.17	.30	.38

International

Contemporaneous cross correlation

	with U.S.		*within country*	
Country	Output	Consumption	Saving-investment rate	Net exports/output-output
Austria	.31	.07	.29	−.42
Finland	.02	−.01	.09	−.36
France	.22	−.18	−.04	−.17
Germany	.42	.39	.42	−.27
Italy	.39	.25	.06	−.62
Switzerland	.27	.25	.38	−.66
United Kingdom	.48	.43	.07	−.21
Europe	.70	.46		
U.S.	1.00	1.00	.68	−.36

Real exchange rate

Source	First-order autocorr.	Std. dev. (ratio to output)	Corr. with relative consumption
CKM	.83	5.50	−.35
BF	.80	4.81	

[47] Source: Backus, Kehoe, and Kydland (1992) unless otherwise noted.

Table 3. Model-generated moments, BKK productivity process

Standard deviation

Variable	Percentage	Relative to y $(Py/\tilde{P})$
y $(Py/\tilde{P})$	.9961 (.7950)	1.00 (1.00)
C $(PC/\tilde{P})$	.6929 (.4681)	.6956 (.5888)
$\tilde{v}N_E$ $(P\tilde{v}N_E/\tilde{P})$	3.6356 (3.5002)	3.6498 (4.5754)
N_E	3.6314	3.6456 (relative to y)
$\tilde{v}N_D$ $(P\tilde{v}N_D/\tilde{P})$	.3316 (.3795)	.3329 (.4773)
N_D	.2697	.2707 (relative to y)
TB/y	1.0178	1.0218
TB $(PTB/\tilde{P})$	.1571 (.2968)	.1577 (.3733)
$\tilde{Q}$	.0278	.0279 (relative to y)

$Corr(\text{Variable}_{t+j}, y_t)$, $j = -5, ..., 5$ (corr. with $P_t y_t/\tilde{P}_t$ if variable deflated with $\tilde{P}$)

Variable ↓	−5	−4	−3	−2	−1	0	1	2	3	4	5
y	−.02	.11	.27	.47	.71	1.00	.71	.47	.27	.11	−.02
$Py/\tilde{P}$	−.02	.10	.27	.47	.71	1.00	.71	.47	.27	.10	−.02
C	−.05	.07	.22	.41	.64	.92	.69	.49	.32	.17	.05
$PC/\tilde{P}$	−.08	.03	.18	.37	.59	.87	.66	.48	.32	.19	.08
$\tilde{v}N_E$	.06	.16	.29	.44	.62	.84	.50	.24	.05	−.08	−.17
$P\tilde{v}N_E/\tilde{P}$	.08	.18	.31	.46	.65	.86	.49	.22	.02	−.12	−.21
N_E	.07	.17	.29	.44	.62	.84	.49	.23	.04	−.10	−.19
$\tilde{v}N_D$	−.42	−.38	−.31	−.20	−.03	.19	.46	.61	.67	.67	.61
$P\tilde{v}N_D/\tilde{P}$	−.34	−.37	−.39	−.39	−.37	−.33	.06	.30	.45	.51	.52
N_D	−.41	−.38	−.31	−.21	−.05	.16	.44	.59	.65	.65	.60
TB/y	−.01	−.13	−.29	−.48	−.71	−.99	−.66	−.41	−.20	−.05	.07
TB	−.17	−.17	−.16	−.14	−.11	−.06	.20	.34	.40	.39	.34
$PTB/\tilde{P}$	−.05	−.13	−.23	−.36	−.50	−.67	−.33	−.10	.05	.14	.18

Table 3, continued

$Corr(\tilde{Q}_{t+j}, \tilde{Q}_t)$, $j = -5, ..., 5$										
-5	-4	-3	-2	-1	0	1	2	3	4	5
.19	.37	.55	.73	.89	1.00	.89	.73	.55	.37	.19

Contemporaneous cross correlation

y^*, y $(P^*y^*/\tilde{P}^*, Py/\tilde{P})$	C^*, C $(P^*C^*/\tilde{P}^*, PC/\tilde{P})$	Saving rate, "Investment" rate
.44 (.21)	.92 (.86)	.95
$\left(PC/\tilde{P}\right)/(P^*C^*/\tilde{P}^*), \tilde{Q}$	$C/C^*, Q$	
$-.99$	.71	

Table 4. Model-generated moments, Baxter productivity process

Standard deviation

Variable	Percentage	Relative to y ($Py/\tilde{P}$)
y ($Py/\tilde{P}$)	1.0521 (.7195)	1.00 (1.00)
C ($PC/\tilde{P}$)	.9136 (.5749)	.8683 (.7990)
$\tilde{v}N_E$ ($P\tilde{v}N_E/\tilde{P}$)	3.6873 (3.3538)	3.5047 (4.6613)
N_E	3.6489	3.4682 (relative to y)
$\tilde{v}N_D$ ($P\tilde{v}N_D/\tilde{P}$)	.3697 (.4693)	.3514 (.6522)
N_D	.3033	.2883 (relative to y)
TB/y	1.3288	1.2630
TB ($PTB/\tilde{P}$)	.4662 (.7583)	.4431 (1.0539)
$\tilde{Q}$	.1297	.1233 (relative to y)

$Corr(\text{Variable}_{t+j}, \text{y}_t)$, $j = -5, ..., 5$ (corr. with $P_t y_t/\tilde{P}_t$ if variable deflated with $\tilde{P}$)

Variable ↓	-5	-4	-3	-2	-1	0	1	2	3	4	5
y	.01	.14	.31	.50	.73	1.00	.73	.50	.31	.14	.01
$Py/\tilde{P}$	.03	.17	.33	.53	.75	1.00	.75	.53	.33	.17	.03
C	.01	.14	.30	.50	.73	1.00	.73	.49	.30	.14	.01
$PC/\tilde{P}$	.02	.16	.32	.52	.74	1.00	.74	.52	.33	.17	.04
$\tilde{v}N_E$	.12	.22	.34	.48	.64	.82	.51	.27	.08	$-.07$	$-.17$
$P\tilde{v}N_E/\tilde{P}$	.18	.27	.37	.48	.59	.70	.41	.18	.01	$-.12$	$-.21$
N_E	.13	.23	.35	.49	.64	.81	.50	.25	.06	$-.08$	$-.18$
$\tilde{v}N_D$	$-.36$	$-.30$	$-.21$	$-.09$	.08	.30	.51	.63	.67	.66	.61
$P\tilde{v}N_D/\tilde{P}$	$-.29$	$-.33$	$-.36$	$-.38$	$-.40$	$-.40$	$-.04$	.21	.38	.47	.51
N_D	$-.36$	$-.32$	$-.24$	$-.13$	.02	.21	.45	.59	.65	.66	.62
TB/y	$-.07$	$-.19$	$-.34$	$-.51$	$-.72$	$-.95$	$-.65$	$-.40$	$-.20$	$-.04$	.08
TB	$-.16$	$-.21$	$-.26$	$-.32$	$-.39$	$-.45$	$-.19$	$-.01$	.13	.21	.26
$PTB/\tilde{P}$	$-.15$	$-.23$	$-.32$	$-.41$	$-.51$	$-.62$	$-.34$	$-.13$	.03	.14	.21

Table 4, continued

$Corr(\tilde{\mathsf{Q}}_{t+j}, \tilde{\mathsf{Q}}_t)$, $j = -5, ..., 5$

-5	-4	-3	-2	-1	0	1	2	3	4	5
.01	.14	.30	.49	.73	1.00	.73	.49	.30	.14	.01

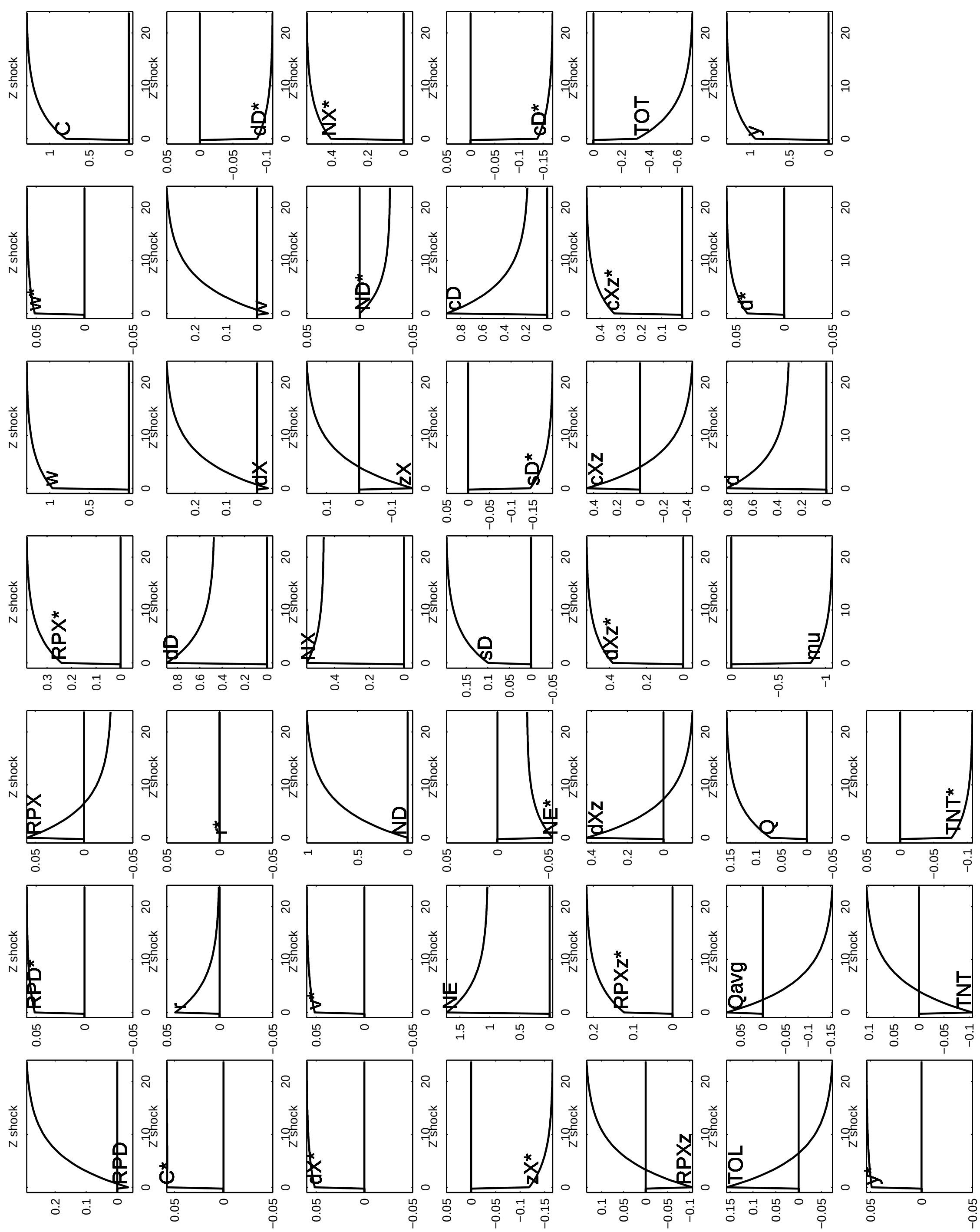
Contemporaneous cross correlation

y^*, y $(P^*y^*/\tilde{P}^*, Py/\tilde{P})$	C^*, C $(P^*C^*/\tilde{P}^*, PC/\tilde{P})$	Saving rate, "Investment" rate
.32 (.44)	.30 (.46)	.47
$\left(PC/\tilde{P}\right)/(P^*C^*/\tilde{P}^*), \tilde{Q}$	$C/C^*, Q$	
$-.98$	.13	

Table C.1. Model summary, bond trading

Euler equations (bonds)	$(C_t)^{-\gamma}(1+\eta B_{t+1}) = \beta(1+r_{t+1})E_t\left[(C_{t+1})^{-\gamma}\right]$ $(C_t)^{-\gamma}(1+\eta B_{*,t+1}) = \beta\left(1+r^*_{t+1}\right)E_t\left[\frac{Q_{t+1}}{Q_t}(C_{t+1})^{-\gamma}\right]$ $(C^*_t)^{-\gamma}\left(1+\eta B^*_{t+1}\right) = \beta(1+r_{t+1})E_t\left[\frac{Q_t}{Q_{t+1}}\left(C^*_{t+1}\right)^{-\gamma}\right]$ $(C^*_t)^{-\gamma}\left(1+\eta B^*_{*,t+1}\right) = \beta\left(1+r^*_{t+1}\right)E_t\left[\left(C^*_{t+1}\right)^{-\gamma}\right]$
Net foreign assets	$B_{t+1} + Q_t B_{*,t+1} = (1+r_t)B_t + Q_t(1+r^*_t)B_{*,t}$ $+\frac{1}{2}(w_t L - Q_t w^*_t L^*) + \frac{1}{2}\left(N_{D,t}\tilde{d}_{D,t} - N^*_{D,t}Q_t\tilde{d}^*_{D,t}\right)$ $+\frac{1}{2}\left(N_{X,t}\tilde{d}_{X,t} - N^*_{X,t}Q_t\tilde{d}^*_{X,t}\right)$ $-\frac{1}{2}\left(N_{E,t}\tilde{v}_t - N^*_{E,t}Q_t\tilde{v}^*_t\right) - \frac{1}{2}(C_t - Q_t C^*_t)$
Bond market equilibrium	$B_{t+1} + B^*_{t+1} = 0$ $B_{*,t+1} + B^*_{*,t+1} = 0$
Labor market equilibrium	$L = \frac{\theta-1}{w_t}\left(N_{D,t}\tilde{d}_{D,t} + N_{X,t}\tilde{d}_{X,t}\right) + \frac{1}{Z_t}(\theta N_{X,t}f_{X,t} + N_{E,t}f_{E,t})$ $L^* = \frac{\theta-1}{w^*_t}\left(N^*_{D,t}\tilde{d}^*_{D,t} + N^*_{X,t}\tilde{d}^*_{X,t}\right) + \frac{1}{Z^*_t}\left(\theta N^*_{X,t}f^*_{X,t} + N^*_{E,t}f^*_{E,t}\right)$

Figure 1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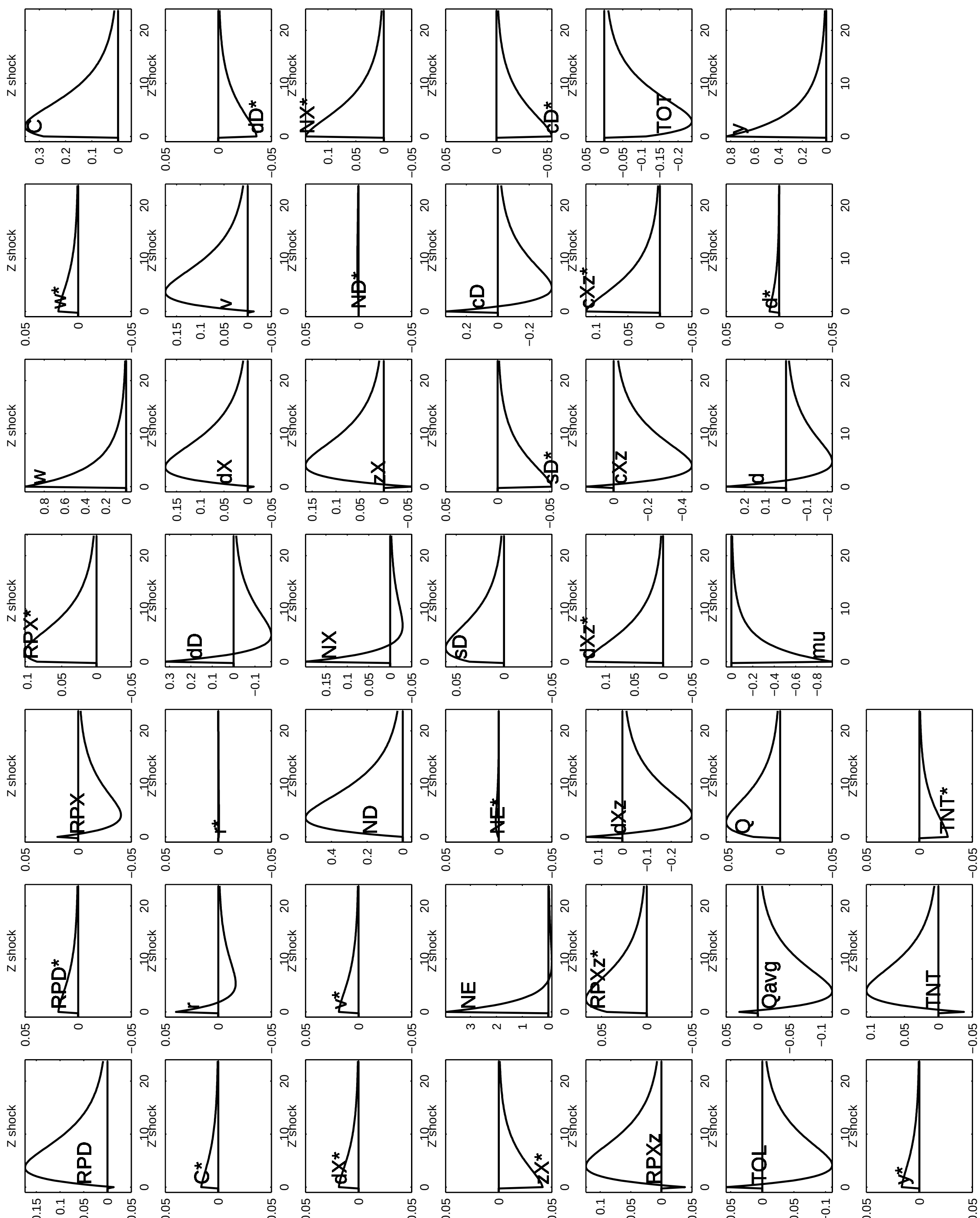
RPD
RPD*
RPX
RPX*
w
w*
C
C*
r
r*
dD
dX
w
dD*
dX*
v*
ND
NX
zX
ND*
NX*
zX*
NE
NE*
sD
sD*
cD
sD*
RPXz
RPXz*
dXz
dXz*
cXz
cXz*
TOT
TOL
Qavg
Q
mu
d
d*
y
y*
TNT
TNT*

Figure 2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RPD*
RPX
RPX*
w
w*
C
C*
r
r*
dD
dX
v
dD*
dX*
v*
ND
NX
zX
ND*
NX*
zX*
NE
NE*
sD
sD*
cD
cD*
RPXz
RPXz*
dXz
dXz*
cXz
cXz*
TOT
TOL
Qavg
Q
mu
d
d*
y
y*
TNT
TNT*

Figure 3

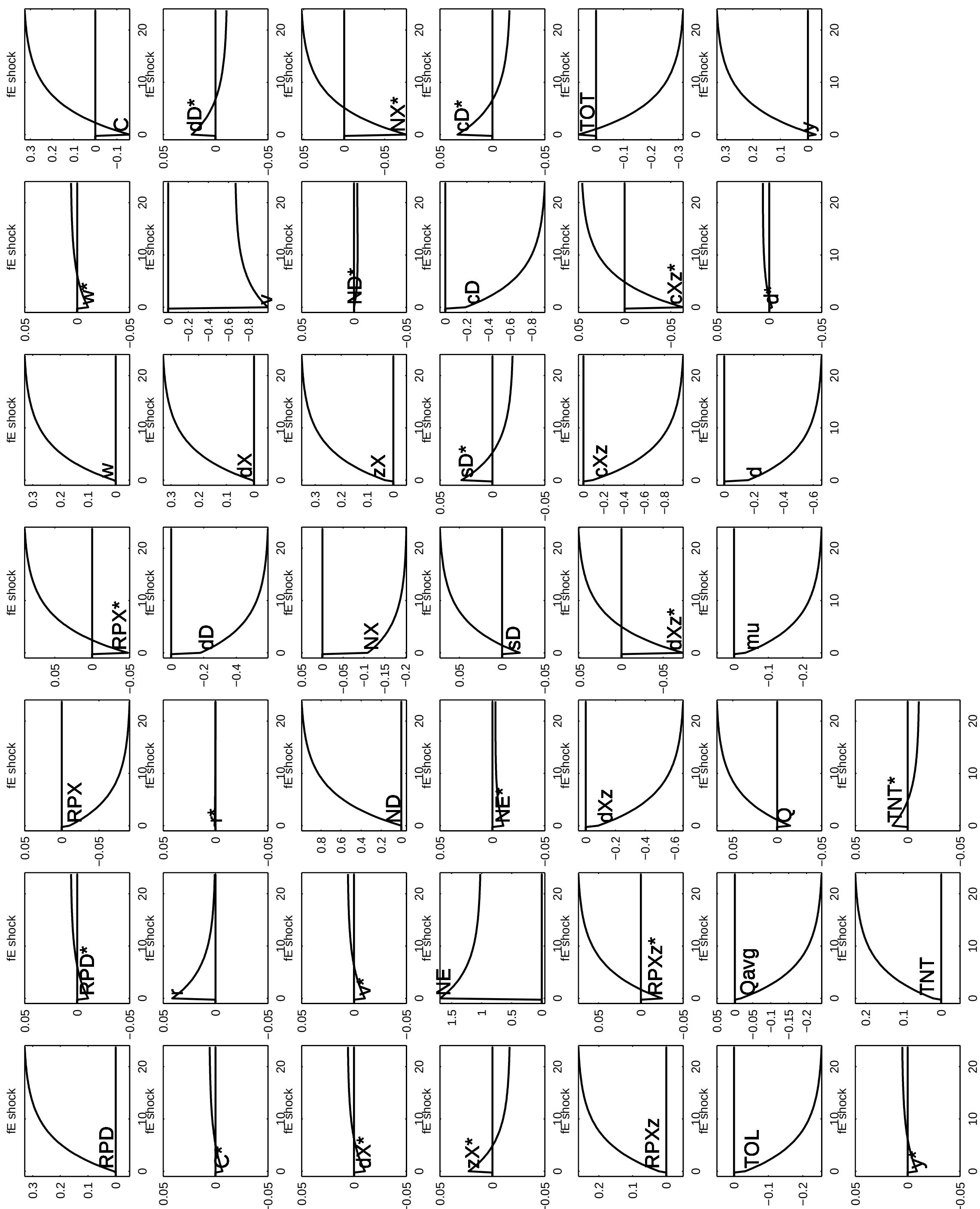
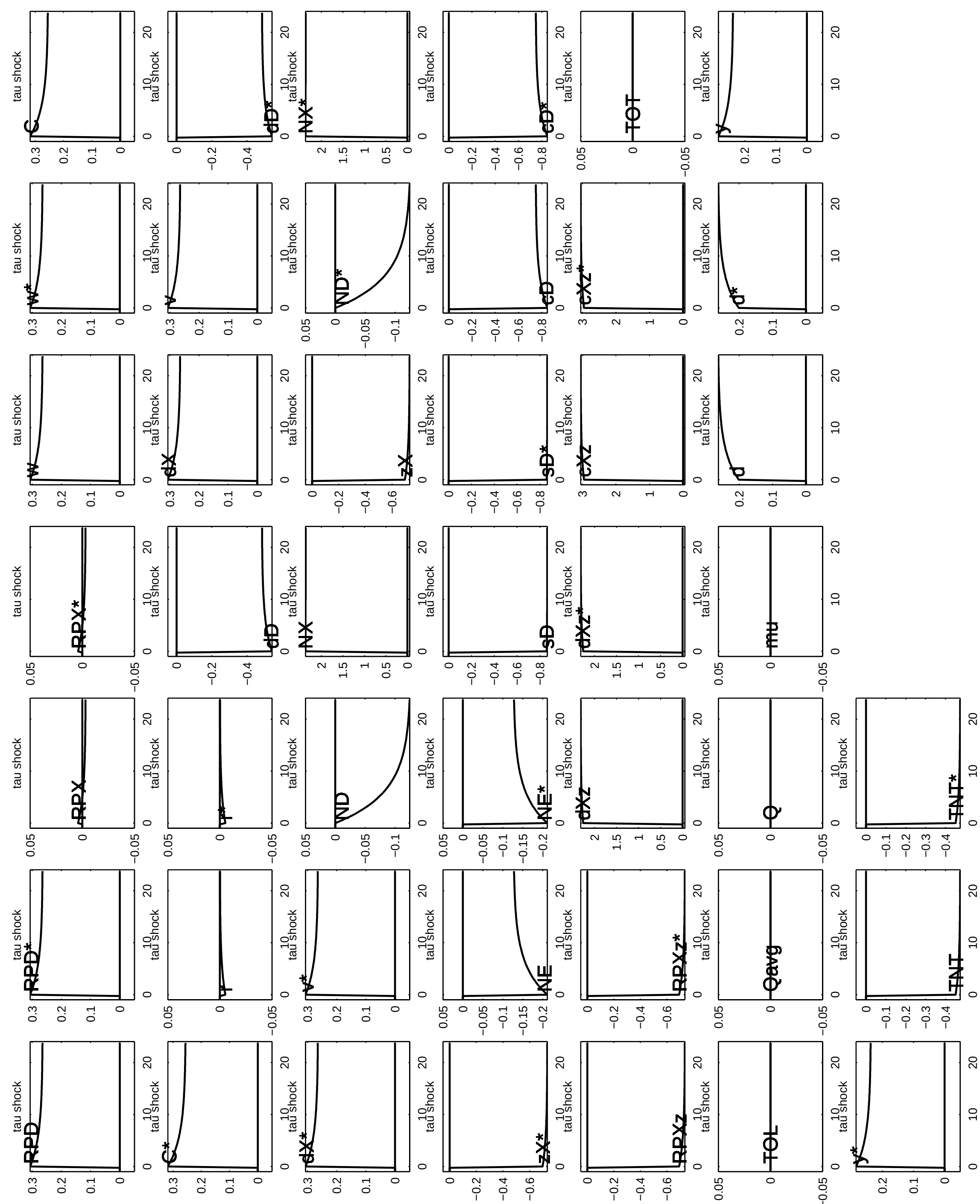
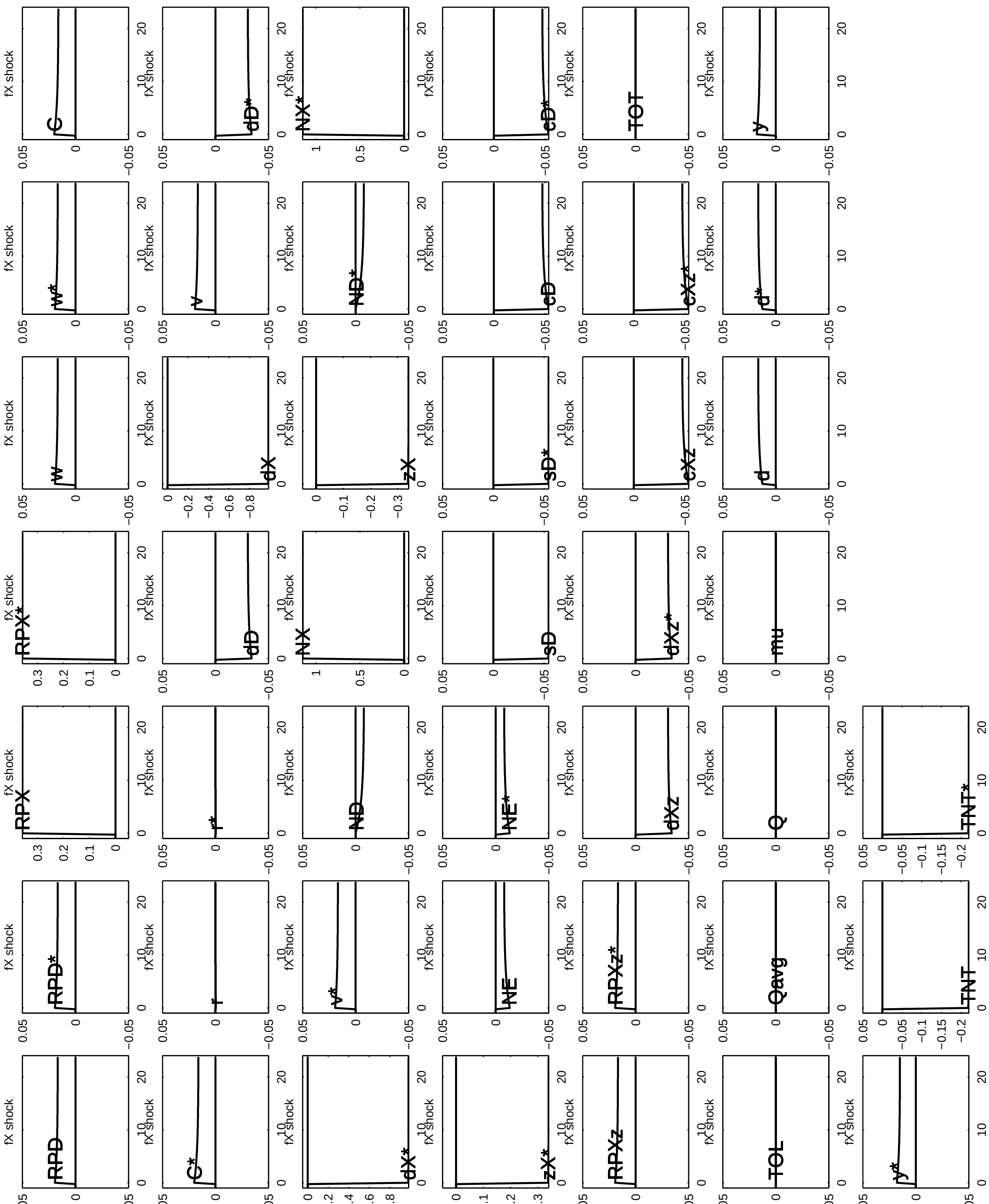


Figure 4



tau shock
C
W*
W
RPX*
RPX
RPD*
RPD
dD*
V
dX
dD
T*
T
C*
NX*
ND*
zX
NX
ND
V*
dX*
cD*
cD
sD*
sD
NE*
NE
zX*
TOT
cXZ*
cXZ
dXz*
dXz
RPXz*
RPXz
y
d*
d
mu
Q
Qavg
TOL
TNT*
TNT
y*

Figure 5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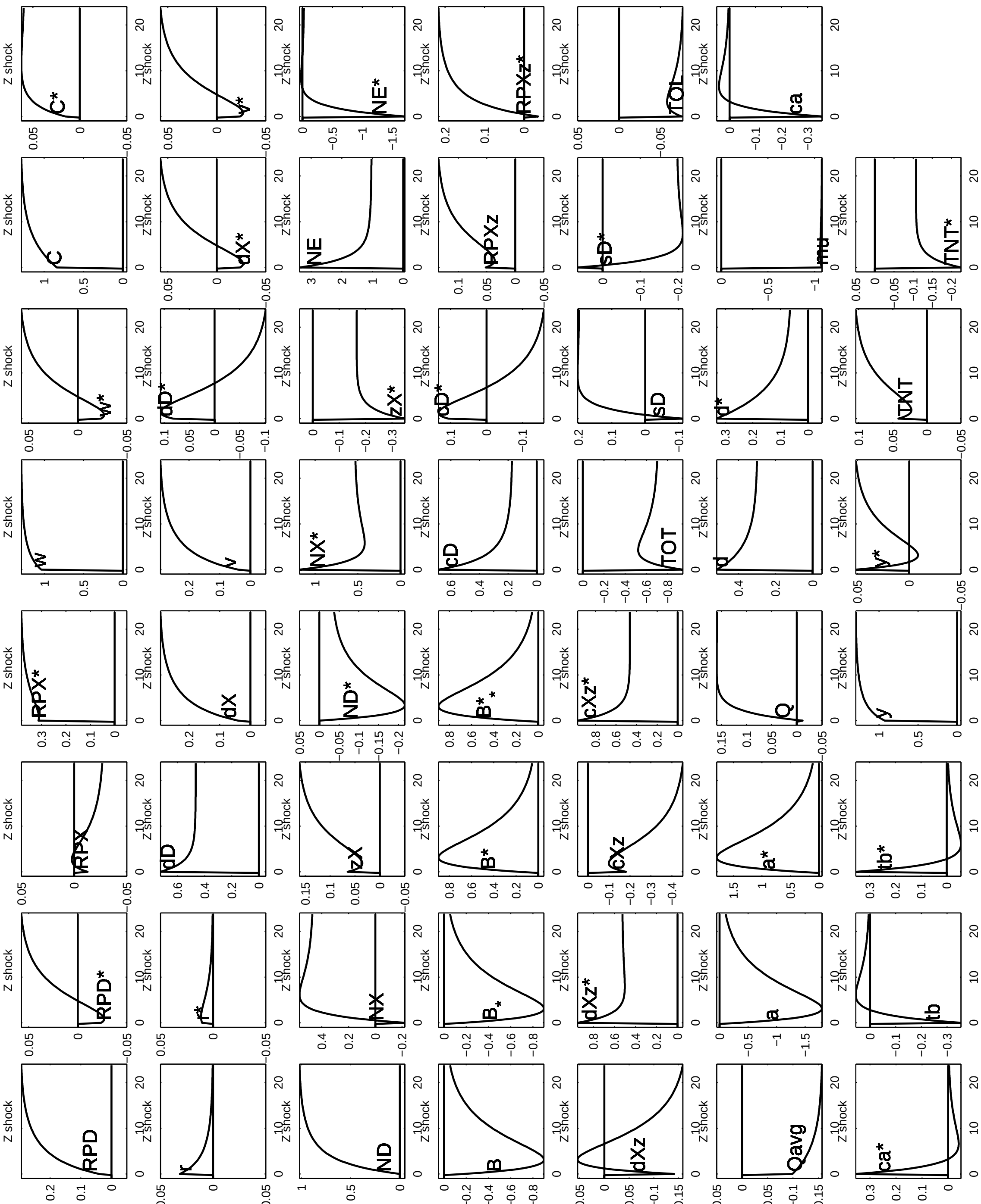
RPD*
RPX
RPX*
w
w*
C
C*
r
r*
dD
dX
v
dD*
dX*
v*
ND
NX
zX
ND*
NX*
zX*
NE
NE*
sD
cD
sD*
cD*
RPXz
RPXz*
dXz
dXz*
cXz
cXz*
TOT
TOL
Qavg
Q
mu
d
d*
y
y*
TNT
TNT*

Figure 6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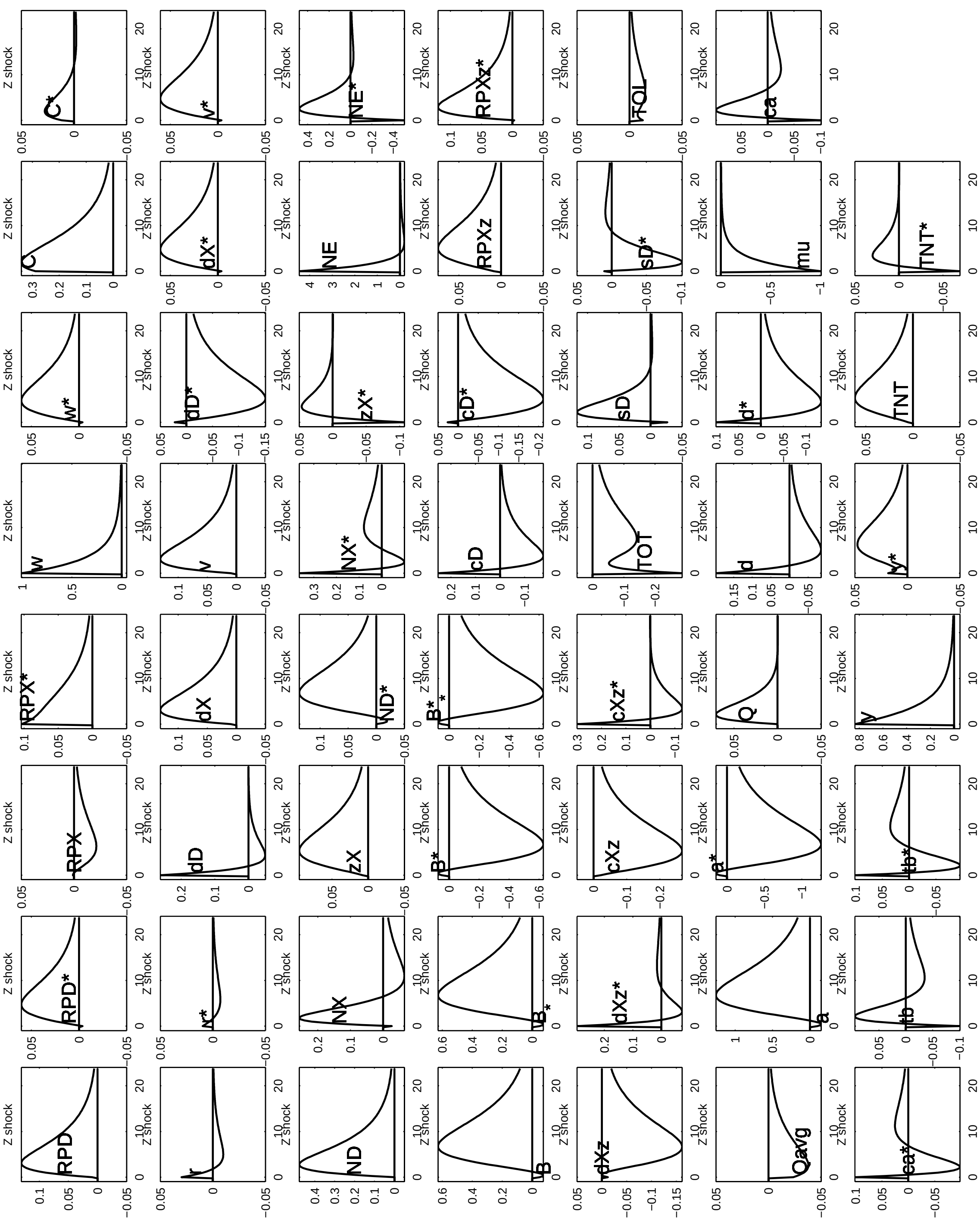
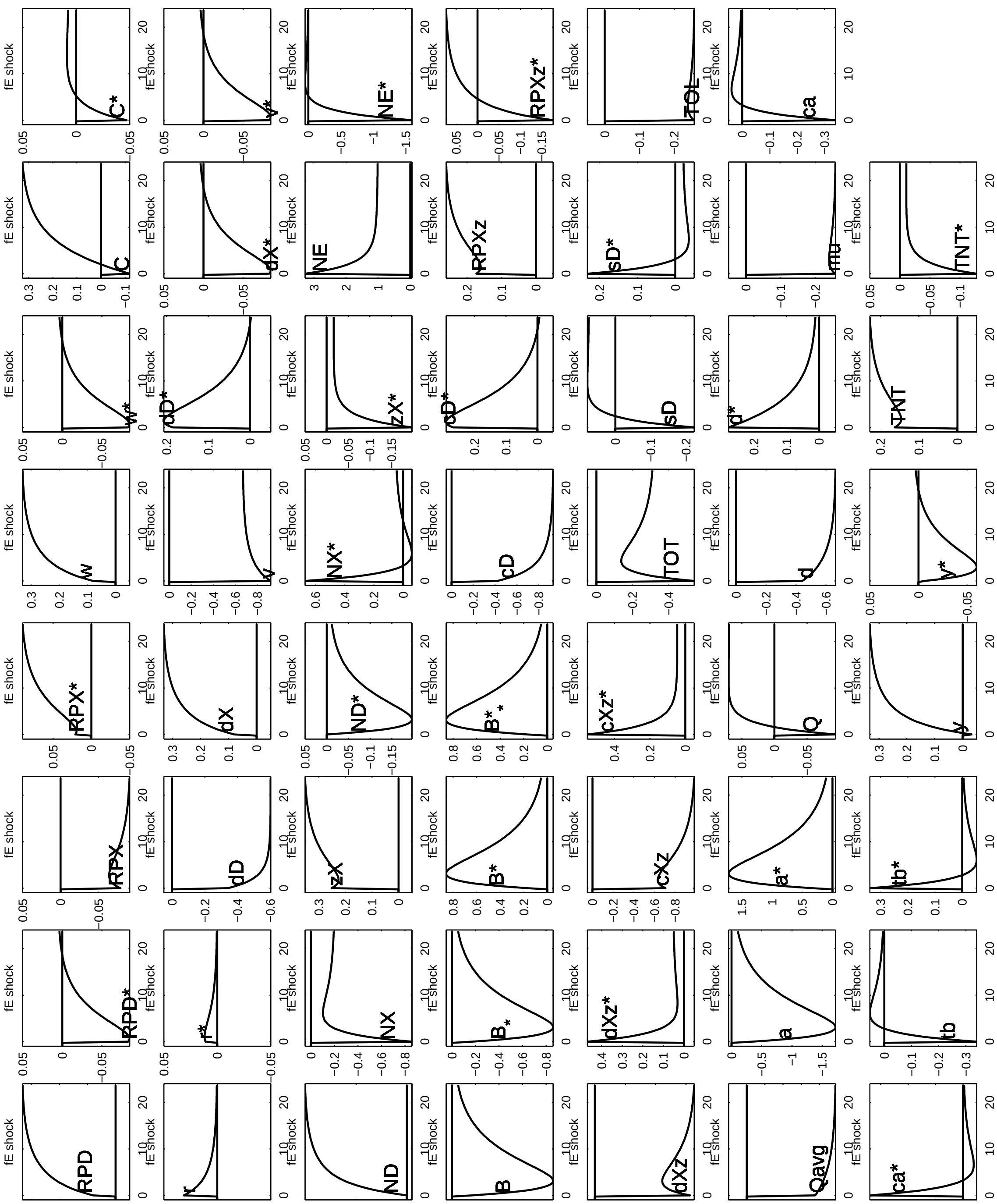
RPD
r
ND
B
dXz
Qavg
ca*
RPD*
r*
NX
B*
dXz*
a
tb
RPX
dD
zX
cXz
a*
tb*
RPX*
dX
ND*
cXz*
Q
y
w
v
NX*
cD
TOT
d
y*
w*
dD*
zX*
cD*
sD
d*
TNT
C
dX*
NE
RPXz
sD*
mu
TNT*
C*
NE*
RPXz*
TOL
ca

Figure 7



Z shock
RPD
RPD*
RPX
RPX*
w
w*
C
C*
r
r*
dD
dX
v
dD*
dX*
v*
ND
NX
zX
ND*
NX*
zX*
NE
NE*
B
B*
cD
cD*
RPXz
RPXz*
dXz
dXz*
cXz
cXz*
TOT
sD
sD*
TOL
Qavg
a
a*
Q
d
d*
mu
ca
ca*
tb
tb*
y
y*
TNT
TNT*

Figure 8


fE shock
RPD
RPD*
RPX
RPX*
w
w*
C
C*
r
r*
dD
dX
v
dD*
dX*
v*
ND
NX
zX
ND*
NX*
zX*
NE
NE*
B
B*
cD
cD*
RPXz
RPXz*
dXz
dXz*
cXz
cXz*
TOT
sD
sD*
TOL
Qavg
a
a*
Q
d
d*
mu
ca
ca*
tb
tb*
y
y*
TNT
TNT*