Econ. 240B, Second Half | Daniel McFadden, 2001

EXERCISE 4. DEMAND SYSTEMS FOR FACTORS OF PRODUCTION

(To be handed in on Nov. 23)

Producers operating in competitive input markets will choose factor inputs to minimize costs at each output level, given factor prices. Suppose manufacturers utilize N unit inputs $\mathbf{z} = (z_1,..,z_N)$ with input prices $\mathbf{p} = (p_1,...,p_N)$, and have a Diewert (generalized Leontief) unit cost function,

$$C(\mathbf{p}) = 0.5 \cdot \sum_{i=1}^{N} \sum_{j=1}^{N} \alpha_{ij}(p_i p_j)^{1/2} + \sum_{i=1}^{N} p_i \gamma_i$$

where the α's are nonnegative parameters with $\alpha_{ij} = \alpha_{ji}$ and the γ_i are disturbances. By Shephard's lemma, the unit input demand functions are given by the derivatives of the unit cost function with respect to the input prices:

$$z_n = \sum_{j=1}^{N} \alpha_{nj}(p_j/p_n)^{1/2} + \gamma_n$$

for n = 1,...,N. This is a system of equations with the property that if factor markets are indeed competitive and the Diewert specification is correct, then the disturbances should have zero conditional means, given the explanatory variables. However, there is likely to be correlation of the disturbances across factors, because optimization errors of producers that lead one unit input to be less than optimal will require some other input to be more than optimal.

The file klem.dat contains 25 annual observations from 1947 through 1971 for U.S. manufacturing industries. The first row of the file gives the variable names, corresponding to the list below. Variables are separated by spaces, and can be read in free format.

VARIABLE DESCRIPTION

YEAR	Two-digit year, beginning with 47 and ending with 71
QY	Quantity of Gross Output (bil. current dol.)
PY	Price Index of Gross Output (1947 = 1)
QK	Quantity of Capital Services (bil. current dol.)
PK	Rental Price Index for Capital Services (1947 = 1)
QL	Quantity of Labor Input (bil. current dol.)
PL	Price Index for Labor Input (1947 = 1)
QE	Quantity of Aggregate Energy Input (bil. current dol.)
PE	Price Index for Aggregate Energy Input (1947 = 1)
QM	Quantity of Non-Energy Intermediate Materials (bil. current dol.)
PM	Price Index for Non-Energy Intermediate Materials (1471 = 1)

QV	Quantity of Value-Added Output (bil. current dol.)
PV	Price Index of Value-Added Output (1947 = 1)
USPOP	US Population (mil.)
CIVPOP16	US Noninstitutionalized Population Age 16+ (mil.)
SALETAX	Effective Rate of Sales and Excise Taxation (fraction)
PROPTAX	Effective Rate of Property Taxation (fraction)
GDP	Gross Domestic Product (bil. current dol.)
PGDP	Price Index for GDP (1992 = 100)

These data are reproduced from a classic Berndt-Wood (1975) paper on demand for energy. The factor price indices are normalized to be one in 1947, and the factor inputs in that year are in billions of 1947 dollars. Then, in each year, the product of a factor price times a factor quantity equals payments to that factor in billions of current dollars.

a. Using these data, and assuming constant returns to scale, estimate the Diewert system of unit factor demand functions for labor, capital, energy, and materials. Impose the symmetry conditions required by economic theory.

b. Test the symmetry hypothesis.

c. Test the hypothesis of input substitutability (i.e., off-diagonal α_{ij} terms zero).

d. (extra credit) Estimate a translog system for the unit demand functions, starting from the unit cost function

$$\log C(\mathbf{p}) = \sum_{i=1}^{N} \gamma_i \log(p_i) + 0.5 \cdot \sum_{i=1}^{N} \sum_{j=1}^{N} \alpha_{ij}(\log(p_i)\log(p_j)) + \sum_{i=1}^{N} \log(p_i)\, g_i$$

with the identifying restrictions $\alpha_{ij} = \alpha_{ji}$ and $\alpha_{ii} = 0$, and the restrictions

$$\sum_{i=1}^{N} \gamma_i = 1, \quad \sum_{i=1}^{N} g_i = 0, \text{ and } \sum_{i=1}^{N} \alpha_{ij} = 0 = \sum_{j=1}^{N} \alpha_{ij}$$

imposed by homogeneity of degree zero in prices. Test the hypothesis of Cobb-Douglas unit cost functions (i.e., $\alpha_{ij} = 0$ for all i,j). The sign of the α_{ij} indicates whether the factors are substitutes (+) or complements (-). Is energy a complement or a substitute for capital?