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Complementarities, Momentum, and the Evolution of Modern Manufacturing

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In the 19th century, the railroad and telegraph were at the center of a set of technological advances, physical investments and managerial innovations that transformed American industry (Alfred Chandler). Later, the automobile and telephone played a similar role in another transformation.

Today, the high-tech industries include computers, telecommunications and electronics. Working on our remarkably powerful computers (even as they rapidly become obsolete), co-authoring papers by electronic mail and fax, and conversing on our portable cellular telephones, we are struck by what appears to be a self-supporting and reinforcing dynamic to the technological improvements across the electronics industries. An advance almost anywhere in the sector seems to call forth more advances across the sector.

These advances are occurring contemporaneously with a broad pattern of other changes, not only in the electronics industries, but in manufacturing more generally, and not just in hardware, but in methods and organization as well. A new paradigm has begun to emerge. In contrast to traditional manufacturing firms, modern firms frequently (1) make greater use of flexible, programmable

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equipment and of computer-aided design and manufacturing technologies, (2) have fewer job classifications, (3) offer more varieties of their major products and/or update their product lines more frequently, (4) put more emphasis on speed in order processing, production and delivery, (5) hold much lower inventories of intermediate and finished goods, (6) rely on subcontractors to supply a greater proportion of the total value added, and (7) overlap design, product and process engineering to speed the introduction of new products. These features of modern manufacturing firms encompass technology choices, marketing strategies, personnel policies, supplier relations, lines of internal communications, and other operational policies in a far-reaching and coherent pattern whose existence, in the words of Michael Piore, poses a “challenge to economic theory.”

In a recent paper (Milgrom and Roberts), two of us proposed a theory to explain the emergence of this new paradigm, arguing that the various characteristics and activities described are mutually complementary and so tend to be adopted together, with each making the others more attractive. In that theory, the falling costs of high-speed data communication, data processing, and flexible, multi-task equipment lead to increases in the directly affected activities, which through a web of complementarities then lead to increases in a set of related activities as well.

Although the costs of flexible machinery and data communication and processing surely have fallen substantially in recent decades, an analysis which takes these changes as exogenous does not constitute a full explanation of the pattern of change that we observe. One must ask: Why are these particular costs falling relative to others? Here we enrich the Milgrom-Roberts analysis by adding a dynamic to it: innovations in the manufacture of basic inputs both arise in response to a growing market for those inputs and simultaneously encourage that growth. This reflects a fundamental complementarity between the level of any activity and investments that reduce its marginal cost. In our model, innovation results in falling marginal costs, which lead to increasing usage of the inputs, which in turn leads to increasing investments in the development of complementary techniques, which further raise the underlying demand for the inputs, leading to more innovations, and so on, just as we have noted within the electronics industry itself.

Significantly, just as this verbal sketch requires no assumptions about the presence or absence of returns to scale, neither does our formal analysis: The momentum of the system of changes we analyze results entirely from the positive feedback effects that each of a group of core activities and practices has on the *other* activities and practices in the group.[1]

Our model also adds to the Milgrom-Roberts model groups of "additional" activities each of which may interact with any one core activity in a very general way but which are not themselves part of the *mutually complementary core group*.

Our formal analysis is summarized in two theorems. Most important is the *Momentum Theorem*, which asserts that once the system begins along a path of growth of the core variables, it will continue forever along that path or, more realistically, until unmodeled forces disturb the system. Second, the *Reduced Forms Theorem* concerns the set of models that have a reduced form to which the Momentum Theorem applies.

I. The Downstream Industry Model and the Reduced Forms Theorem

We suppose that there is a representative firm in the "downstream manufacturing industry" whose profit function in period t depends on the current state of (public) knowledge, represented by a vector $\theta(t)\in\mathbb{R}^k$, on a vector of core decision variables $x = (x^I,x^O)\in\mathbb{R}^{m+n}$ where the variables $x^I = (x_1,\ldots,x_m)$ denote purchased *inputs* and the variables $x^O = (x_{m+1},\ldots,x_{m+n})$ denote the *other* core decision variables, and finally on various variables outside the core group and denoted by $\{y^i(t);1\leq i\leq m+n\}$. The firm's payoff takes the form:

$$\rho(x(t),\theta(t)) + \sum_j \phi_j(x_j(t),y^j(t)) - R(x_I(t)),$$

where $R(x_I(t))$ is the amount paid for inputs. If ρ is smooth, the assumptions that the components of x are mutually complementary and that their effectiveness is enhanced by technical knowledge are represented by the inequalities $\partial^2\rho/\partial x_i\partial x_j \geq 0$ for all $i \neq j$ and $\partial^2\rho/\partial x_i\partial\theta_j \geq 0$ for all i and j, where $1 \leq i,j \leq m+n$. That is, the marginal product of any component of x is nondecreasing in the levels of the other arguments and in the level of technological knowledge. More generally, the first assumption is that ρ is *supermodular*, that is, for all x,x', and θ, $\rho(x,\theta)+\rho(x',\theta) \leq \rho(\mathrm{Max}(x,x'),\theta) + \rho(\mathrm{Min}(x,x'),\theta)$, where Max and Min are taken component-by-component. The second is that ρ has *increasing differences*, that is, if for all $x \geq x'$, the difference $\rho(x,\theta) - \rho(x',\theta)$ is non-decreasing in θ. We emphasize that there are no assumptions made about the own second partial derivatives, $\partial^2\rho/\partial x_i^2$, so that the profit function may be convex in x_i over some ranges and concave over others. No assumptions are made about returns to scale.

The y^i's represent other variables, each of which interacts with at most one core variable. For example, if one of the core decision variables x_i is the level of inventories to hold, the y^i vector might include decisions about where to hold the inventory or how large the storage room should be. The restriction is that none of these should affect the returns to other core variables, such as the flexibility of equipment or the number of job classifications.

We assume that each decision variable x_i or y^i is constrained to lie in some compact set $S(x_i)$ or $S(y^i)$, which may be finite or infinite. Our main conclusion about these non-core variables is that their presence has no qualitative effect on the analysis.

Theorem 1 (**Reduced Forms**). Suppose ρ and each ϕ_j (j = 1,...,m+n) is continuous. Then the reduced form (gross) profit function of the manufacturing sector given by

$$\pi(x,\theta) = \text{Max}_y\ \rho(x,\theta) + \textstyle\sum_j \phi_j(x_j,y^j), \qquad (*)$$

is continuous, supermodular in x, and has increasing differences in (x,θ).

Proof. Let $G(x) \equiv \text{Max} \sum_j \phi_j(x_j,y^j)$. Then, because of the separability of the y variables, $G(z)+G(z') = G(\text{Max}(z,z')) + G(\text{Min}(z,z'))$. So, for all θ, $[\pi(z,\theta)+\pi(z',\theta)] - [\pi(\text{Max}(z,z'),\theta) + \pi(\text{Min}(z,z'),\theta)] = [\rho(z,\theta)+\rho(z',\theta)] + [G(z)+G(z')] - [\rho(\text{Max}(z,z'),\theta) + \rho(\text{Min}(z,z'),\theta)] - [G(\text{Max}(z,z')) + G(\text{Min}(z,z'))] = [\rho(z,\theta)+\rho(z',\theta)] - [\rho(\text{Max}(z,z'),\theta) + \rho(\text{Min}(z,z'),\theta)] \leq 0$. The last inequality holds because $\rho(x,\theta)$ is supermodular in x for any given θ. Finally, from $\pi(z,\theta)-\pi(z',\theta) = \rho(z,\theta) - \rho(z',\theta) + G(z) - G(z')$, we conclude that $\pi(x,\theta)$ should also have increasing differences in (x,θ) since $\rho(x,\theta)$ has increasing differences in (x,θ). □

II. The Upstream Industry and the Contracting Equilibrium

We also suppose that there is a representative firm in the "upstream," input producing sector. Its profit function is $R(x_I(t))-C(x^I(t),T(t),\eta(t))$, where $\eta(t)$ is the "knowledge" vector describing know-how in the input industry in period t and T(t) is the level of technology it uses in producing its output. We assume that -C is supermodular in (x,T) and has increasing differences in (x,T) and η. Thus, the core aspects of the technology are separable or complementary ($\partial^2 C/\partial T_k \partial T_j \leq 0$) and there are no diseconomies of scope in producing the various inputs ($\partial^2 C/\partial x^I_i \partial x^I_j \leq 0$). Also, increases in both public knowledge $\eta(t)$ and the firm's own technology (T(t)) reduce its marginal cost of production: $\partial^2 C/\partial x^I_j \partial T_k \leq 0$ and $\partial^2 C/\partial x^I_j \partial \eta_k \leq 0$ for all j and k. Finally, accumulated public knowledge is assumed to reduce the marginal cost of technological improvements: $\partial^2 C/\partial T_j \partial \eta_k \leq 0$.[2]

Given the limited space, we set aside the model of consumers and questions of the nature and existence of equilibrium. Rather, we assume that the upstream firms and their suppliers arrange terms that are efficient for themselves, ignoring whatever effects their activities may have on the economy-wide accumulation of knowledge. This latter assumption – that firms ignore the effect of their efforts on public knowledge – is indicative of a free rider problem and is most reasonable when the typical firm is not too large. Then, the industry equilibrium involves choosing x and T to maximize the objective:

$$\pi(x(t),\theta(t)) - C(x^I(t),T(t),\eta(t)) \ , \qquad (**)$$

where π is the reduced form profit function given in (*).

III. Knowledge Accumulation Dynamics and the Momentum Theorem

The knowledge $\theta(t)$ and $\eta(t)$ at date t is assumed to be freely available to all firms. Suppose that $\theta(t+1) = f(\theta(t),\eta(t),x(t),T(t))$, and $\eta(t+1) = g(\theta(t),\eta(t),x(t),T(t))$, where both f and g are non-decreasing functions. That is, higher levels of the core activities in the downstream sector and higher levels of technology in the upstream sector combine with a higher initial state of knowledge today to produce a higher state of knowledge tomorrow. The model incorporates the possibility that learning-by-doing and high levels of activity in one industry increase learning in the other.

Theorem 2 (**Momentum**). Suppose that for every given value of (θ,η), there is a unique (x,T) that maximizes (**). If there is any time t such that $\theta(t) \geq \theta(t-1)$ and $\eta(t) \geq \eta(t-1)$, then for all times $s \geq t$, $x(s) \geq x(s-1)$, $T(s) \geq T(s-1)$, $\theta(s) \geq \theta(s-1)$ and $\eta(s) \geq \eta(s-1)$.

Proof. By Theorem 1, $\pi(x,\theta)-C(x^I,T,\eta)$ is supermodular in (x,T) and has increasing differences in $(x,T;\theta,\eta)$. Therefore, given our hypothesis that $\theta(t) \geq \theta(t-1)$ and $\eta(t) \geq \eta(t-1)$, we conclude that $x(t) \geq x(t-1)$ and $T(t) \geq T(t-1)$ according to the Topkis's monotonicity theorem (Donald Topkis).[3] $\theta(t+1) \geq \theta(t)$ and $\eta(t+1) \geq \eta(t)$ then follow immediately from the fact that f and g are non-decreasing. The theorem then follows by induction. □

Although we have not explored it here, our model does allow the possibility of multiple steady states. Nevertheless, because our formal model has no durable capital in the firms and because our representative firms always contract efficiently, the possibility that expectations affect the path of the economy is excluded. Instead, following Paul David and Steven Durlauf, we emphasize the role of history, featuring the momentum of the economic system.

IV. Conclusion

The Momentum Theorem shows that complementarities among a group of core activities and processes can account for the emergence of a persistent pattern of change, even without any of the usual assumptions in the growth literature about economies of scale (see Paul Romer). Our method, emphasizing complementarities over issues of scale, also promises to clarify the logic of growth models and to allow a far richer modeling of the multi-faceted processes of growth and development.

In Chandler's account of nineteenth century American economic growth, for example, the emergence of the large industrial enterprise was accompanied not only by improvements in communications (telegraph) and transportation (railroads) that helped to create national markets, but also by innovations in finance (e.g., bond markets), management methods (e.g., cost accounting), large-scale manufacturing technologies (e.g., continuous process technologies), and so on. Each of these improvements and innovations were complementary to further growth of large enterprises, and the expanding scale of these enterprises correspondingly encouraged continuing technological, organizational and managerial advances.

Closely related to our ideas are some longstanding analyses of economic development, where the need to manage complementarities among investment projects has been noted by some economists (e.g., Albert Hirschman). The questions addressed in these analyses are not merely ones of whether to develop or how to develop, but also *in which direction* to develop. An analysis of complementarities – richly conceived – seems indispensable to giving a satisfactory answer to these questions.

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ENDNOTES

1. Indeed, our entire analysis is a purely ordinal one, that is, no step in the argument is affected if the variables are subject to arbitrary monotone rescalings. Since returns to scale is a cardinal concept, all of our conclusions apply without any assumptions about returns to scale.

2. Generally, using high technology equipment may require both "knowledge" $\eta(t)$ and cash investment $I(t)$, for example: $T(t) = \eta(t)h(I(t))$, where $h'>0$ and $h''<0$. In this example, the cost of achieving technology level T using knowledge η is $I = h^{-1}(T/\eta)$, which implies the required inequality: $\partial^2 I/\partial T \partial \eta < 0$.

3. Topkis's original treatment is very general and abstract. A simpler account of Topkis's Theorem, restricted to applications in $\mathbb{R}^N$, can be found in the paper by Milgrom and Roberts.