Competition for Exclusive Customers: Comparing Equilibrium and Welfare under One-Part and Two-Part Pricing*

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Abstract

This paper compares one-part pricing and two types of two-part pricing in a general discrete-continuous choice model, providing more extensive welfare results than prior literature. Under two-part pricing, firms may set fixed fees with or without "unit-price commitment." When unit-price commitment is present, both fixed fees and unit prices are set before consumers choose their exclusive suppliers. In the absence of unit-price commitment, consumers choose exclusive suppliers based on known fixed fees and anticipated future unit prices, resulting in high unit prices consistent with models of "aftermarket monopolization." Of the three possible pricing policies, we find that two-part pricing with unit-price commitment is firms' dominant unilateral pricing policy, and it also yields the highest joint profits (under appropriate demand assumptions). In terms of both consumer and social welfare, two-part pricing without unit-price commitment is dominated by the other pricing policies. With sufficiently elastic usage demand and sufficiently small customer-specific fixed costs, one-part pricing produces the highest consumer and social welfare, but the lowest firm profits, of the three pricing policies.

JEL classification: L1, D4 *Keywords*: two-part pricing, aftermarket monopolization, Bertrand competition, discrete-continuous random-utility model.

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1. Introduction

A significant volume of commerce takes place through long-lived supply relationships where customers purchase products exclusively from a single supplier. When consumers are bound to a supplier after their initial choice, firms compete vigorously to sign up customers. Such "competition for the customer" can alter the standard conclusions for prices, profits, and consumer welfare that emerge from the more common "competition in the market."

A principal objective of this paper is to examine the social welfare implications of competition for exclusive customer relationships, where that competition takes place through one-part or differing two-part pricing arrangements. Our modeling framework also analyzes "fore-market competition" with "aftermarket monopolization," where consumers must choose their exclusive suppliers on the basis of fixed fees (*e.g.*, initial purchase prices) before unit prices are determined for "consumables," repair services, upgrades, and other products and services that may be related to the intensity of product usage.

Exclusive supply relationships are a frequently observed phenomenon, perhaps as a means of protecting returns on relationship-specific investments, a result of high switching costs, or an attempt to lessen competition. Examples of consumer goods that are sold under exclusive supply arrangements include arguably communication services such as fixed and mobile phone services, Internet access, and cable and satellite TV. Membership in a heath maintenance organization is invariably exclusive as well.

Component systems offer additional examples of exclusive supply relationships in which consumers purchase a durable good and its complementary "consumables" from the same firm. This situation arises when the seller of equipment (*e.g.*, printers, cameras, personal computers, home video games) is also the sole source of compatible supplies, parts, add-ons, and repair services.[1] In addition to the aforementioned products, software also may be subject to exclusive supply relationships, where users pay for an initial version of the product and then must buy upgrades from the same supplier.

To analyze competition with exclusive customers, we build a non-cooperative model in which firms sell differentiated products and consumers purchase a variable amount of the product from a single supplier. We then examine Bertrand-Nash equilibrium prices, profits, consumer welfare, and social welfare when firms adopt either one-part pricing or one of two alternative two-part pricing policies. A consumer's outlay is directly proportional to his product "usage" under one-part (*i.e.*,

[1] For some of these component systems, consumers have the option of purchasing the consumable from an independent firm, as when a home appliance can be repaired with parts or service supplied by firms other than the manufacturer.

linear) pricing, while two-part pricing additionally imposes a fixed fee that is paid for "access" to the good (or representing the initial purchase price for a durable good).[2]

We distinguish between two cases of two-part pricing: one where firms commit to a specified unit price before consumers choose their suppliers, and another where no such commitment is made. In the latter case, consumers choose their exclusive suppliers on the basis of observed fixed fees and anticipated future unit prices. Since consumers are "locked into" their suppliers before unit prices are determined, the latter form of two-part pricing is consistent with models of aftermarket monopolization, where consumers are vulnerable to high prices for consumables and product upgrades after paying an initial purchase price to obtain (or access) the product.

When firms can commit to two-part pricing, they set unit prices equal to marginal production cost and fixed fees obey an inverse-elasticity rule. If firms do not commit in advance to the unit price, a "bargain-then-rip-off" pricing pattern emerges: unit prices are set at the monopoly level appropriate for a fixed customer base, preceded by intense fore-market competition for customers using fixed fees. This includes the possibility of fixed fees that are below customer-specific fixed costs, representing an inducement for consumers to "sign up" with the supplier.

Our analysis finds that if firms could non-cooperatively choose their pricing policy among one-part pricing, two-part pricing with unit-price commitment, and two-part pricing without unit-price commitment before setting actual price levels, then it is a dominant unilateral strategy to choose two-part pricing with unit price commitment. By inducing the lowest unit prices of the three pricing policies, this particular pricing policy produces the largest amount of total surplus per customer that firms then exploit through their choice of fixed fees (subject to oligopoly competition). In this fashion, we show that the unilateral incentive for firms to prefer two-part pricing over one-part pricing extends from Coase's (1946) monopoly result to the oligopoly case.

Interestingly, under a rather straightforward demand assumption (*i.e.*, a firm's "market penetration rate" is log concave with respect to its product price), two-part pricing with unit-price commitment is also the *cooperatively* preferred choice of pricing policy, leading to higher firm profits than the other two alternatives. Thus, when firms do not commit to unit prices, the reasons for the lack of commitment may have more to do with market uncertainties regarding costs, demand, and technological innovation than a desire to monopolize aftermarkets. Our results also can be interpreted as meaning that, in the face of consumer "lock-in" and an inability to commit to

[2] Two-part pricing schemes also may take the form of bundled pricing, as when a mobile phone service provider charges for airtime on its network that is inseparable from gaining access to that network. Hence, a two-part price could apply where there is a fixed monthly charge for gaining access to the network and a per-minute usage charge. A two-part (or three-part) pricing scheme also arises when the price of the mobile handset is bundled with the recurring charges for accessing and using the network.

aftermarket prices, firms would earn greater profits by encouraging aftermarket competition that drives prices for consumables sufficiently close to their marginal cost.

Both two-part pricing with unit-price commitment and one-part pricing provide higher levels of consumer and social welfare than two-part pricing without unit-price commitment. Thus, consumers and society also are made worse off by the inability to commit to unit prices.

When each consumer's "usage demand" is sufficiently price sensitive (and there are no customer-specific fixed costs), we find that consumer and social welfare is greatest, but firm profits are lowest, under one-part pricing. As the usage demand for each customer becomes increasingly price sensitive, optimal unit prices under one-part pricing decline toward marginal cost. Although the total surplus per customer is less than that achieved under two-part pricing with unit-price commitment, consumer surplus is higher and more consumers participate in the market in equilibrium. Consequently, consumer and social welfare are higher under one-part pricing.

This result points out how evaluating the impact of nonlinear pricing on consumer and social welfare may change based on whether consumers have the ability to refrain from purchasing the good in question. Previous studies have obtained opposite results regarding social welfare, assuming either that the market is "covered" in that all consumers make purchases of the relevant product, or that the degree of product differentiation is small or approaching zero.

Our paper also analyzes how pricing is affected when consumers patronizing a given supplier have differing usage demands. In these circumstances, unit prices may be below marginal cost and firms may earn lower margins from high-volume customers than low-volume customers under two-part pricing with unit-price commitment. This result arises when high-volume customers are relatively more likely than low-volume customers to substitute suppliers (or refrain from purchasing from any supplier) in response to an increase in the aggregate price of the good. Our findings differ from the standard monopoly result involving price discrimination, as unit prices depart from marginal cost as a means of price discriminating among consumers with differing degrees of product substitutability that is related to their intensity of product usage. Our results also would predict that a monopolist using a two-part pricing policy would set unit prices below marginal cost if high-volume customers exhibit greater dispersion in their product utilities (*i.e.*, reservation prices) than low-volume customers, even if the expected product utility was greater for those customers relative to low-volume customers.

This paper contributes to a growing literature that analyzes nonlinear pricing under oligopoly (see Stole, 2005, for an excellent survey). Armstrong and Vickers (2001) compare consumer and social welfare under competition with discriminatory and uniform pricing when consumers are heterogeneous. They derive several of their welfare comparisons involving one-part and two-part pricing by observing what happens as the degree of product differentiation approaches zero (*i.e.*, the price elasticity of "access demand" approaches infinity). Rochet and Stole (2002) examined similar issues in a model with a specific form of consumer heterogeneity.

The generality of the results from these papers and others is limited when the market is assumed to be completely covered or the extent of product heterogeneity is identical (or approaching zero) across customers of different usage intensities. Recently, Yin (2004) has compared the welfare effects of one-part pricing and two-part pricing with unit-price commitment under a Hotelling construct, considering how welfare is affected in equilibrium if the market is fully covered or incompletely covered (*i.e.*, localized monopoly). Yin also makes a welfare comparison of these pricing policies under logit demand, finding that two-part pricing with commitment yields greater social welfare than one-part pricing when all consumers must purchase the differentiated product and there are no customer-specific fixed costs. Our results show that this result does not apply under more general demand assumptions, including a situation where consumers do not necessarily purchase the good in question.

Our analysis extends and generalizes the above literature in several important ways. First, we consider a multi-firm oligopoly rather than the duopoly considered in much of the prior literature. Second, our analysis also includes two-part pricing without unit-price commitment, an empirically relevant case which has been largely ignored. Third, our model allows for the presence of customer-specific fixed costs (also considered by Armstrong and Vickers, 2001), which affects the welfare comparison between one and two-part pricing. Fourth, our model is based on a general construct of stochastic consumer preferences with a "no-purchase" option,[3] that subsumes prior modeling approaches (including logit and Hotelling-type models where there is a distribution of consumer reservation prices at each location) and extends to any distribution of preferences that satisfies our assumption of log-concave "access demand." Lastly, our model uniquely distinguishes between the role of the price elasticity of "access demand" and the price elasticity of "usage demand" in determining product prices and making welfare comparisons under one-part and two-part pricing.

The antitrust literature on aftermarkets is also relevant to our analysis (see, among others, Borenstein, MacKie-Mason and Netz, 1995; and Emch, 2003). Much of that literature is concerned with the competitive effects of specific unilateral practices, and less with the welfare implications of competition for customers under different pricing policies. One contribution of this paper is the finding that a commitment not to engage in aftermarket monopolization (or the presence of aftermarket competition) may benefit firms as well as consumers.

The remainder of the paper is organized as follows. Section 2 describes the structure of the game, the discrete-continuous choice problem facing consumers, and the profit-maximization prob-

[3] Besanko, Perry and Spady (1990) and Anderson, de Palma, and Thisse (1992) were among the first to use discrete-choice models to capture demand behavior in differentiated oligopolies.

lem facing firms. Section 3 examines Bertrand-Nash equilibrium under two-part pricing where firms commit in advance to unit prices, as well as the case where they can freely adjust unit prices after consumers have selected their suppliers. Section 4 compares profits and welfare under both cases of two-part pricing. Section 5 examines equilibrium one-part prices, comparing profits and welfare with those obtained under both forms of two-part pricing. In Section 6, the analysis considers how two-part prices change when consumers differ in their usage demands. Finally, Section 7 offers concluding remarks.

2. The Model

2.1. Game Structure

The game played by consumers and firms unfolds as follows. First, each one of a finite number of firms sets a tariff for a single differentiated product. Next, each one of a continuum of consumers learns their idiosyncratic tastes for all product varieties, and each consumer selects one particular variety or makes no purchase at all. If they choose a variety, consumers then decide how much of it to consume (*i.e.*, use). Finally, firms satisfy the usage demands of their base of exclusive customers. Throughout the game, firms' costs and product attributes are fixed and common knowledge.

Firms compete noncooperatively for (exclusive) customers, setting one-part or two-part prices. A Bertrand-Nash equilibrium arises when firms issue their best response to rivals' pricing decisions. When firms cannot commit to unit prices in advance, consumers perfectly forecast the equilibrium unit prices that will ensue.

2.2. Individual Demand for Product Access and Usage

We adopt the discrete-continuous model of consumer behavior developed by Dubin and McFadden (1984), Hanemann (1984), and others that generalizes the standard qualitative choice framework. Consumers select a single firm $j \in \{1, 2, ..., J\}$ as their exclusive supplier and then purchase a variable quantity of the chosen supplier's product. Consumers alternatively may choose a "no-purchase" option denoted as $j = 0$. There are many reasons for this exclusivity of supplier choice, including the presence of exclusive supply contracts or the existence of prohibitive costs (or product incompatibilities) in sourcing from two or more suppliers.[4]

[4] Other explanations for this pricing pattern have been offered in the literature on supplier switching costs. See, for example, Farrell and Klemperer (2005).

Firms set a two-part tariff, consisting of a "fixed fee" r_j and a "unit price" p_j.[5] We will also consider one-part pricing which amounts to assuming that $r_j = 0$.

Let $u_{ij}(x_{ij}, z_{ij})$ be the direct utility to consumer i from consuming x_{ij} units of product j and z_{ij} units of a composite outside good. Working backwards, if consumer i has selected product j, he then chooses quantity $x_{ij}(p_j, y_i - r_j)$ of that product, and the quantity $z_{ij}(p_j, y_i - r_j)$ of the outside good, subject to his income constraint. The resulting indirect utility from consuming variety j is given by:

$$v_{ij}(p_j, y_i - r_j) = \max_{x_{ij}, z_{ij}} \left\{ u_{ij}(x_{ij}, z_{ij}) : p_j x_{ij} + z_{ij} \leq y_i - r_j \right\}. \tag{2.1}$$

Note that the product's fixed fee is simply deducted from income. The utility of the no-purchase option amounts to the utility derived from consumption of the composite good alone, which is expressed as $v_{i0} = u_{i0}(y_i)$.

Consumers are assumed to be identical up to an additive utility disturbance, with equal incomes $y_i = y$ (or the same constant marginal utility of income). Under these assumptions, we can express the total utility of purchasing from firm j as the sum of the deterministic indirect utility $V_j(p_j, r_j)$ and the realization of the additive idiosyncratic disturbance, ε_{ij}:

$$v_{ij}(p_j, y_i - r_j) = V_j(p_j, r_j) + \varepsilon_{ij}. \tag{2.2}$$

Here ε_{ij} is consumer i's realization of an i.i.d. draw from a continuously differentiable distribution with c.d.f. G_j and p.d.f. g_j. As with any other alternative, the no-purchase option may have a random disturbance term.

Since consumers are identical before they draw a random disturbance, we drop any reference to individual consumers in what follows (*i.e.*, ε_j and ε_k replace ε_{ij} and ε_{ik}, respectively), and assume without loss of generality that consumers are of unit mass. Note that, due to additive separability, the disturbances do not affect usage levels. Later in the paper, we consider the case where there are two types of consumers with differing usage intensities.

The "penetration rate" (*i.e.*, "access demand") for firm j's product is measured as the fraction of the *entire* consumer population (including those who opt not to purchase) that chooses firm j as their supplier. It is given by:

$$\begin{aligned}
\pi_j(\mathbf{p}, \mathbf{r}) &= Pr\left\{ V_j(p_j, r_j) + \varepsilon_j \geq V_k(p_k, r_k) + \varepsilon_k, \forall k \neq j \right\} \\
&= \int Pr\left\{ \varepsilon_k \leq V_j(p_j, r_j) - V_k(p_k, r_k) + \varepsilon_j, \forall k \neq j \mid \varepsilon_j \right\} Pr\{\varepsilon_j\} d\varepsilon_j \\
&= \int \prod_{k \neq j} G_k(V_j(p_j, r_j) - V_k(p_k, r_k) + \varepsilon_j) g_j(\varepsilon_j) d\varepsilon_j, \qquad (2.3)
\end{aligned}$$

[5] In specific settings, the fixed fee may be paid each period. This differs from the usual form of a two-part tariff in which the fixed fee is paid once. Our model comprises a single period, which effectively eliminates this distinction.

where $\mathbf{p} = (p_j)$ and $\mathbf{r} = (r_j)$ are J-vectors of firms' unit prices and fixed fees, respectively, and where we have invoked statistical independence across the idiosyncratic disturbances. Thus, the penetration rate of firm j depends on the unit prices and fixed fees chosen by all suppliers through their effect on the indirect utilities (*i.e.*, the $V_k(p_k, r_k)$'s as well as $V_j(p_j, r_j)$). The probability of choosing the no-purchase option is given by:

$$\pi_0(\mathbf{p}, \mathbf{r}) = Pr\left\{\varepsilon_k \leq -V_k(p_k, r_k) + \varepsilon_0, \forall k \neq 0\right\},$$

where the deterministic part of the no-purchase indirect utility is normalized to zero (*i.e.*, $V_0 = 0$).

Since an increase in either its fixed fee or unit price necessarily lowers the indirect utility associated with consuming a particular product variety (*i.e.*, $\partial V_j/\partial r_j, \partial V_j/\partial p_j < 0$), it holds that:

$$\begin{aligned} \frac{\partial \pi_j(\mathbf{p}, \mathbf{r})}{\partial r_j} &= \frac{\partial V_j}{\partial r_j} \int \left[\sum_{k \neq j} g_k(V_j - V_k + \varepsilon_j) \prod_{l \neq j,k} G_l(V_j - V_l + \varepsilon_j)\right] g_j(\varepsilon_j) d\varepsilon_j \qquad (2.4) \\ &= \frac{\partial V_j}{\partial r_j} \int \left[\sum_{k \neq j} \frac{g_k(V_j - V_k + \varepsilon_j)}{G_k(V_j - V_k + \varepsilon_j)} \prod_{l \neq j} G_l(V_j - V_l + \varepsilon_j)\right] g_j(\varepsilon_j) d\varepsilon_j < 0, \\ \frac{\partial \pi_j(\mathbf{p}, \mathbf{r})}{\partial p_j} &= \frac{\partial V_j}{\partial p_j} \int \left[\sum_{k \neq j} g_k(V_j - V_k + \varepsilon_j) \prod_{l \neq j,k} G_l(V_j - V_l + \varepsilon_j)\right] g_j(\varepsilon_j) d\varepsilon_j \qquad (2.5) \\ &= \frac{\partial V_j}{\partial p_j} \int \left[\sum_{k \neq j} \frac{g_k(V_j - V_k + \varepsilon_j)}{G_k(V_j - V_k + \varepsilon_j)} \prod_{l \neq j} G_l(V_j - V_l + \varepsilon_j)\right] g_j(\varepsilon_j) d\varepsilon_j < 0. \end{aligned}$$

From the above equations, it necessarily follows that:

$$\frac{\partial \pi_j(\mathbf{p}, \mathbf{r})}{\partial p_j} = \left(\frac{\partial V_j/\partial p_j}{\partial V_j/\partial r_j}\right) \frac{\partial \pi_j(\mathbf{p}, \mathbf{r})}{\partial r_j}. \qquad (2.6)$$

This result will be quite useful later in this paper.

To ensure that second-order conditions are satisfied, and that a unique symmetric equilibrium exists, we make the following two assumptions:

$$\frac{\partial^2 \ln \pi_j(\mathbf{p}, \mathbf{r})}{\partial r_j^2} < 0, \qquad (A1)$$

$$\frac{\partial^2 \ln \pi_j(\mathbf{p}', \mathbf{r}')}{\partial r_j \partial r'} = \sum_k \frac{\partial^2 \ln \pi_j(\mathbf{p}', \mathbf{r}')}{\partial r_j \partial r_k} < 0, \qquad (A2)$$

where $\mathbf{p}' = (p', ..., p')$ and $\mathbf{r}' = (r', ..., r')$.

Assumption (A1), which states that each firm's penetration rate (*i.e.*, access demand) is *log concave* with respect to its own fixed fee, is helpful in satisfying second-order conditions for profit-maximizing prices.[6] According to assumption (A2), a given increase in each firm's fixed fee leads to a greater percentage decrease in its penetration rate when all firms are charging relatively high prices (holding constant the price, and thus the utility, of the "no-purchase" option). Similar to concavity conditions that are imposed in oligopoly models where consumers do not make a discrete supplier choice (*e.g.*, standard Cournot models with homogeneous products), we impose restrictions on the concavity of firm demand in our discrete-choice model in order to satisfy second-order conditions and ensure the uniqueness and stability of equilibrium.

Our analysis frequently examines behavior in a *symmetric* equilibrium with a common distribution of disturbances for all varieties (*i.e.*, $g_j = g$ and $G_j = G$ for all $j \neq 0$), where fixed fees and unit prices also are the same across firms (*i.e.*, $r_j = r'$ and $p_j = p'$ for all $j \neq 0$). Consequently, assumptions (A1) and (A2) place restrictions on idiosyncratic preferences as described by the probability density $g(\cdot)$. Note that an increase in r_j causes the difference in indirect utility, $V_j(p_j, r_j) - V_k(p_k, r_k)$, to decline for all $k \neq j$. However, from firm j's perspective, when all firms increase their common fixed fee (*i.e.*, r' increases), the difference declines only between variety j's indirect utility and the indirect utility offered by the no-purchase option (*i.e.*, $V_j(p_j, r_j) - V_0(p_0, r_0)$). Under symmetric distributions, $\partial^2 \ln \pi_j(\mathbf{p}, \mathbf{r})/\partial r_j^2$ is thus a positive multiple of $\partial^2 \ln \pi_j(\mathbf{p}', \mathbf{r}')/\partial r_j \partial r' = \sum_{k \neq 0} \partial^2 \ln \pi_j(\mathbf{p}', \mathbf{r}')/\partial r_j \partial r_k$.[7] Consequently, satisfying assumption (A1) ensures that assumption (A2) is also satisfied. Based on the above assumptions and appropriate boundary conditions, a unique, symmetric, and (locally) stable equilibrium exists.[8]

Referring to equation (2.4), and assuming that the marginal utility of income is constant (*i.e.*,

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[6] A sufficient, but not necessary, condition for log concavity is that π_j is concave in r_j.

[7] Let $g_j = g$ and $G_j = G$ for all j. When the indirect utility is the same for the no-purchase option as for the J varieties of the product (*i.e.*, $V_j(p_j, r_j) = V_k(p_k, r_k)$, $\forall\ k \neq j$), it necessarily holds that $\partial^2 \ln \pi_j(\mathbf{p}', \mathbf{r}')/\partial r_j{}^2 = J(\partial^2 \ln \pi_j(\mathbf{p}', \mathbf{r}')/\partial r_j \partial r')$.

[8] For further discussion of the existence and uniqueness of equilibrium in discrete-choice models, see Caplin and Nalebuff (1991) and Anderson, de Palma, and Nesterov (1995). Given the continuity of firm profit functions with respect to own prices and rival prices, and the concavity of a firm's own profits with respect to its own fixed fee (implied by assumption (A1)) and unit price, an equilibrium necessarily exists if the set of feasible price combinations is compact and convex (by Kakutani's fixed point theorem). Let unit prices be nonnegative and bounded above by a "maximum" level which exceeds the level that would maximize the profits earned from product usage for any given customer. Let a feasible fixed fee be one that exceeds a specified level associated with nonpositive profits. Then, assuming that a "maximum" fixed fee level exists that is jointly and individually suboptimal for all firms (where the fee exceeds the level that is individually profit-maximizing), an internal equilibrium necessarily exists. When competition is symmetric, the uniqueness of the symmetric equilibrium is assured by assumption (A2).

$\partial V_j/\partial r_j = -\partial V_j/\partial y = -\gamma, \forall j$), there is a presumption that assumptions (A1) and (A2) will be satisfied under symmetry if the likelihood ratio, $g(\varepsilon_k)/G(\varepsilon_k)$, is sufficiently monotone decreasing in ε_k. Since the difference, $V_j(p_j, r_j) - V_k(p_k, r_k) + \varepsilon_j$, is declining in r_j, the term, $g(V_j - V_k + \varepsilon_j)/G(V_j - V_k + \varepsilon_j)$ is increasing in r_j when $g(\cdot)/G(\cdot)$ is monotone decreasing. Particular examples that satisfy assumptions (A1) and (A2) include the uniform distribution and the extreme-value distribution underlying the logit demand specification.

Thus, our modeling framework subsumes prior approaches that compare one-part and two-part pricing under more restrictive assumptions, such as logit demand or Hotelling-type competition with diverse consumer reservation prices, while also applying to any other demand specification that conforms with the general assumptions above. In further contrast to much of the prior literature examining the welfare aspects of nonlinear pricing, our results apply to any J-firm oligopoly, and we permit consumers to not purchase the good in question rather than forcing the market to be "covered."

2.2.1. Usage Demand

For purposes of our analysis, it is important, of course, to evaluate a customer's consumption level (*i.e.*, "usage demand") for product j, denoted as $x_j(p_j, r_j)$. Using Roy's Identity, and noting that income net of the fixed fee for product j is $y - r_j$, we can express each customer's usage demand for product j as follows:

$$x_j(p_j, r_j) = -\frac{\partial V_j/\partial p_j}{\partial V_j/\partial y} = \frac{\partial V_j/\partial p_j}{\partial V_j/\partial r_j}. \tag{2.7}$$

Since the consumer population is assumed to be of unit mass, the *market* demand for the usage of product j is simply $\pi_j(\mathbf{p}, \mathbf{r})x_j(p_j, r_j)$. Unless otherwise specified, we assume that usage demand is declining in the unit price (*i.e.*, $\partial x_j(p_j, r_j)/\partial p_j < 0$ for p_j, r_j such that $x_j(p_j, r_j) > 0$). One particular example would be the case of linear usage demand, where $V_j(p_j, r_j) = \xi_j + \gamma \left[y - r_j - \alpha_j p_j + \frac{1}{2}\beta_j p_j^2 \right]$.

Lastly, we can substitute equation (2.7) into equation (2.6) to obtain the following:

$$\frac{\partial \pi_j(\mathbf{p}, \mathbf{r})}{\partial p_j} = x_j(p_j, r_j)\frac{\partial \pi_j(\mathbf{p}, \mathbf{r})}{\partial r_j}. \tag{2.8}$$

This result states that the impact on a firm's penetration rate of marginally raising its unit price is equal to the impact of marginally raising its fixed fee, multiplied by the amount of product usage per customer.

3. Equilibrium With and Without Unit-Price Commitment

We start by characterizing the equilibrium with two-part price competition. Distinct from previous literature that assesses the welfare aspects of two-part pricing, we consider two cases which differ in their treatment of unit prices. In the first case, firms commit to unit prices in advance of consumers' choosing their exclusive suppliers. In the second case, firms set unit prices after consumers select suppliers. This distinction is appealing for analyzing certain types of behavior, such as the possibility of "aftermarket monopolization." In particular, our analysis assesses whether a strategy of two-part pricing without unit-price commitment, and the aftermarket monopolization associated with this strategy, is desirable either for firms or consumers relative to alternative pricing policies, such as two-part pricing with unit-price commitment or one-part pricing.

3.1. Profit Maximization and Nash Equilibrium

By assumption, costs can be decomposed into customer-specific "fixed" costs and variable usage costs, both of which exhibit constant returns to scale. We let f_j denote firm j's constant fixed cost per customer, and m_j denote its constant marginal usage cost. Without loss of generality, we assume that product-specific fixed costs are zero, or are entirely sunk.

Each firm's profits are a function of the prices set by itself and its rivals, as well as the predetermined product attributes and the distribution of consumer preferences. Profits equal revenues from fixed fees and usage charges less associated costs:

$$\Gamma_j(\mathbf{p}, \mathbf{r}) = \pi_j(\mathbf{p}, \mathbf{r}) \left[r_j - f_j + (p_j - m_j) x_j(p_j, r_j) \right]. \tag{3.1}$$

The term in square brackets is the firm's "average net revenue per user" or "$ANRPU$," which represents revenues per customer *net* of associated fixed costs and variable usage costs.

We characterize competition in these markets by a Nash equilibrium in prices, where the number of firms is fixed exogenously. Firms set their fixed fees and unit prices simultaneously. When they cannot commit to unit prices, firms choose their fixed fees simultaneously, and then set unit prices after consumers have selected their exclusive suppliers. Prices $(\mathbf{p}^*, \mathbf{r}^*)$ form a pure-strategy Nash equilibrium when, for each j, $\Gamma_j(\mathbf{p}^*, \mathbf{r}^*) \geq \Gamma_j(p_j, \mathbf{p}^*_{-j}, r_j, \mathbf{r}^*_{-j})$, for all (p_j, r_j) . As formulated, and using assumptions (A1) and (A2), our profit functions have sufficient continuity and concavity to ensure that a unique, symmetric Nash equilibrium exists.

3.2. Equilibrium With Unit-Price Commitment

We first consider the case where both the fixed fee and the unit price are set in advance of the consumer's purchase decision. Differentiating equation (3.1) with respect to r_j and p_j, we obtain the following first-order conditions for optimal pricing:

$$\frac{\partial \Gamma_j(\mathbf{p},\mathbf{r})}{\partial r_j} = \frac{\partial \pi_j}{\partial r_j}\left[r_j - f_j + (p_j - m_j)x_j\right] + \pi_j\left[1 + (p_j - m_j)\frac{\partial x_j}{\partial r_j}\right] = 0, \tag{3.2}$$

$$\frac{\partial \Gamma_j(\mathbf{p},\mathbf{r})}{\partial p_j} = \frac{\partial \pi_j}{\partial p_j}\left[r_j - f_j + (p_j - m_j)x_j\right] + \pi_j\left[x_j + (p_j - m_j)\frac{\partial x_j}{\partial p_j}\right] = 0. \tag{3.3}$$

These conditions take a familiar form, but prices affect both the number of customers patronizing firm j (through $\partial\pi_j/\partial r_j$ and $\partial\pi_j/\partial p_j$), as well as the quantity of the product that each consumer uses (through $\partial x_j/\partial r_j$ and $\partial x_j/\partial p_j$). Later, we consider the case where consumers are "locked in" to their providers before unit prices are set, implying that $\partial\pi_j/\partial p_j = 0$.

Following standard consumer theory, the term $\partial x_j/\partial p_j$ can be decomposed into a substitution effect and an income effect:

$$\frac{\partial x_j}{\partial p_j} = \frac{\widetilde{\partial x_j}}{\partial p_j} + x_j\frac{\partial x_j}{\partial r_j}, \tag{3.4}$$

where $\widetilde{\partial x_j}/\partial p_j$ denotes the Hicksian substitution effect around the utility level $u_j = V_j(p_j, r_j)$, and $\partial x_j/\partial r_j$ is the corresponding income effect (noting that $dy = -dr_j$). Substituting equation (2.8), which states that $\partial\pi_j/\partial p_j = x_j(\partial\pi_j/\partial r_j)$, and equation (3.4) into equation (3.3), we obtain:

$$\frac{\partial \Gamma_j(\mathbf{p},\mathbf{r})}{\partial p_j} = x_j\left\{\frac{\partial \pi_j}{\partial r_j}\left[r_j - f_j + (p_j - m_j)x_j\right] + \pi_j\left[1 + (p_j - m_j)\frac{\partial x_j}{\partial r_j}\right]\right\} + \pi_j(p_j - m_j)\frac{\widetilde{\partial x_j}}{\partial p_j} = 0. \tag{3.5}$$

Inserting the first-order condition for the optimal fixed fee (see equation (3.2)) into the large bracketed expression, the above equation simplifies as follows:

$$\frac{\partial \Gamma_j(\mathbf{p},\mathbf{r})}{\partial p_j} = \pi_j(p_j - m_j)\frac{\widetilde{\partial x_j}}{\partial p_j} = 0,$$

which requires that the profit-maximizing unit price, $\overline{p}$, must satisfy

$$\overline{p}_j = m_j. \tag{3.6}$$

Regardless of the two-part tariffs set by its rivals, a firm's best response is always to set its unit price equal to marginal cost. This result is similar to those obtained by Gasmi, Moreaux, and Sharkey (2000) with homogeneous products, and more generally by Armstrong and Vickers (2001) and Rochet and Stole (2002) with differentiated products.

To determine the optimal fixed fee, we substitute equation (3.6) into equation (3.2), which implies that:

$$\frac{\partial \Gamma_j(\mathbf{p}, \mathbf{r})}{\partial r_j} = \frac{\partial \pi_j}{\partial r_j}(r_j - f_j) + \pi_j = 0. \tag{3.7}$$

With slight rearrangement, this becomes:

$$ANRPU_j = r_j - f_j = \left(-\frac{\partial \ln \pi_j(m_j, \mathbf{p}_{-j}, r_j, \mathbf{r}_{-j})}{\partial r_j}\right)^{-1}. \tag{3.8}$$

This expression will prove useful later when making profit and welfare comparisons. Dividing equation (3.8) by r_j, we obtain

$$\frac{r_j - f_j}{r_j} = \left(-\frac{\partial \pi_j}{\partial r_j}\frac{r_j}{\pi_j}\right)^{-1} = \frac{1}{\eta_j}, \tag{3.9}$$

where η_j is the elasticity of firm j's "access demand" (*i.e.,* penetration rate) with respect to its fixed fee.

With the unit price equal to marginal cost, firm profits depend solely on the price-cost margin associated with the fixed fee (*i.e.*, $(r_j - f_j)/r_j$), which equals the inverse of the firm's elasticity of access demand under profit-maximizing behavior. We summarize below:

Proposition 3.1. *In a Bertrand-Nash equilibrium with two-part price competition and unit-price commitment, each firm sets its unit price equal to its marginal cost, and the fixed fee obeys an inverse access-demand elasticity rule (as described by equation (3.9)).*

We develop intuition for this result by recalling the case of monopoly two-part pricing with homogeneous consumers. In that case, Coase (1946) found that a monopolist maximized profit by setting its unit price at marginal cost which, in turn, maximized total surplus per customer. The monopolist then adjusted the fixed fee to extract nearly all of the available consumer surplus.

Some of this intuition carries over to two-part price competition (with unit-price commitment) among differentiated oligopolists. Although firms still set unit prices to maximize total surplus per customer, the fixed fee does not extract as much consumer surplus as in the monopoly case. Firms instead use their fixed fees to compete for customers; decreases in the fixed fee increase the number of customers patronizing a given firm at the expense of a lower price-cost markup per customer (*i.e.*, lower $ANRPU$). The optimal fixed fee depends on the firm's elasticity of access demand, producing a relationship between the optimal price-cost margin and the elasticity of demand that is analogous to those found in one-part pricing oligopoly models without discrete purchase decisions. As product differentiation lessens (*e.g.,* as the number of firms increases), we would expect the elasticity of access demand facing any single firm to increase, causing the equilibrium fixed fee and price-cost margin to fall.

3.3. Equilibrium Without Unit-Price Commitment

Suppose that firms are unable to commit in advance to prices for usage. After a customer signs an exclusive contract and pays the fixed fee r_j, firms set their unit prices recognizing that their customers are "captive." Firms cannot steal customers from their rivals because customers are under long-term contracts or face prohibitive costs of switching suppliers. Consumers, however, perfectly anticipate profit-maximizing unit prices, implying that supplier choices are made based on so-called "total cost of ownership."

These conditions are consistent with markets that display aftermarket monopolization where customers are "locked in" to their suppliers. While suppliers can charge relatively high prices for usage, they must compete fiercely for their "installed" customer bases. Examples of such product markets include telephone and cable television service, as well as component systems (*e.g.*, video game systems, computers, printers, or software) where the component supplier is the source of consumables, parts, service, add-ons, or product upgrades.

Under these conditions, firm j considers its customer base π_j to be fixed when setting unit prices. Therefore, it sets its unit price to maximize usage profits only:

$$\max_{p_j} \pi_j(\mathbf{p}, \mathbf{r}) \left[(p_j - m_j)x_j(p_j, r_j)\right]. \tag{3.10}$$

The profit-maximizing unit price, $\widehat{p}_j$, obeys the familiar monopoly mark-up rule:

$$\frac{\widehat{p}_j - m_j}{\widehat{p}_j} = \left(-\frac{\partial x_j}{\partial p_j}\frac{\widehat{p}_j}{\widehat{x}_j}\right)^{-1} = \frac{1}{\epsilon_j}, \tag{3.11}$$

where $\epsilon_j = (-\partial x_j/\partial p_j)(p_j/x_j)$ is the elasticity of usage demand for firm j's customers with respect to the unit price. Since idiosyncratic consumer preferences enter the indirect utility function additively (as described by equation (2.2)), and are therefore independent of unit price, all customers of a given firm choose the same usage level, $\widehat{x}_j = x_j(\widehat{p}_j, r_j)$.

Of course, rational consumers will anticipate monopoly usage pricing when evaluating suppliers. Substituting $\widehat{p}_j$ into equation (3.1), and differentiating with respect to the fixed fee, we obtain the first-order condition:

$$\frac{\partial \pi_j}{\partial r_j}\left[r_j - f_j + (\widehat{p}_j - m_j)\widehat{x}_j\right] + \pi_j\left[1 + (\widehat{p}_j - m_j)\frac{\partial \widehat{x}_j}{\partial r_j}\right] = 0. \tag{3.12}$$

When usage demand is independent of income (*i.e.*, $\partial x_j/\partial r_j = 0$), the above first-order condition can be expressed as follows:

$$ANRPU_j = r_j - f_j + (\widehat{p}_j - m_j)\widehat{x}_j = \left(-\frac{\partial \ln \pi_j(\widehat{p}_j, \mathbf{p}_{-j}, r_j, \mathbf{r}_{-j})}{\partial r_j}\right)^{-1}. \tag{3.13}$$

Dividing equation (3.13) through by r_j, and substituting from equation (3.11), we obtain:

$$\begin{aligned} \frac{r_j - f_j}{r_j} &= \left(-\frac{\partial \pi_j}{\partial r_j}\frac{r_j}{\pi_j}\right)^{-1} - \frac{(\widehat{p}_j - m_j)\widehat{x}_j}{r_j} \\ &= \frac{1}{\eta_j} - \left(\frac{\widehat{p}_j\widehat{x}_j}{r_j}\right)\frac{1}{\epsilon_j}, \end{aligned} \tag{3.14}$$

where $\eta_j = (-\partial \pi_j/\partial r_j)(r_j/\pi_j)$ is again the elasticity of supplier j's access demand with respect to its fixed fee. From this result, we obtain the following proposition:

Proposition 3.2. *In a Bertrand-Nash equilibrium with two-part price competition, but no unit-price commitment, each firm sets unit price at the monopoly level given its customer base (as described by equation (3.11)). When usage demand is independent of income, the equilibrium fixed fee obeys a "modified inverse elasticity" rule where the markup of the fixed fee over the per-customer fixed cost depends inversely on the price elasticity of access demand (i.e., η_j) and the price elasticity of usage demand (i.e., ϵ_j), as described by equation (3.14). Under assumptions (A1) and (A2), the fixed fee in this no-commitment case is lower than the fixed fee in the commitment case.*

Proof. See Appendix.

Note that, since the elasticity of usage demand enters negatively into equation (3.14), it is possible that the fixed fee is less than the fixed cost per customer, $r_j < f$. This situation arises when the price elasticity of usage demand is sufficiently small relative to the price elasticity of access demand. When the price elasticity of usage demand is relatively low—so that $1/\epsilon_j$ is large in absolute value—the markup on usage is relatively large. Effectively, access becomes a "loss leader" in a firm's product bundle: the loss per customer is more than made up by profits from usage. We have a "bargain-then-rip-off" situation, where customers are enticed with a very attractive fixed fee, perhaps one below cost (*i.e.*, a bargain) and subsequently charged high unit prices once they are "locked in" (*i.e.*, a rip-off).[9] This asymmetry is not surprising because the lack of unit-price commitment creates intense competition with respect to the fixed fee when both firms and consumers know that unit usage prices will inevitably be set at monopoly levels.

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[9] An example of this may be when wireless telephony providers offer a free (or heavily discounted) phone to new customers who choose their service. Indeed, when the fixed cost per customer is small, firms may find it optimal to pay consumers to sign up with their service, reaping profits later through high usage charges to these captive customers.

4. Welfare Effects of Unit-Price Commitment

The welfare effects of unit-price commitment under two-part pricing are not immediately clear. Commitment to unit prices leads to socially optimal product usage, but only relative to the equilibrium assignment of consumers to participating firms. The absence of commitment may result in deadweight loss from the monopoly pricing of usage, but competition for customers puts strong downward pressure on fixed fees that conceivably could result in higher overall market penetration (*i.e.*, a lower proportion of consumers choosing the "no-purchase" option).

We resolve this tension in the following proposition that shows, in a important class of cases, two-part pricing with unit-price commitment dominates two-part pricing without unit-price commitment from the standpoint of both firms and consumers.

Proposition 4.1. *Assume that usage demand is independent of income, and assumptions (A1) and (A2) hold. In the symmetric N-firm Bertrand-Nash equilibrium, industry profits, consumer welfare, and social welfare are all higher under two-part pricing with unit-price commitment as compared to two-part pricing without unit-price commitment.*

Proof. See Appendix for details. The proof proceeds as follows. First, it is shown that in a symmetric equilibrium, the indirect utility offered to each consumer is greater with unit-price commitment. Consequently, each firm serves more consumers in a symmetric equilibrium given the presence of the no-purchase option. Since total surplus per customer is also greater because unit prices are set at marginal cost, it necessarily holds that social welfare–measured as the sum of consumer surplus and industry profits–is greater with unit-price commitment. Secondly, when assumptions (A1) and (A2) hold, it is shown that each firm's profits per customer (*i.e.*, $ANRPU$) are higher with unit-price commitment, implying that total firm profits also are higher.■

The well-known logit demand model satisfies assumptions (A1) and (A2) (as do uniformly distributed preferences), which implies that:

Corollary 4.2. *In the symmetric logit model, two-part pricing with unit-price commitment dominates two-part pricing without unit-price commitment in terms of profits, consumer welfare, and social welfare.*

We have found that when firms compete for customers and consumers fully anticipate the total cost of product ownership, aggressive upfront competition for customers through the fixed fee is insufficient to offset the social loss created by aftermarket monopolization with high unit prices. Another way to view the above results is that, in a situation where they are unable to commit to unit prices, firms would benefit from the presence of aftermarket competition that drives unit prices close to marginal cost. Hence, for the firms offering the "base" product, "second sourcing" (and "third" and "fourth" sourcing) of consumables, parts, and repair services that generates sufficiently robust aftermarket competition will raise profits, consumer welfare, and social welfare relative to aftermarket monopolization.

The intuition behind Proposition 4.1 (and Corollary 4.2) is that if firms offer consumers the same indirect utility under unit-price commitment as that arising in the equilibrium without unit-price commitment, they would earn higher margins per customer because of the additional surplus generated by setting unit prices at marginal cost rather than monopoly levels. Facing a higher margin per customer (but the same number of customers if consumer utility is the same), firms under two-part pricing with commitment have incentive to further reduce their fixed fees in order to attract additional customers. Hence, in equilibrium, consumers receive greater utility and social welfare is higher under two-part pricing with commitment. Moreover, if access demand is log-concave (*i.e.*, satisfies assumptions (A1) and (A2)), the higher total surplus earned under two-part pricing with unit-price commitment is divided between firms and consumers in equilibrium so that firms also earn a higher margin per customer.

Under these conditions, the interest of firms and consumers are aligned with respect to their preferences for unit-price commitment. Since the absence of unit-price commitment is not a profit-maximizing pricing policy, the rationale for not making unit-price commitments may rest on strategic factors other than a desire by firms to monopolize "aftermarkets." These factors might include dynamically changing costs, consumer preferences, or technology, which may provide firms with substantial option value in delaying decisions regarding prices and the nature of future "consumables" or product upgrades.

The above analysis also shows that unit-price commitment constitutes a non-cooperative equilibrium in an extended game where firms choose their pricing policy before setting specific price levels. Clearly, the profit-maximizing response for any firm to its rivals' choices of two-part pricing policies (*i.e.*, with or without unit-price commitment) is a pricing rule satisfying equations (3.2) and (3.3), which requires a commitment to marginal-cost unit pricing and a fixed-fee choice that satisfies equation (3.9). By analogous reasoning, firms unilaterally prefer two-part pricing with commitment to one-part pricing, since the latter pricing policy imposes an additional constraint that a firm's fixed fee must equal zero.

Thus, regardless of rivals' choice of pricing policy, a firm's profit-maximizing unilateral response is to employ a strategy of two-part pricing with unit-price commitment where unit prices are set at marginal cost. Firms also would *cooperatively* choose two-part pricing with unit-price commitment as their preferred pricing policy, since they earn higher joint profits relative to two-part pricing without unit-price commitment (assuming that access demand is log concave). We summarize below:

Proposition 4.3. *Let firms non-cooperatively choose their preferred pricing policy among one-part pricing and two-part pricing with or without unit-price commitment, prior to setting their specified price levels (subject to a Bertrand-Nash pricing equilibrium). Two-part pricing with unit-price commitment is the dominant strategy and the Nash-equilibrium pricing policy. Under assumptions (A1) and (A2), it also holds that two-part pricing with unit-price commitment leads to higher joint profits than two-part pricing without unit-price commitment (in a symmetric N-firm Bertrand-Nash pricing equilibrium).*

Among other results in the next section, we show that two-part pricing with unit-price commitment also provides higher joint profits than one-part pricing.

5. One-Part Versus Two-Part Pricing

For various reasons, firms may find it difficult to engage in two-part pricing. Consumers may be reluctant to pay an upfront access fee when the supplier does not make binding commitments to provide service at a specified quality level or guarantee the longevity of its service. In these situations, firms may be forced to set a usage charge only.

Technically, this one-part pricing arrangement is equivalent to two-part pricing where the fixed fee is forced to equal zero (*i.e.*, $r_j = 0$, $\forall j$), and where firms commit in advance to the aftermarket unit price. One also might think of one-part pricing as a lease rate for a durable good (or a subscription license for software), while a two-part price amounts to a fixed purchase charge and a separate fee for consumables, upgrades, and repair and maintenance services.

Subject to one-part pricing, the firm's profit function is specified as follows:

$$\Gamma_j(\mathbf{p}, \mathbf{0}) = \pi_j(\mathbf{p}, \mathbf{0}) \left[-f_j + (p_j - m_j)x_j(p_j)\right]. \tag{5.1}$$

The associated first-order condition is:

$$\frac{\partial \Gamma_j(\mathbf{p}, \mathbf{0})}{\partial p_j} = \frac{\partial \pi_j}{\partial p_j}\left[-f_j + (p_j - m_j)x_j\right] + \pi_j \left[x_j + (p_j - m_j)\frac{\partial x_j}{\partial p_j}\right] = 0. \tag{5.2}$$

When firms make positive profits, it is clear from equation (5.2) that the optimal unit price under one-part pricing is between marginal cost (which is optimal under two-part pricing with commitment) and the monopoly price (which is optimal under two-part pricing without commitment).

To see this result, let $p_j^1 = m_j$, where p_j^1 is the optimal unit price under one-part pricing. Then, firm j would necessarily lose f_j per customer under one-part pricing (see equation (5.1)), implying that profits increase by raising the unit price. If instead $p_j^1 = \widehat{p}_j$, where $\widehat{p}_j$ satisfies the monopoly price condition $x_j + (p_j - m_j)(\partial x_j/\partial p_j) = 0$ (see equation (3.11)), it then holds that:

$$\frac{\partial \Gamma_j(\mathbf{p}, \mathbf{0})}{\partial p_j} \mid_{p_j^1 = \widehat{p}_j} = \frac{\partial \pi_j}{\partial p_j} \left[-f_j + (p_j - m_j) x_j \right].$$

When firm j makes positive profits (*i.e.*, $-f_j + (p_j - m_j)x_j > 0$), the above expression is necessarily negative in sign. Given that $\partial \Gamma_j(\mathbf{p}, \mathbf{0})/\partial p_j$ is positive for $p_j^1 = m_j$ and negative for $p_j^1 = \widehat{p}_j$, and that $\partial^2 \Gamma_j(\mathbf{p}, \mathbf{0})/\partial p_j^2 < 0$ when second-order conditions are satisfied, the optimal one-part unit price is between marginal cost and the monopoly unit price.

Proposition 5.1. *In a Bertrand-Nash equilibrium with one-part pricing, each firm sets its unit price between the marginal cost level associated with two-part pricing with unit-price commitment and the monopoly level associated with two-part pricing without unit-price commitment.*

Note that the first-order condition represented by equation (5.2) can be restated as follows:

$$\begin{aligned} \frac{p_j - m_j}{p_j} &= \frac{\frac{f_j}{p_j x_j} \left(\frac{\partial \pi_j}{\partial p_j} \frac{p_j}{\pi_j} \right) - 1}{\frac{\partial \pi_j}{\partial p_j} \frac{p_j}{\pi_j} + \frac{\partial x_j}{\partial p_j} \frac{p_j}{x_j}} \\ &= \frac{\frac{f_j}{p_j x_j} \eta_j^p + 1}{\eta_j^p + \epsilon_j}, \end{aligned} \tag{5.3}$$

where $\eta_j^p = (-\partial \pi_j/\partial p_j)(p_j/\pi_j)$ is the elasticity of firm j's *access* demand (*i.e.,* market penetration rate) with respect to its *unit* price, and $\epsilon_j = (-\partial x_j/\partial p_j)(p_j/x_j)$ is again the elasticity of firm j's usage demand with respect to its unit price. Thus, as expected, the price-cost margin under one-part pricing depends on both the elasticity of usage demand and the elasticity of access demand with respect to the unit price.

Refer to equation (5.3). When customer-specific fixed costs equal zero (*i.e.*, $f_j = 0$), the price-cost margin under one-part pricing shrinks to zero as the price elasticity of usage demand increases to infinity (*i.e.*, $\epsilon_j \to \infty$), regardless of the extent of inter-brand competition as represented by the price elasticity of access demand, η_j^p. When customer-specific fixed costs are positive, firms are not viable unless usage demand behavior allows them to recover fixed costs through the optimal setting of unit prices (at levels in excess of marginal cost).

A natural question arises as to whether consumers and producers are better off under one-part pricing or two-part pricing with and without unit-price commitment. Based on the above results, the following proposition and corollary can be obtained:

Proposition 5.2. *Assume that usage demand is independent of income, and that assumptions (A1) and (A2) hold. In the symmetric N-firm Bertrand-Nash equilibrium, it holds that:*

(a) Relative to one-part pricing, profits are higher under two-part pricing with unit-price commitment, but consumer utility and the number of consumers using the differentiated product may be lower or higher. Thus, as compared to one-part pricing, consumer and social welfare may be lower or higher under two-part pricing with commitment.

(b) Relative to one-part pricing, profits per customer may be lower or higher under two-part pricing without unit-price commitment, but consumer utility and the number of consumers using the differentiated product are lower. Thus, as compared to one-part pricing, fewer consumers are served and total surplus per customer is lower (due to higher unit prices) under two-part pricing without commitment, implying that both consumer and social welfare are also lower.

Proof. See Appendix.

Corollary 5.3. *When customer-specific fixed costs are absent (or sufficiently small), one-part pricing produces higher consumer and social welfare but lower profits than either form of two-part pricing as the price elasticity of usage demand approaches infinity.*

Proof. See Appendix.

Based on Proposition 5.2, consumers and society are definitely better off under one-part pricing relative to two-part pricing without unit-price commitment, and they may benefit from one-part pricing relative to two-part pricing with unit-price commitment. In fact, when the elasticity of usage demand is sufficiently high and customer-specific fixed costs are sufficiently low, consumer and social welfare are higher and firm profits are lower under one-part pricing relative to either form of two-part pricing (see Corollary 5.3). Firms benefit most from two-part pricing with commitment, while the profitability of two-part pricing without commitment relative to one-part pricing depends on the magnitude of the usage and access demand elasticities.

The rationale for the above results is that, under one-part pricing, the optimal unit price must consider not only the price sensitivity of consumers in choosing among product varieties (and the no-purchase option), but also the price sensitivity of usage demand. When the price elasticity of usage demand is sufficiently high, unit prices are sufficiently low under one-part pricing to induce lower profits and higher consumer welfare relative to either form of two-part pricing. By contrast, under two-part pricing, the fixed fee is set to maximize profits subject to inter-brand substitution and consumers' option not to purchase. When substitutability toward these alternatives is relatively low—implying that the price elasticity of access demand is relatively low—the total consumer payment increases, leading to lower consumer welfare and higher firm profits relative to one-part pricing.

To make our intuition more precise, we recall that $\partial\pi_j/\partial p_j = x_j(\partial\pi_j/\partial r_j)$, which allows us to restate the first-order condition for optimal one-part pricing (see equation (5.2)) as follows:

$$\frac{\partial\Gamma_j(\mathbf{p},\mathbf{0})}{\partial p_j} = \frac{\partial\ln\pi_j}{\partial r_j}[-f_j + (p_j - m_j)x_j] + \left[1 + \frac{p_j - m_j}{p_j}\left(\frac{\partial x_j}{\partial p_j}\frac{p_j}{x_j}\right)\right] = 0. \quad (5.4)$$

In turn, the above expression can be further restated in terms of average net revenue per user:

$$ANRPU_j = \left(-\frac{\partial\ln\pi_j}{\partial r_j}\right)^{-1}\left[1 + \frac{p_j - m_j}{p_j}\left(\frac{\partial x_j}{\partial p_j}\frac{p_j}{x_j}\right)\right]. \quad (5.5)$$

Compare this last expression to the optimal $ANRPU$ under two-part pricing with unit-price commitment, as described by equation (3.8). When the elasticity of usage demand equals zero (*i.e.*, $\epsilon_j = (-\partial x_j/\partial p_j)(p_j/x_j) = 0$), the two conditions are equivalent.[10] Hence, one-part pricing and two-part pricing with (and without) unit-price commitment lead to the same level of industry profits and consumer surplus. Social welfare is the same in both cases.

By contrast, when the elasticity of usage demand approaches infinity (*i.e.*, $\epsilon_j \to \infty$) and customer-specific fixed costs equal zero (*i.e.*, $f_j = 0$), the optimal unit price under one-part pricing approaches marginal cost regardless of consumer price sensitivity in choosing between varieties (see equation 5.3)). Since there is no fixed fee, and since the unit price approaches the marginal-cost level obtained under two-part pricing with unit-price commitment, consumer and social welfare are necessarily higher under one-part pricing when compared with two-part pricing with commitment. This is because total surplus per customer is virtually identical under the two pricing policies, but more consumers purchase the differentiated product under one-part pricing. At the same time, one-part pricing necessarily produces lower profits than either form of two-part pricing.

Thus, if the usage demand elasticity equals zero, one-part pricing and two-part pricing produce the same social welfare level when there are no customer-specific fixed costs. However, as the usage demand elasticity approaches infinity, one-part pricing dominates two-part pricing in terms of consumer and social welfare, but it produces lower profits. Under appropriate conditions, it is possible that one-part pricing also dominates two-part pricing in terms of consumer and social welfare, but suffers in terms of profits, at intermediate usage elasticity levels (in the absence of customer-specific fixed costs).

[10] The decline in indirect utility resulting from a marginal increase in the unit price, p_j, is the same as that resulting from an x_j increase in the fixed fee level, r_j. When usage demand is insensitive to changes in the unit price, and x_j is thus constant, firms can offer consumers the same indirect utility (and consequently attain the same market penetration rate) under one-part and two-part pricing while earning the same profit per customer (*i.e. ANRPU).* This is not the case when usage demand is sensitive to changes in price. See Appendix for further details.

The above welfare results depart from earlier results that compare one-part and two-part pricing, such are Gasmi, Moreaux, and Sharkey (2000), Armstrong and Vickers (2001), and Yin (2004). Those papers make more restrictive assumptions regarding consumer demand (*e.g.*, a Hotelling or logit demand model with no outside purchase option) or the extent of product differentiation (*e.g.*, products are "nearly homogeneous," as when Hotelling "travel costs" are quite small or approach zero). None of these prior models considers the possibility of two-part pricing with aftermarket monopolization, and many ignore the potential presence of customer-specific fixed costs.

In our more general demand framework, we show that firms and consumers unequivocally benefit from two-part pricing where unit-price commitments are made in advance of consumers' choosing their suppliers. However, one-part pricing provides higher levels of consumer and social welfare than either form of two-part pricing when usage demand is sufficiently elastic and customer-specific fixed costs are small or absent. By contrast, Yin finds that two-part pricing with unit-price commitment provides higher levels of social welfare than one-part pricing assuming that markets are fully covered. Armstrong and Vickers find that discriminatory pricing (*e.g.*, two-part tariffs) produces higher social welfare in a Hotelling framework when product differentiation is sufficiently small. We show that these results are not sustained when consumers can refrain from purchasing the product in question, some product differentiation exists (*i.e.*, there is a finite access demand elasticity), and usage demand is sufficiently elastic.

6. Two-Part Pricing with Heterogeneous Usage

Throughout the prior analysis, we assumed that consumers had common usage demands. In this section, consumers now differ in their usage demands, but firms continue to set a single two-part tariff for all customers. Even with advance commitments to unit prices, firms now depart from marginal-cost pricing as they competitively discriminate among customer types with differing usage and brand preferences.

Let there be two types of consumers, A and B, with differing usage demands, that have mass n_A and n_B, respectively. It is assumed that firms cannot observe directly consumer types, which would enable them to engage in third-degree price discrimination. Modifying slightly the original random-utility model described by equation (2.2), we assume that consumer i's utility for product j is as follows:

$$v_{ij}(p_j, r_j) = V_j^t(p_j, r_j) + \varepsilon_{ij}^t,$$

where $t = A, B$ denotes consumer i's type. We also assume that the usage demand of type-B

consumers exceeds that of type-A consumers, as expressed by:[11]

$$x_j^A(p_j, r_j) < x_j^B(p_j, r_j) \text{ for all } (p_j, r_j) \text{ such that } x_j^A(p_j, r_j) > 0. \tag{6.1}$$

Based on the expected purchases of the two consumer types, A and B, firm profits can be expressed as follows:

$$\Gamma_j(\mathbf{p}, \mathbf{r}) = \Sigma_{t=A,B}\; n_t \pi_j^t(\mathbf{p}, \mathbf{r}) \left[r_j - f_j + (p_j - m_j) x_j^t(p_j)\right], \tag{6.2}$$

where the costs of serving the two types of customers are assumed to be identical. Under two-part pricing with commitment, each firm sets its prices to satisfy the following first-order conditions (assuming again that usage demand is independent of income, $\partial x_j / \partial r_j = 0$):

$$\frac{\partial \Gamma_j(\mathbf{p}, \mathbf{r})}{\partial r_j} = \Sigma_{t=A,B}\; n_t \left\{ \frac{\partial \pi_j^t}{\partial r_j} \left[r_j - f_j + (p_j - m_j) x_j^t\right] + \pi_j^t \right\} = 0, \tag{6.3}$$

$$\frac{\partial \Gamma_j(\mathbf{p}, \mathbf{r})}{\partial p_j} = \Sigma_{t=A,B}\; n_t \left\{ \frac{\partial \pi_j^t}{\partial p_j} \left[r_j - f_j + (p_j - m_j) x_j^t\right] + \pi_j^t \left[x_j^t + (p_j - m_j) \frac{\partial x_j^t}{\partial p_j}\right] \right\} = 0. \tag{6.4}$$

Without loss of generality, let $x_j^B(p_j) = [1 + \lambda(p_j)] x_j^A(p_j)$, where $\lambda(p_j) > 0$ for all p_j such that $x_j^A(p_j) > 0$. We can substitute this expression, as well as equation (2.8) (*i.e.*, $\partial \pi_j / \partial p_j = x_j(\partial \pi_j / \partial r_j)$, into equation (6.4). Then, by substituting equation (6.3) into equation (6.4), we obtain the following result:

$$\lambda(p_j) x_j^A n_B \left\{ \frac{\partial \pi_j^B}{\partial r_j} \left[r_j - f_j + (p_j - m_j) x_j^B\right] + \pi_j^B \right\} = -(p_j - m_j) \left[n_A \pi_j^A \frac{\partial x_j^A}{\partial p_j} + n_B \pi_j^B \frac{\partial x_j^B}{\partial p_j} \right]. \tag{6.5}$$

The optimal pricing solution must simultaneously satisfy both equations (6.3) and (6.5), implying that the optimal unit price may be *above* or *below* marginal cost. To see this, note that equation (6.3) can be rewritten as follows:

$$\frac{\partial \Gamma_j(\mathbf{p}, \mathbf{r})}{\partial r_j} = \Sigma_{t=A,B}\; n_t \pi_j^t \left\{ \frac{-\eta_j^t}{r_j} \left[r_j - f_j + (p_j - m_j) x_j^t\right] + 1 \right\} = 0, \tag{6.3a}$$

[11] To illustrate, consider the specification of random utility that is quadratic in unit prices:

$$V_j^t(p_j, r_j) + \varepsilon_{ij}^t = \xi_j + \gamma \left[y - r_j - \alpha_j^t p_j + \frac{1}{2} \beta_j^t p_j^2 \right] + \varepsilon_{ij}^t.$$

Under this specification, usage demand has the linear form, $x_j^t(p_j) = \alpha_j^t - \beta_j^t p_j$. Condition (6.1) holds, for instance, when $\alpha_j^A < \alpha_j^B$ and $\beta_j^A = \beta_j^B$.

where $\eta_j^t \equiv (-\partial \pi_j^t / \partial r_j)(r_j / \pi_j^t)$ is type t consumers' elasticity of access demand for product variety j with respect to its fixed fee. If type B consumers are proportionately more price sensitive in their access decisions (*i.e.,* $\eta_j^A < \eta_j^B$), equations (6.3a) and (6.5) will be satisfied jointly only if

$$\frac{-\eta_j^B}{r_j}\left[r_j - f_j + (p_j - m_j)x_j^B\right] + 1 < 0.$$

Since the above expression corresponds in sign to the bracketed term on the left side of equation (6.5), the equilibrium unit price must be below marginal cost (*i.e.*, $p_j < m_j$) for that equation to be satisfied (given that $\partial x_j^t / \partial p_j < 0$). If we assume alternatively that $p_j \geq m_j$, then both equations cannot be satisfied.

Otherwise, if type B consumers are proportionately less sensitive to a change in the fixed fee (*i.e.,* $\eta_j^A > \eta_j^B$), then equations (6.3a) and (6.5) can be jointly satisfied only if the equilibrium unit price is greater than marginal cost (*i.e.*, $p_j > m_j$). Unit price equals marginal cost only if the elasticity of access demand is the same for both consumer types. We summarize this discussion in the following proposition:

Proposition 6.1. *With consumers who differ in their usage demands, equilibrium unit prices under two-part pricing with unit-price commitment can deviate from marginal cost. Unit prices are less (greater) than marginal cost if high-volume consumers have a larger (smaller) elasticity of access demand with respect to increases in the fixed fee than low-volume customers. If both types of consumers have equal elasticities of access demand, then unit prices are optimally set at marginal cost.*

The intuition behind the above proposition is as follows. With heterogeneous usage preferences and a single two-part tariff structure, an equilibrium fixed fee will not extract maximum profits from each consumer type. If high-volume consumers are *less* price responsive than low-volume consumers in selecting a supplier, then the fixed fee "undershoots" the level that would be optimal for high-volume consumers alone. Further profits are extracted from high-volume consumers if total charges (net of costs) could be raised to that consumer type relative to low-volume consumers. Setting price above marginal cost achieves this objective.

By contrast, if high-volume consumers are relatively *more* price responsive than low-volume consumers in choosing a supplier, then the equilibrium fixed fee "overshoots" the optimal level for extracting profits from high-volume consumers only. Consequently, the optimal unit price is set below marginal cost to compensate for this "overshooting" effect. So, in this case, firms select their unit prices in a manner that allows them to price discriminate among consumers that exhibit differing degrees of product substitutability, where those differences in substitutability are related to the intensity of product usage.

Note that competition may effectively eliminate the ability of firms to use a single two-part tariff to extract higher profits from high-volume consumers that have a relatively high willingness to pay for the good in general. In the above example, even if high-volume consumers generally receive higher indirect utility from using the good (regardless of the supplier), competition for these customers may cause less rent to be extracted relative to low-volume customers with a lower willingness to pay for the good. This situation arises if high-volume customers are more price sensitive in choosing suppliers than low-volume customers, leading firms to set unit prices below marginal cost.

Lastly, it should be mentioned that the dispersion of consumer preferences, as opposed to the mere impact of competition, is a large driver of the above results. In models where consumers using the same quantity of the product derive identical utility, a monopolist operating under a single two-part pricing scheme optimally sets unit prices above marginal cost in order to extract higher surplus from high-volume customers with higher willingness to pay for the good. In our model with stochastic consumer preferences among purchasers of the same product quantity, a monopolist may be unable to extract higher rents from high-volume customers even if those customers have a higher *expected* willingness to pay for the good. With a potential "no purchase" option, the monopolist may optimally set unit prices below marginal cost if high-volume customers are more price sensitive than low-volume customers in deciding whether to purchase the product.[12] This might be the case, for example, if high-volume customers have substantially greater dispersion in their indirect utilities (*i.e.*, reservation prices) when compared to low-volume customers.

7. Conclusion

In this article, we analyzed price competition when consumers make variable purchases exclusively from a single supplier of a differentiated product. With consumer behavior described by a generalized discrete-continuous choice model, we examined the connection between competition for customers and competition for sales. Specifically, we examined the differences in firm profits, consumer utility, and social welfare experienced under one-part pricing and two-part pricing, where firms under two-part pricing might or might not commit to specified unit prices in advance of consumers' deciding on an exclusive supplier.

Under two-part pricing with unit-price commitment, firms set a high upfront fee (*e.g.*, initial purchase price in the case of durable goods or component systems), but charge cost-based unit

[12] See Oi (1971). When a monopolist can post a menu of different two-part tariffs, consumers with heterogeneous usage preferences can be sorted more efficiently. Nonetheless, the monopolist cannot "perfectly" extract all potential rent because of costs involved in inducing self-selection.

prices for product usage (*e.g.*, consumables, repair services, add-ons, or product upgrades in the case of durable goods or component systems). Without unit-price commitment, a bargain-then-rip-off pricing pattern arises where a low upfront fee, possibly below customer-specific fixed costs, is combined with a high unit price. This type of pricing is also consistent with models of aftermarket monopolization, where consumers buy consumables, repair services, add-ons, or upgrades at high prices subsequent to an initial product purchase.

We find that two-part pricing with unit-price commitment is the dominant unilateral profit-maximizing pricing policy when compared to one-part pricing and two-part pricing without unit-price commitment. It is also jointly profit-maximizing provided that access demand is log-concave in price. Moreover, two-part pricing with commitment provides higher consumer and social welfare relative to two-part pricing without commitment.

Consequently, neither firms nor consumers benefit from a pricing policy where firms do not commit to unit prices, implying that the lack of commitment stems from either an inability to make such a commitment or other factors such as cost, demand, or technology uncertainties that firms prefer to wait to be resolved. Another way of viewing our results is that firms which are unable to make aftermarket pricing commitments would earn greater profits by inducing aftermarket competition that drives unit prices sufficiently close to marginal cost.

Along with the above findings, our analysis shows that one-part pricing provides greater consumer and social welfare than two-part pricing without unit-price commitment. When consumers' usage demand becomes sufficiently price elastic and there are no customer-specific fixed costs, one-part pricing provides greater consumer and social welfare, but less profit, than either form of two-part pricing. Thus, producer and social interests diverge regarding the choice of two-part or one-part pricing, particularly when the usage demand elasticity is relatively high and customer-specific fixed costs are low. This finding contrasts with welfare results obtained previously using models that assume consumers cannot refrain from purchasing the product in question or that product heterogeneity is "small" (or approaching zero).

Lastly, the introduction of heterogeneous usage demand overturns our earlier finding that unit prices are set at marginal cost under two-part pricing with commitment. When high-usage customers are relatively more price sensitive than low-usage customers in choosing among suppliers, firms set unit prices below marginal cost. In this situation, firms under competition earn lower margins on high-usage customers that may have a greater general willingness to pay for the product itself. This finding contrasts markedly with results previously obtained under monopoly.

The examination of nonlinear pricing under imperfect competition is a fertile area for research, and we believe our approach provides answers to some critical questions. Our equilibrium results offer testable hypotheses regarding the structure of pricing in important markets such as communications and media services, consumer and business hardware-software products, and many personal services. In particular, this paper shows how the interplay between fore-market and aftermarket competition affects observed prices for product access and usage. Our discrete-continuous formulation could prove useful in answering other questions as well, such as extending simulation models of horizontal mergers to allow consumers to make multiple purchases from a single supplier subject to nonlinear pricing.[13]

Despite the realism of our formulation of differentiated-product oligopoly, some features have been sacrificed to get sharp results. In particular, we impose symmetry in order to make certain welfare comparisons. Firms are assumed to adopt the same kind of pricing policy when, in fact, mixtures of one-part and two-part pricing occur in practice. Our model also does not permit customers to switch suppliers after the initial selection. The option of switching suppliers would likely lessen the aftermarket monopolization that arises in the no-commitment case, and firms may invest in reputations for not behaving opportunistically toward their installed base when supplier-customer interaction is repeated continuously. These are just a few of the fruitful avenues to pursue in future research.

[13] The models of Werden and Froeb (1994) and others employ a logit demand framework to represent oligopoly equilibrium, and while consumers are exclusive with a single supplier in these models, they do not allow for nonlinear pricing because they take usage demand to be perfectly inelastic.

Appendix

Proof of Proposition 3.2

The text discussion has established all results of this proposition, except that the fixed fee under two-part pricing without unit-price commitment is lower relative to the case of two-part pricing with unit-price commitment. Let $g_j(p_j, r_j)$ represent the inverse of the price elasticity of access demand. In other words,

$$g_j(p_j, r_j) \equiv \left(-\frac{\partial \pi_j(p_j, r_j)}{\partial r_j} \frac{r_j}{\pi_j(p_j, r_j)}\right)^{-1}, .$$

where we have suppressed the notation for the constant rival prices, $(\mathbf{p}_{-j}, \mathbf{r}_{-j})$. Letting $\overline{r}_j$ represent the optimal fixed fee (and $\overline{p}_j = m_j$ the optimal unit price) under two-part pricing with unit-price commitment, it holds from equation (3.9) that:

$$\frac{\overline{r}_j - f_j}{\overline{r}_j} = g_j(\overline{p}_j, \overline{r}_j).$$

By contrast, the optimal fixed fee $\widehat{r}_j$ in the absence of unit-price commitment satisfies (see equation (3.14)):

$$\frac{\widehat{r}_j - f_j}{\widehat{r}_j} = g_j(\widehat{p}_j, \widehat{r}_j) + \frac{\widehat{p}_j \widehat{x}_j}{\widehat{r}_j} \left(\frac{\partial x_j}{\partial p_j} \frac{\widehat{p}_j}{\widehat{x}_j}\right)^{-1},$$

where $\widehat{p}_j$ is the optimal unit price, $\widehat{x}_j = x_j(\widehat{p}_j)$, and $(\partial x_j/\partial p_j)(\widehat{p}_j/\widehat{x}_j) < 0$.

Now, assume that the marginal utility of income is constant (*i.e.*, $\partial V_j/\partial r_j = \gamma, \forall r_j$), and thus usage demand is independent of income. If the indirect utility, V_j, is the same under both two-part pricing with and without unit-price commitment, then π_j, $\partial \pi_j/\partial r_j$, and $\partial \ln \pi_j/\partial r_j$ are the same under both pricing policies (see equations (2.3) and (2.4)). However, given that unit prices are higher without commitment, V_j can be the same in both cases only if r_j is relatively lower under two-part pricing without unit-price commitment.

If, by contrast, r_j is the same in both cases, and $\partial \ln \pi_j/\partial r_j$ is decreasing in r_j (by assumption (A1)), then

$$-\frac{\partial \ln \pi_j(\widehat{p}_j, r_j)}{\partial \ln r_j} > -\frac{\partial \ln \pi_j(\overline{p}_j, r_j)}{\partial \ln r_j} \forall r_j,$$

which implies that

$$g_j(\widehat{p}_j, r_j) < g_j(\overline{p}_j, r_j) \ \forall r_j.$$

Hence, if

$$\frac{\overline{r}_j - f_j}{\overline{r}_j} = g(\overline{p}_j, \overline{r}_j),$$

it necessarily follows that:

$$\frac{\overline{r}_j - f_j}{\overline{r}_j} > g(\widehat{p}_j, \overline{r}_j) + \frac{\widehat{p}_j \widehat{x}_j}{\overline{r}_j} \left(\frac{\partial \widehat{x}_j}{\partial p_j} \frac{\widehat{p}_j}{\widehat{x}_j} \right)$$

Since $r_j - f_j$ is increasing and $\partial \ln \pi_j(p_j, r_j)/\partial r_j$ (and, by extension, $r_j g(p_j, r_j)$) is decreasing in r_j, the optimal fixed fee without commitment, $\widehat{r}_j$, is necessarily less than the fixed fee with commitment, $\overline{r}_j$. Hence, $\widehat{r}_j < \overline{r}_j$. ■

Proof of Proposition 4.1

Once again, let "−"denote "two-part pricing with unit-price commitment," and "∧" denote "two-part pricing without unit-price commitment." Since we examine the case of symmetric competition, the subscript j is frequently dropped for expositional convenience.

Assume that profits per customer (*i.e.*, $ANRPU$) are the same in the no-commitment and commitment cases, which implies that the following condition holds:

$$\widehat{r} - f + (\widehat{p} - m)x(\widehat{p}) = \overline{r} - f + (\overline{p} - m)x(\overline{p}),$$

where $\widehat{p} > m$, $\overline{p} = m$, and $x(\widehat{p}) < x(\overline{p})$ (since $\widehat{p} > \overline{p}$ and $\partial x(p)/\partial p < 0$). Making the appropriate substitutions into the above equation, we can alternately express the *equal* $ANRPU$ assumption as follows:

$$\widehat{r} = \overline{r} - (\widehat{p} - m)x(\widehat{p}).$$

However, *if profits per customer are the same in the no-commitment and commitment cases (i.e.,* $\overline{ANRPU}_j = \widehat{ANRPU}_j$*), it necessarily holds that* $V_j(\widehat{p}, \widehat{r}_j) < V_j(\overline{p}, \overline{r})$ *(i.e.,* $V_j(\widehat{p}, \overline{r} - (\widehat{p} - m)x(\widehat{p})) < V_j(m, \overline{r})$*).* To arrive at this result, we use Roy's Identity to obtain the following:

$$\begin{aligned} V_j(\widehat{p}, r) - V_j(m, r) &= \int_m^{\widehat{p}} (\partial V_j(p, r)/\partial p) dp \\ &= \int_m^{\widehat{p}} -\gamma x(p) dp_j \\ &< -\gamma(\widehat{p} - m)x(\widehat{p}), \end{aligned} \tag{7.1}$$

where $\gamma = \partial V_j/\partial y = -\partial V_j/\partial r$. The inequality follows from the fact that $\partial x(p)/\partial p < 0$.

If usage volume is independent of income (*i.e.*, the marginal utility of income is constant), then the indirect utility function is additively separable with respect to the unit price and the fixed fee.

Hence, to compare indirect utility under different unit price and fixed fee combinations, we merely add the impact on indirect utility from differences in the unit price to the impact on indirect utility from differences in the fixed fee. Thus, it holds that:

$$\begin{aligned} V_j(\widehat{p},\widehat{r}_j) - V_j(\overline{p},\overline{r})_{|equal\ ANRPU} &= V_j(\widehat{p},\overline{r} - (\widehat{p} - m)x(\widehat{p})) - V_j(m,\overline{r}) \\ &= V_j(\widehat{p},\overline{r}) - V_j(m,\overline{r}) + \gamma(\widehat{p} - m)x(\widehat{p}) < 0, \end{aligned}$$

where the inequality is obtained by substituting equation (7.1). In other words, under the *equal* $ANRPU$ assumption, it necessarily holds that indirect utility is higher under two-part pricing with unit-price commitment relative to two-part pricing without unit-price commitment.[14]

Let us assume instead that *indirect utility is equal* under both forms of two-part pricing (*i.e.*, $V_j(\overline{p},\overline{r}) = V_j(\widehat{p},\widehat{r})$), which requires that profits per customer are necessarily higher under two-part pricing with unit-price commitment (*i.e.*, $\overline{ANRPU}_j > \widehat{ANRPU}_j$). Now, if $V_j(\overline{p},\overline{r}) = V_j(\widehat{p},\widehat{r})$ $\forall j$ (and the marginal utility of income is constant), then $\pi_j(\overline{\mathbf{p}},\overline{\mathbf{r}}) = \pi_j(\widehat{\mathbf{p}},\widehat{\mathbf{r}})$, $\partial\pi_j(\overline{\mathbf{p}},\overline{\mathbf{r}})/\partial r_j = \partial\pi_j(\widehat{\mathbf{p}},\widehat{\mathbf{r}})/\partial r_j$, and $\partial \ln \pi_j(\overline{\mathbf{p}},\overline{\mathbf{r}})/\partial r_j = \partial \ln \pi_j(\widehat{\mathbf{p}},\widehat{\mathbf{r}})/\partial r_j$ (see equations (2.3) and (2.4)), where $\overline{\mathbf{p}} = (\overline{p},...,\overline{p})$, $\widehat{\mathbf{p}} = (\widehat{p},...,\widehat{p})$, $\overline{\mathbf{r}} = (\overline{r},...,\overline{r})$, and $\widehat{\mathbf{r}} = (\widehat{r},...,\widehat{r})$.

If prices are set optimally under two-part pricing with unit-price commitment, then the following first-order condition is satisfied (see equation (3.8)):

$$\overline{ANRPU}_j = \left(-\frac{\partial \ln \pi_j(\overline{\mathbf{p}},\overline{\mathbf{r}})}{\partial r_j}\right)^{-1}.$$

However, since our *equal utility* assumption implies that $\overline{ANRPU}_j > \widehat{ANRPU}_j$ and that $\partial \ln \pi_j(\overline{\mathbf{p}},\overline{\mathbf{r}})/\partial r_j$ $= \partial \ln \pi_j(\widehat{\mathbf{p}},\widehat{\mathbf{r}})/\partial r_j$, it necessarily holds that

$$\widehat{ANRPU}_j < \left(-\frac{\partial \ln \pi_j(\widehat{\mathbf{p}},\widehat{\mathbf{r}})}{\partial r_j}\right)^{-1}.$$

The above result is *inconsistent* with the first-order condition for the optimal fixed fee under two-part pricing without unit-price commitment (see equation (3.13)), which states that $\widehat{ANRPU}_j = (-\partial \ln \pi_j(\widehat{\mathbf{p}},\widehat{\mathbf{r}})/\partial r_j)^{-1}$. Given that $\widehat{ANRPU}_j$ is increasing in the fixed fee and $(-\partial \ln \pi_j(\widehat{\mathbf{p}},\mathbf{r}')/\partial r_j)^{-1}$ is decreasing in r' (since $\partial^2 \ln \pi_j(\widehat{\mathbf{p}},\mathbf{r}')/\partial r_j \partial r' < 0$ by assumption (A2)), this first-order condition

[14] This result can be proved more generally by a revealed preference argument. Given that $\overline{p} = m$ and that $\widehat{r} = \overline{r} - (\widehat{p} - m)x(\widehat{p})$, the same bundle, consisting of $x(\widehat{p})$ units of variety j of the differentiated good and $y - \overline{r} - mx(\widehat{p})$ units of the numeraire good, can potentially be purchased in both the no-commitment and the commitment cases. However, in the commitment case, each consumer maximizes his utility by instead purchasing $x(m)$ units of variety j, indicating that the consumer realizes higher utility from this bundle than the bundle consumed in the no-commitment case.

can be satisfied (and a symmetric equilibrium attained) only if *the equilibrium fixed fee under two-part pricing without commitment exceeds the level that yields the same indirect utility as under two-part pricing with commitment*. Thus, in equilibrium, $V_j(\widehat{p},\widehat{r}) < V_j(\overline{p},\overline{r})$.

Consequently, both indirect utility and the number of product purchasers are necessarily higher with unit-price commitment than without unit-price commitment (because higher indirect utility implies that fewer consumers choose the no-purchase option). Since total surplus per customer, consumer indirect utility, and the number of product purchasers are all higher, *consumer and social welfare must be higher under two-part pricing with unit-price commitment as opposed to two-part pricing without unit-price commitment*. Total surplus (*i.e.*, firm profits plus consumer surplus) per customer is higher under two-part pricing with commitment as a result of setting unit prices at marginal cost, as opposed to the monopoly level when there is no unit-price commitment.

Now, assume once again that *profits per customer (i.e., ANRPU) are equal* under both pricing policies. If $\overline{ANRPU}_j = \widehat{ANRPU}_j$, then the fixed fee is necessarily higher under two-part pricing without commitment (or lower under two-part pricing with commitment) than in the *equal utility* case. Since $\partial \ln \pi_j(\widehat{\mathbf{p}},\mathbf{r}')/\partial r_j$ is decreasing in r' by assumption (A2), it follows that $\partial \ln \pi_j(\widehat{\mathbf{p}},\widehat{\mathbf{r}})/\partial r_j < \partial \ln \pi_j(\overline{\mathbf{p}},\overline{\mathbf{r}})/\partial r_j$ under the *equal ANRPU* assumption. Hence, if prices are set optimally under two-part pricing with commitment, it must hold under the "equal $ANRPU$" assumption that:

$$\overline{ANRPU}_j = \left(-\frac{\partial \ln \pi_j(\overline{\mathbf{p}},\overline{\mathbf{r}})}{\partial r_j}\right)^{-1}$$

and

$$\widehat{ANRPU}_j > \left(-\frac{\partial \ln \pi_j(\widehat{\mathbf{p}},\widehat{\mathbf{r}})}{\partial r_j}\right)^{-1}.$$

Given that $\widehat{ANRPU}_j$ is increasing in the fixed fee and $(-\partial \ln \pi_j(\widehat{\mathbf{p}},\mathbf{r}')/\partial r_j)^{-1}$ is decreasing in r' (since $\partial^2 \ln \pi_j(\widehat{\mathbf{p}},\mathbf{r}')/\partial r_j \partial r' < 0$ by assumption (A2)), the equilibrium fixed fee under two-part pricing without unit-price commitment, which satisfies $\widehat{ANRPU}_j = (-\partial \ln \pi_j(\widehat{\mathbf{p}},\widehat{\mathbf{r}})/\partial r_j)^{-1}\ \forall j$, must be below the level that yields the same profits per customer as under two-part pricing with unit-price commitment. Since profits per customer and the number of product purchasers are both higher under two-part pricing with unit-price commitment (because indirect utility is greater), *total firm profits are higher under unit-price commitment.* ■

Proof of Corollary 4.2

In the case of logit demand, the demand for firm j's product is expressed as follows:

$$\pi_j(\mathbf{p},\mathbf{r}) = \frac{\exp V_j(p_j,r_j)}{1+\sum_{k=1}^{J}\exp V_k(p_k,r_k)},$$

where $V_0 = 0$. Consequently, it holds that:

$$\frac{\partial \ln \pi_j(\mathbf{p}, \mathbf{r})}{\partial r_j} = (1 - \pi_j)\frac{\partial V_j}{\partial r_j},$$

and

$$\frac{\partial^2 \ln \pi_j(\mathbf{p}, \mathbf{r})}{\partial r_j^2} = (1 - \pi_j)\frac{\partial^2 V_j}{\partial r_j^2} - \frac{\partial \pi_j}{\partial r_j}\frac{\partial V_j}{\partial r_j},$$

$$\frac{\partial^2 \ln \pi_j(\mathbf{p}', \mathbf{r}')}{\partial r_j \partial r'} = (1 - \pi_j)\frac{\partial^2 V_j}{\partial r_j^2} - \frac{\partial \pi_j}{\partial r'}\frac{\partial V_j}{\partial r_j}.$$

The marginal utility of income is nonincreasing (*i.e.*, $\partial^2 V_j/\partial r_j^2 = \partial^2 V_j/\partial y^2 \leq 0$). Since $\partial V_j/\partial r_j < 0$, $\partial \pi_j/\partial r_j < 0$, and $\partial \pi_j/\partial r' < 0$ (because the no-purchase option becomes more attractive when all suppliers raise their fixed fees), it necessarily holds that $\partial^2 \ln \pi_j/\partial r_j^2 < 0$ and $\partial^2 \ln \pi_j/\partial r_j \partial r' < 0$. Consequently, both assumptions (A1) and (A2) are satisfied. ■

Proof of Proposition 5.2

This proof is analogous to the welfare and profits comparison involving two-part pricing with and without unit-price commitment. We initially examine indirect utility per customer under the different pricing policies. First, we show that indirect utility per customer is higher under one-part pricing relative to two-part pricing *without* commitment, implying that more consumers purchase the differentiated product under one-part pricing. This result ensures that consumer and social welfare are also higher under one-part pricing, since total surplus per customer is necessarily higher (due to lower unit prices) and the number of customers is larger.

Secondly, we show that the indirect utility per customer may be higher or lower under one-part pricing relative to two-part pricing *with* commitment. Thus, even though total surplus per customer is higher under two-part pricing with commitment (due to lower unit prices), more customers may be served under one-part pricing. Consequently, social welfare may be relatively higher under one-part pricing. We show that welfare is, in fact, higher when the elasticity of usage demand is sufficiently large (and customer-specific fixed costs are sufficiently small).

After comparing indirect utility under the three pricing policies, we next compare profits per customer. First, we show that profits per customer are lower in equilibrium under one-part pricing relative to two-part pricing *with* commitment. This effectively ensures that total firm profits are higher under two-part pricing with commitment.[15]

[15] Firms earn greater per-customer profits under two-part pricing with commitment relative to one-part pricing. Under this pricing policy, they also may serve more customers relative to one-part pricing, which necessarily ensures greater overall firm profits.

Secondly, we show that profits per customer under one-part pricing may be higher or lower than under two-part pricing *without* commitment, depending on the price elasticity of usage demand. Although the number of product purchasers is necessarily greater under one-part pricing, this result implies that total firm profits may still be higher under two-part pricing without commitment. [Profits per customer under one-part pricing necessarily fall to zero as the usage demand elasticity increases to infinity, while profits under two-part pricing remain positive (assuming that the access demand elasticity is finitely valued).]

Indirect Utility Comparison

Let "–" again denote "two-part pricing with unit-price commitment," "∧" denote "two-part pricing without unit-price commitment," and "1" denote "one-part pricing." Assuming that *profits per customer are the same* in all three cases (*i.e.*, $\overline{ANRPU_j} = ANRPU_j^1 = \widehat{ANRPU}_j$), it can be readily shown (*e.g.*, by revealed preference) that $V_j(\overline{p}, \overline{r}) > V_j(p^1, 0) > V_j(\widehat{p}, \widehat{r})$ because unit prices are lowest under two-part pricing with commitment and highest under two-part pricing without commitment.

Let us assume instead that *indirect utility is equal under all three pricing policies.* Hence, $V_j(\overline{p}, \overline{r}) = V_j(p^1, 0) = V_j(\widehat{p}, \widehat{r})$, which necessarily requires that $\overline{ANRPU_j} > ANRPU_j^1 > \widehat{ANRPU}_j$. Now, if $V_j(\overline{p}, \overline{r}) = V_j(p^1, 0) = V_j(\widehat{p}, \widehat{r})$, and if the marginal utility of income is constant (*i.e.*, $\partial V_j/\partial y = -\partial V_j/\partial r = \gamma \ \forall r_j$), it necessarily follows under our *equal utility* assumption that (see equations (2.3) and (2.4)):

$$\frac{\partial \ln \pi_j(\overline{\mathbf{p}}, \overline{\mathbf{r}})}{\partial r_j} = \frac{\partial \ln \pi_j(\mathbf{p^1}, \mathbf{0})}{\partial r_j} = \frac{\partial \ln \pi_j(\widehat{\mathbf{p}}, \widehat{\mathbf{r}})}{\partial r_j}.$$

Now, when prices are set optimally under one-part pricing, it holds that (see equation (5.5)):

$$ANRPU_j^1 = \left(-\frac{\partial \ln \pi_j(\mathbf{p^1}, \mathbf{0})}{\partial r_j}\right)^{-1} \left(1 + \frac{(p^1 - m)}{p^1}\left(\frac{\partial x}{\partial p}\frac{p^1}{x^1}\right)\right),$$

Alternatively, firms under two-part pricing with commitment may serve fewer customers in (a symmetric) equilibrium than under one-part pricing. This situation requires that each firm offer lower indirect utility per consumer relative to one-part pricing. Since total surplus per customer is higher under two-part pricing with commitment relative to one-part pricing (because unit prices equal marginal cost), and since rival firms are offering lower indirect utility to consumers relative to one-part pricing in this equilibrium, each firm *could* have served the same number of customers as under one-part pricing while earning higher per-customer profits.

However, revealed preference indicates that each firm under two-part pricing with commitment earns even greater profits by instead choosing to serve fewer customers at an even higher profit level per customer. Thus, two-part pricing with commitment necessarily produces higher profits for each firm than one-part pricing, even if fewer customers are served.

where $x^1 = x(p^1)$. Since our *equal utility* assumption implies that $ANRPU_j^1 > \widehat{ANRPU}_j$ and that $\partial \ln \pi_j(\mathbf{p^1}, \mathbf{0})/\partial r_j = \partial \ln \pi_j(\widehat{\mathbf{p}}, \widehat{\mathbf{r}})/\partial r_j < 0$, and since $[(p^1 - m)/p^1]\ (\partial x/\partial p)\ (p^1/x^1) < 0$, it necessarily follows from the above equation that:

$$\widehat{ANRPU}_j < \left(- \frac{\partial \ln \pi_j(\widehat{\mathbf{p}}, \widehat{\mathbf{r}})}{\partial r_j} \right)^{-1}.$$

Under optimal two-part pricing without unit-price commitment, the above condition must instead hold with equality (see equation (3.13)), implying that the indirect utility per customer cannot be the same as under one-part pricing. Given that $\widehat{ANRPU}_j$ is increasing in the fixed fee, and that $(-\partial \ln \pi_j(\widehat{\mathbf{p}}, \mathbf{r}')/\partial r_j)^{-1}$ is decreasing in r' (by condition (A2)), the equilibrium fixed fee under two-part pricing without commitment must exceed the level that yields the same indirect utility per customer as under one-part pricing. *Consequently, in equilibrium, indirect utility per customer is higher under one-part pricing relative to two-part pricing without commitment (i.e., $V_j(p^1, 0) > V_j(\widehat{p}, \widehat{r}_j)$).*

Next, since $ANRPU_j^1 < \overline{ANRPU_j}$ and $\partial \ln \pi_j(\mathbf{p^1}, \mathbf{0})/\partial r_j = \partial \ln \pi_j(\overline{\mathbf{p}}, \overline{\mathbf{r}})/\partial r_j$ under our *equal utility* assumption, the following condition must hold when prices are set optimally under one-part pricing:

$$\begin{aligned} ANRPU_j^1 &= \left(- \frac{\partial \ln \pi_j(\mathbf{p^1}, \mathbf{0})}{\partial r_j} \right)^{-1} \left(1 + \frac{(p^1 - m)}{p^1} \left(\frac{\partial x}{\partial p} \frac{p^1}{x^1} \right) \right) \\ &= \left(- \frac{\partial \ln \pi_j(\overline{\mathbf{p}}, \overline{\mathbf{r}})}{\partial r_j} \right)^{-1} \left(1 + \frac{(p^1 - m)}{p^1} \left(\frac{\partial x}{\partial p} \frac{p^1}{x^1} \right) \right) \\ &< \overline{ANRPU_j}. \end{aligned}$$

From the last inequality, it follows that

$$\overline{ANRPU_j} + \left(\frac{\partial \ln \pi_j(\overline{\mathbf{p}}, \overline{\mathbf{r}})}{\partial r_j} \right)^{-1} > \left(- \frac{\partial \ln \pi_j(\overline{\mathbf{p}}, \overline{\mathbf{r}})}{\partial r_j} \right)^{-1} \left(\frac{(p^1 - m)}{p^1} \left(\frac{\partial x}{\partial p} \frac{p^1}{x^1} \right) \right),$$

where

$$\left(- \frac{\partial \ln \pi_j(\overline{\mathbf{p}}, \overline{\mathbf{r}})}{\partial r_j} \right)^{-1} \left(\frac{(p^1 - m)}{p^1} \left(\frac{\partial x}{\partial p} \frac{p^1}{x^1} \right) \right) < 0,$$

given that $\partial \ln \pi_j(\overline{\mathbf{p}}, \overline{\mathbf{r}})/\partial r_j < 0$, $p^1 > m$, and $\partial x/\partial p < 0$. Hence,

$$\overline{ANRPU_j} + \left(\frac{\partial \ln \pi_j(\overline{\mathbf{p}}, \overline{\mathbf{r}})}{\partial r_j} \right)^{-1} <, =, > 0.$$

Given that optimal behavior under two-part pricing with unit-price commitment requires that $\overline{ANRPU_j} = (-\partial \ln \pi_j(\overline{\mathbf{p}}, \overline{\mathbf{r}})/\partial r_j)^{-1}$ (see equation (3.8)), the equilibrium fixed fee under this pricing policy may be above, equal to, or below the level that yields the same indirect utility level as under one-part pricing. *Consequently, indirect utility per customer may be higher, equal, or lower under one-part pricing relative to two-part pricing with commitment.*

In the absence of customer-specific fixed costs, the optimal one-part price approaches marginal cost (*i.e.*, $p_j^1 \to m_j$) as the elasticity of usage demand approaches infinity (*i.e.*, $(\partial x/\partial p)(p/x) \to -\infty$), implying that $ANRPU_j^1$ approaches zero (see equation (5.3)). In this case, our *equal utility* assumption implies that $\overline{ANRPU}_j$ also approaches zero, which necessarily means that $\overline{ANRPU}_j < (-\partial \ln \pi_j(\overline{\mathbf{p}},\overline{\mathbf{r}})/\partial r_j)^{-1}$(for $0 < -\partial \ln \pi_j/\partial r_j < \infty$). Based on assumption (A2), the equilibrium fixed fee under two-part pricing with unit-price commitment (satisfying $\overline{ANRPU}_j = (-\partial \ln \pi_j(\overline{\mathbf{p}},\overline{\mathbf{r}})/\partial r_j)^{-1} \; \forall j$) necessarily exceeds the level that yields the same indirect utility per customer as under one-part pricing. Hence, as the elasticity of usage demand approaches infinity, indirect utility per customer (and social welfare) is necessarily higher under one-part pricing.

As the elasticity of usage demand approaches zero (*i.e.*, $(\partial x/\partial p)(p/x) \to 0$), then $ANRPU_j^1 \to \overline{ANRPU}_j$ under our *equal utility* assumption (see above discussion). The first-order conditions under two-part pricing with commitment and one-part pricing also "converge" (see equations (3.8) and (5.5)), as does the equilibrium levels of indirect utility per customer and profits under these pricing policies.

Profits Comparison

Now, assume that equilibrium *profits per customer are equal* under all three pricing policies, implying that $\overline{ANRPU}_j = ANRPU_j^1 = \widehat{ANRPU}_j$. Consequently, $V_j(\overline{p},\overline{r}) > V_j(p^1,0) > V_j(\widehat{p},\widehat{r})$, and $\pi_j(\overline{\mathbf{p}},\overline{\mathbf{r}}) > \pi_j(\mathbf{p^1},\mathbf{0}) > \pi_j(\widehat{\mathbf{p}},\widehat{\mathbf{r}})$. Under assumption (A2), it necessarily follows that:

$$\frac{-\partial \ln \pi_j(\overline{\mathbf{p}},\overline{\mathbf{r}})}{\partial r_j} < \frac{-\partial \ln \pi_j(\mathbf{p^1},\mathbf{0})}{\partial r_j} < \frac{-\partial \ln \pi_j(\widehat{\mathbf{p}},\widehat{\mathbf{r}})}{\partial r_j}. \tag{7.2}$$

This result stems from the fact that, if indirect utility is equal under the different pricing policies (and the marginal utility of income is constant), then $\partial \ln \pi_j/\partial r_j$ is also equal under the different policies. However, under our *equal ANRPU* assumption, the fixed fee under two-part pricing with(without) commitment is necessarily below(above) the level that produces utility equal to that attained under one-part pricing. Hence, if access demand satisfies assumption (A2), we obtain the above inequality.

When prices are set optimally under one-part pricing, it holds that:

$$ANRPU_j^1 = \left(-\frac{\partial \ln \pi_j(\mathbf{p^1},\mathbf{0})}{\partial r_j}\right)^{-1}\left(1+\frac{(p^1-m)}{p^1}\left(\frac{\partial x}{\partial p}\frac{p^1}{x^1}\right)\right).$$

Given our *equal ANRPU* assumption (*i.e.*, $\overline{ANRPU}_j = ANRPU_j^1 = \widehat{ANRPU}_j$) and the results from equation (7.2), it necessarily follows that

$$\overline{ANRPU}_j < \left(-\frac{\partial \ln \pi_j(\overline{\mathbf{p}},\overline{\mathbf{r}})}{\partial r_j}\right)^{-1}.$$

Since optimal behavior under two-part pricing with unit-price commitment requires instead that $\overline{ANRPU}_j = (-\partial \ln \pi_j(\overline{\mathbf{p}},\overline{\mathbf{r}})/\partial r_j)^{-1}\ \forall j$, the equilibrium fixed fee necessarily exceeds the level that yields the same profits per customer as under one-part pricing (using assumption (A2)). *Consequently, profits per customer must be lower under one-part pricing relative to two-part pricing with commitment.*

Because our *equal* $ANRPU$ assumption implies that $-\partial \ln \pi_j(\mathbf{p^1}, \mathbf{0})/\partial r_j < -\partial \ln \pi_j(\widehat{\mathbf{p}}, \widehat{\mathbf{r}})/\partial r_j$ (see equation 7.2), it holds that:

$$\begin{aligned}
\widehat{ANRPU}_j &= ANRPU_j^1 \\
&= \left(-\frac{\partial \ln \pi_j(\mathbf{p^1}, \mathbf{0})}{\partial r_j}\right)^{-1} \left(1 + \frac{(p^1 - m)}{p^1}\left(\frac{\partial x}{\partial p}\frac{p^1}{x^1}\right)\right) \\
&> \left(\frac{-\partial \ln \pi_j(\widehat{\mathbf{p}}, \widehat{\mathbf{r}})}{\partial r_j}\right)^{-1} \left(1 + \frac{(p^1 - m)}{p^1}\left(\frac{\partial x}{\partial p}\frac{p^1}{x^1}\right)\right).
\end{aligned}$$

From this inequality, it follows that

$$\widehat{ANRPU}_j + \left(\frac{\partial \ln \pi_j(\widehat{\mathbf{p}}, \widehat{\mathbf{r}})}{\partial r_j}\right)^{-1} > \left(\frac{-\partial \ln \pi_j(\widehat{\mathbf{p}}, \widehat{\mathbf{r}})}{\partial r_j}\right)^{-1} \left(\frac{(p^1 - m)}{p^1}\left(\frac{\partial x}{\partial p}\frac{p^1}{x^1}\right)\right),$$

where

$$\left(\frac{-\partial \ln \pi_j(\widehat{\mathbf{p}}, \widehat{\mathbf{r}})}{\partial r_j}\right)^{-1} \left(\frac{(p^1 - m)}{p^1}\left(\frac{\partial x}{\partial p}\frac{p^1}{x^1}\right)\right) < 0,$$

given that $\partial \ln \pi_j(\widehat{\mathbf{p}}, \widehat{\mathbf{r}})/\partial r_j < 0$, $p^1 > m$, and $\partial x/\partial p < 0$. Hence,

$$\widehat{ANRPU}_j + \left(\frac{\partial \ln \pi_j(\widehat{\mathbf{p}}, \widehat{\mathbf{r}})}{\partial r_j}\right)^{-1} <,=,> 0.$$

Optimal behavior under two-part pricing without unit-price commitment requires that $\widehat{ANRPU}_j = (-\ \partial \ln \pi_j(\widehat{\mathbf{p}}, \widehat{\mathbf{r}})/\partial r_j)^{-1}\ \forall j$ (see equation (3.13)), implying that the equilibrium fixed fee under two-part pricing may be above, equal to, or below the level which yields the same profits per customer as under one-part pricing. *Consequently, profits per customer may be higher, equal, or lower under one-part pricing relative to two-part pricing without commitment.*

As the elasticity of usage demand approaches infinity (*i.e.*, $(\partial x/\partial p)(p/x) \to -\infty$), $ANRPU_j^1$ approaches zero. Hence, $\widehat{ANRPU}_j < (-\ \partial \ln \pi_j(\widehat{\mathbf{p}}, \widehat{\mathbf{r}})/\partial r_j)^{-1}$ under our *equal* $ANRPU$ assumption (for $0 < -\partial \ln \pi_j / \partial r_j < \infty$). Given assumption (A2), the equilibrium fixed fee under two-part pricing without commitment (satisfying $\widehat{ANRPU}_j = (-\,\partial \ln \pi_j(\widehat{\mathbf{p}}, \widehat{\mathbf{r}})/\partial r_j)^{-1}$) must be above the level that yields the same profits as under one-part pricing. Consequently, as the usage demand elasticity approaches infinity, total firm profits are necessarily lower under one-part pricing relative to two-part pricing without unit-price commitment. ■

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