## Econ 131

Spring 2023
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Final Exam

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## Exam Instructions:

- Explanation should be written using pens (we recommend black or blue ink, as these often scan the best). No pencils, except for graphs.
- Show your work. Credit will only be awarded on the basis of what is written on the exam.
- Sign the academic honesty pledge. Cheating will be dealt with harshly.

Student Name:

Student ID Number:

Affirm the academic honesty pledge below. For those writing on a non-printed copy, please just write "Academic Honesty Pledge as on exam", and sign your name.
If you do not affirm this pledge, your exam will be marked invalid.

## 0. ACADEMIC HONESTY PLEDGE

I confirm that I have abided by all academic honesty rules for UC Berkeley and Economics 131. I confirm that I did not see this exam before my official exam start time. I confirm that I have not shared and will not share this exam with anyone else. I confirm that I haven't copied from anybody else's exam.

Signature:

## 1. True/False/Uncertain (20 points, 2 points per question.)

Explain your answer fully based on what was discussed in class, since all the credit is based on the explanation. Your grade depends entirely on the substance of your justification, not on whether you are correct in writing "True" or "False". Note that it is possible to answer each question for full credit with three sentences or fewer, and answers longer than ten lines long will not be graded.
(a) The spike of retirement hazard at age 62 in the United States is driven by the US social security system.
Solution: TRUE: There is indeed a spike of retirement hazard at age 62 in the United States that is driven by the early retirement age for social security. We know it is a causal effect of social security for 2 reasons: (1) there is no other policy change at age 62 . (2) the spike at age 62 did not exist when the early retirement age for social security was 65 (before 1970), and emerged shortly afterwards (see slides).
(b) Longer unemployment benefits lead to longer unemployment spells.

Solution: TRUE. There is very compellingly identified empirical evidence of positive effects of longer unemployment benefits on unemployment spells, as we saw in class. For duration, we saw the Card-Chetty-Weber (2007) study using a regression discontinuity in Austria where you get up to 30 weeks of benefits when you have been employed for $36+$ months in last 5 years (instead of up to 20 weeks). There is a jump in duration of unemployment based on number of months previously employed precisely at the 36 threshold.
(c) The US time series of top $1 \%$ income shares estimated using (pre-tax) income reported on tax returns since 1913 shows that top incomes are very sensitive to the top marginal tax rate of the individual income tax.

Solution: TRUE: As we saw in class, there is a strong negative correlation between top income shares and top tax rates which is true in the big picture and also along specific tax reform episodes most notably the tax reform act of 1986.
(d) Absent social security, workers would simply save for their retirement themselves as predicted by the life-cycle model.

Solution: FALSE: It is true that the rational life-cycle model predicts that workers should save for retirement. In practice however, absent social security, many people are unable to
save by themselves and have to keep working into old age or get supported by their family (generally children). Therefore, in practice, government provided retirement benefits are needed and are indeed ubiquitous.
(e) A funded social security system is preferable to an unfunded social security system because it delivers bigger retirement benefits relative to taxes paid while working.

Solution: PARTLY TRUE: When the system is in place, a funded system's rate of return is given by the market rate of return on financial assets $r$ while the implicit rate of return of an unfunded system is given by $n+g$, population growth rate + per person earnings growth. Normally $r>n+g$ therefore the statement is true in steady state with the important caveat that $r$ might be lower than $n+g$ for some people if their investments delivers poor returns (as $r$ is risky). However, an unfunded system delivers benefits to the first generation who did not pay taxes, so for the first generation, the statement is false.
(f) Medicare health insurance in the United States saves lives.

Solution: True based on the study by Card et al. that we discussed in class and looks at likelihood of survival after emergency room admissions by age. There is a visible jump in survival when one crosses 65, which is the age of eligibility for Medicare.
(g) In the life-cycle model where people work and save when young and live off their savings and returns on savings when old, the government should not tax capital income.

Solution: True, this is the Atkinson-Stiglitz 1976 theorem discussed in class. It says that the the optimal tax rate on capital income should be zero. Using a labor tax on earnings is sufficient.
(h) You decide to create a start-up company that offers insurance against spilling liquid on latpops, and you believe that the probability that an individual spills liquid on their laptop in any given year is $1 \%$. Taking this into account, you decide to set the premium a $\$ 10$ per year for all your clients. You find out that among your clients, the probability of this incident occurring is actually $10 \%$. This is an example of adverse selection.

Solution: True/uncertain. While people who are more likely to spill liquid on their laptops are more willing to take up insurance, there may also be moral hazard effects.
(i) Berkeley decides to purchase 500 bicycles to begin offering a bike share program that is free to the public. This is an example of a pure public good (Bike share programs offer bicycles that are parked in public spaces and can be used for transportation).

Solution: False. Bicycles are a rival good.
(j) A reduction in the tax rate on interest income will increase overall savings.

Solution: False. As shown in class with a simple two period model, a reduction in a the tax rate on interest income will have two offsetting effects so it is unclear what the effects on overall savings would be. Specifically, the price effect makes second period consumption less expensive so individuals will substitute toward future consumption and increase saving. The wealth effect leads to an increase in present consumption and less savings since wealth has increased and period 2 consumption is a normal good.

## 2. Local Public Goods (20 points)

The city of Berkeley has decided to build a new BART station near the Greek Theater to improve access to public transportation on the east side of the university campus. The city of Berkeley plans to fund the functioning of this new BART station, and notably improves the frequency of the BART trains going through the station, from the contributions of only two types of individuals: UCB students (S) and UCB professors (P). Each of the two types of individuals have a utility function over private goods $\left(x_{i}\right)$ and total number of BART trains going through the new station everyday $(B)$, of the form:

$$
\begin{equation*}
U_{i}\left(x_{i}, B\right)=\left(x_{i}-3\right)(B-4) \tag{1}
\end{equation*}
$$

The total number of BART trains going through the station everyday $B$, is the sum of the number of trains paid for by each type of individuals, the students and the professors: $B=b_{s}+b_{p}$. The students have an income of $\$ 5$ and the professors have an income of $\$ 11$. Both the private good and a BART train have a price of $\$ 1$.
(a) How many BART trains will serve the new BART station everyday if the city of Berkeley does not intervene? How many are paid for by the students? By the professors? (4 points)

For the students, we can substitute $x_{s}=5-b_{s}$ and $B=b_{s}+b_{p}$, so $U_{s}=\ln \left(5-b_{s}-3\right)+$ $\ln \left(b_{s}+b_{p}-4\right)$. Now we can attempt to maximize the students' utility to find an interior solution:

$$
\begin{aligned}
\frac{\partial U_{s}}{\partial b_{s}} & =0 \\
\frac{1}{5-b_{s}-3} & =\frac{1}{b_{s}+b_{p}-4} \\
2-b_{s} & =b_{s}+b_{p}-4 \\
\Rightarrow \quad b_{s}^{*} & =\frac{6-b_{p}}{2}
\end{aligned}
$$

This is the students best response function for $b_{s}$ when the professors spends $b_{p}$ on the new $B A R T$ train system. For the professors, we can substitute $x_{p}=11-b_{p}$ and $B=b_{s}+b_{p}$, so $U_{p}=\ln \left(11-b_{p}-3\right)+\ln \left(b_{s}+b_{p}-4\right)$. We can attempt to maximize the professors' utility to find similarly the professors' best response function:

$$
\begin{aligned}
\frac{\partial U_{p}}{\partial b_{p}} & =0 \\
\frac{1}{11-b_{p}-3} & =\frac{1}{b_{s}+b_{p}-4} \\
8-b_{p} & =b_{s}+b_{p}-4 \\
\Rightarrow \quad b_{p}^{*} & =\frac{12-b_{s}}{2}
\end{aligned}
$$

Since their incomes are different, the best response functions are not symmetric. By solving this system of two equations, we have:

$$
\begin{aligned}
2 b_{p}^{*} & =12-\frac{6-b_{p}}{2} \\
2 b_{p}^{*} & =12-3+\frac{b_{p}}{2} \\
2 b_{p}^{*} & =9+\frac{b_{p}}{2} \\
\frac{3 b_{p}^{*}}{2} & =9 \\
b_{p}^{*} & =\frac{18}{3} \\
b_{p}^{*} & =6
\end{aligned}
$$

and $b_{s}^{*}=0$, so $B^{*}=b_{p}^{*}+b_{s}^{*}=6$.
(b) What is the socially optimal number of BART trains? If your answer differs from (a), explain why. (4 points)

Using $M R S_{c}+M R S_{d}=M R T$, we see that $M R S_{s}=\frac{\partial U_{s} / \partial b_{s}}{\partial U_{s} / \partial x_{s}}=\frac{x_{s}-3}{B-4}$ and $M R S_{p}=$ $\frac{\partial U_{p} / \partial b_{p}}{\partial U_{p} / \partial x_{p}}=\frac{x_{p}-3}{B-4}$. In addition, $M R T=1$, because the price of the private good and the public good are both equal to $\$ 1$. When we also substitute $x_{s}=5-b_{s}$ and $x_{p}=11-b_{p}$, and solve for $B$, we have $B_{o p t}^{*}=7$.
(c) Why do we observe a difference between (a) and (b) ? (2 points)

Because the students freeride on the professors. The private market therefore undersupply the public good.
(d) Suppose the city of Berkeley is not happy with the private equilibrium and decides to provide an additional BART train in addition to what the students and the professors choose to provide on their own. This extra train is paid for by imposing a $\$ 1$ lump-sum tax on the professors only. What is the new total number of trains? How does your answer compare to (a)? Did the city of Berkeley achieve the social optimum with this plan? Why or why not? (4 points)

If we take into account the tax and the added train, the students' utility function is now: $U_{s}=\ln \left(5-b_{s}-3\right)+\ln \left(b_{s}+b_{p}+1-4\right)$. The professors' utility function become: $U_{p}=$ $\ln \left(11-1-b_{p}-3\right)+\ln \left(b_{s}+b_{p}+1-4\right)$. Taking similar steps as in part (a), we have: $b_{s}^{*}=0, b_{p}^{*}=5$, so $B^{*}=b_{s}^{*}+b_{p}^{*}+1=6$, same as in (a). The lump-sum tax is fully applied to the public good, the students who are not taxed keep freeriding on the professors and the professors decrease their contributions to the BART system by the amount of the tax.
(e) Propose instead a mechanism the city of Berkeley could use to achieve the social optimum. (4 points)

One possibility for the city of Berkeley is to provide all the 7 BART trains, and tax the students and professors accordingly, i.e. proportionally to their respective income. The students will pay $5 / 16$ ths and the professors will pay 11/16ths respectively. Although there would be no private provision of BART trains, the number of trains will now equal the social optimum.
(f) Does the BART system fit the characteristics of a pure public good? Why or why not? (2 points)

The BART system is excludable as only people that pay can access it. It is generally non rival, unless at peak hours when it gets very crowded. It is therefore not a pure public good, unless it becomes free (maybe soon for Berkeley students?) and remains not too crowded.

## 3. Unemployment Insurance ( 20 points)

A government is thinking about establishing an unemployment insurance system and has asked for an expert assessment. Everybody in the country makes a wage of $\$ \mathbf{1 0 0}$ when employed, and earns 0 when unemployed.

Individuals derive utility from consumption $c$, and have probability $q$ of becoming unemployed. There are three types of individuals with different preferences and probability of becoming unemployed:

- Type 1: $q_{1}=50 \%$ and $U(c)=\sqrt{c}$
- Type 2: $q_{2}=10 \%$ and $U(c)=\sqrt{c}$
- Type 3: $q_{3}=10 \%$ and $U(c)=c$
(Parts $(f)$ and ( $g$ ) are worth 3 points, all other exercises are worth 2 points)
(a) What is the expected utility of each type without insurance?

$$
\mathrm{E}[U]=(1-q) \cdot U(100)+q \cdot U(0)
$$

- Type 1: $\mathrm{E}\left[U_{1, u}\right]=0.5 \sqrt{100}=5$
- Type 2: $\mathrm{E}\left[U_{2, u}\right]=0.9 \sqrt{100}=9$
- Type 3: $\mathrm{E}\left[U_{3, u}\right]=0.9 \cdot 100=90$
(b) Explain why only types 1 and 2 would benefit from insurance?

Types 1 and 2 have a concave utility function which means they are risk averse and would benefit from insurance. Type 3 is risk neutral.

The government first considers leaving the provision of insurance to the market. Assume for the following exercises that insurance companies make zero profits (i.e. insurance premiums are actuarially fair). Insurances charge premium $p$ and pay out benefits $b$ in case of unemployment.
(c) Assuming $q$ is fixed and does not depend on $b$, what is the optimal level of benefits $b$ for types 1 and 2? How is this called?
(Hint: No calculation required.)
The optimal level is $b=\$ 100$. This is full insurance, because types 1 and 2 are risk averse and premiums are actuarially fair.
(d) What prices $\left(p_{1}, p_{2}, p_{3}\right)$ will insurance companies charge to each type (we assume here companies can observe each individual's type)?

- Type 1: $p_{1}=q_{1} \cdot b=0.5 \cdot \$ 100=\$ 50$
- Type 2: $p_{2}=q_{2} \cdot b=0.1 \cdot \$ 100=\$ 10$
- Type 3: $p_{3}=p_{2}=\$ 10$
(e) How would your answer to (c) change if $q$ depended positively on $b$ (i.e. $\frac{\partial q}{\partial b}>0$ )? What is the name of this type of phenomenon?
(Hint: No calculation required.)
Full insurance is no longer optimal, this is an example of moral hazard.
Assume for the remaining exercises that insurance companies and the government cannot observe a given individual's type. However, they do know there are the same number of individuals of each type in the population (and for each type, they know $q$ and $U(c)$ ).
(f) How much is each type willing to pay for insurance?
(Hint: What premium p makes each type indifferent between getting full insurance or no insurance at all? For reference: $0.5^{2}=0.25$ and $0.9^{2}=0.81$.)

For types 1 and 2:

$$
\begin{aligned}
& \mathrm{E}\left[U_{u}\right]=\mathrm{E}\left[U_{i}\right] \Rightarrow\left(1-q_{j}\right) \sqrt{100}=\sqrt{100-p_{\max , 1}} \\
& \Rightarrow\left(1-q_{j}\right)^{2} \cdot 100=100-p_{\max , 1}
\end{aligned}
$$

- Type 1: $p_{\max , 1}=100-0.5^{2} \cdot 100=\$ 75$
- Type 2: $p_{\max , 2}=100-0.9^{2} \cdot 100=\$ 19$
- Type 3: $p_{\max , 3}=\$ 10$
(g) What is the long term equilibrium price of insurance? Who gets insured in equilibrium?
We know that type 3 will not get insured in any pooled equilibrium, so we can start calculating the premium when only type 1 and 2 are part of the market.

$$
p=\bar{q} \cdot 100=\frac{0.5+0.1}{2} \cdot 100=\$ 30
$$

This price is too high for type $2\left(>p_{\max , 2}\right)$, so that only type 1 will remain in the market. The price will then be $\$ 50$.
(h) Should the government provide unemployment insurance instead of leaving it to the market? Why?

Yes, because there is under-provision of insurance. This is a consequence of adverse selection.
(i) How would your answer to (g) and (h) change if type 2 had utility $U_{2}(c)=\log (c)$ instead? (Hint: $\lim _{x \rightarrow 0} \log (x)=-\infty$ )

In that case $p_{\max , 2}=\infty$ so that both type 1 and 2 will get insured at price $\$ 30$. Since type 3 does not gain utility from being insured, there is no longer need for the government to intervene.

