Econ 230B

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Problem Set 2

Solution

1. Social Security Privatization (3 pts)

a) (0.5 pts) $b = (1+n)\tau w$

$c_1^t + c_2^t/(1+r) = (1 - \tau(r-n)/(1+r))w$

b) (0.5 pt)

$$\max_s u(w - d - s) + \delta u((1+r)s + (1+n)d)$$

implies FOC:

$$u'(w - d - s) = \delta(1+r)u'((1+r)s + (1+n)d)$$

Taking the total derivative on both sides gives:

$$\delta u''(c_2)(1+r)[(1+r)ds + (1+n)dd] = -u''(c_1)[dd + ds]$$

$$\frac{ds}{dd} = -\frac{\delta u''(c_2)(1+r)(1+n) + u''(c_1)}{\delta u''(c_2)(1+r)^2 + u''(c_1)}$$

which is between 0 and 1 when $n < r$ and equal to 1 when $n = r$.

c) (0.5 pt) Generation $t_0 - 1$ is hurt. Generation t_0 onward gets a higher return on savings r (instead of n) and hence are better off.

d) (1 pt) Debt per capita a_t evolves according to $(1+n)a_{t+1} = (1+r)a_t - T_t$, where T_t is payment made by each old person in generation t.

To keep debt constant, $a_{t+1} = a_t = a_{t_0} = d$, we need $T_t = (r-n)d$.

So the maximization problem for any future generation will be:

$$\max_s u(w(1-\tau) - s) + \delta u((1+r)(s + \tau w) - (r-n)d)$$

Using the fact that $\tau w = d$, this is equivalent to:

$$\max_s u(w - d - s) + \delta u((1+r)s - (1+n)d)$$

which is exactly the same maximization problem as in b) showing the welfare of future generations is not affected.

e) (0.5 pt) Consider the initial path (w_t, k_t, r_t) with the PAYG system. Suppose the system is switched to the funded system as in d). Then, as in d), we can show that taking (w_t, r_t) as given and as in the PAYG system, the savings decision of the individual remains the same, so that the savings decision s_t will be such that $k_{t+1} = s_t/(1+n)$ and hence the wage rate and the interest rate will indeed be as the individual expect.

So the initial macro-economic equilibrium PAYG path (w_t, k_t, r_t) remains an equilibrium in the reformed system. So indeed nothing is changed in the general equilibrium.

2. Optimal Retirement Savings Subsidies for Hyperbolic Individuals (2.5pts)

a) (0.5 pt) Self-1 solves

$$\max_c \theta u(c) + \beta(y - c)$$

The first order condition is: $\theta u'(c) = \beta$.

b) (0.5 pt) Self-0 solves

$$\max_c \theta u(c) + (y - c)$$

The first order condition is: $\theta u'(c) = 1$.

c) (0.5 pt) With matching $k = (1+m)(y-T-c)$ hence $c + k/(1+m) = y - T$. Self-1 solves

$$\max_c \theta u(c) + \beta(1+m)(y - T - c)$$

The first order condition is: $\theta u'(c) = \beta(1+m)$.

d) (0.5 pt) At best, self-0 can achieve the utility of question a). A match at rate m such that $\beta(1+m) = 1$ allows self-0 to reach this first best utility level. Then $m = 1/\beta - 1$ is the optimal match rate.

e) (0.5 pt) At best, self-0 can achieve the utility of question a). A match at rate m such that $\beta(1+m)$ allows self-0 to reach this first best utility level. Then $m = 1/\beta - 1$ is one of the optimal contract that allows self-0 to reach its first best utility.

3. Optimal labor and capital income taxation (2.5 pts)

a) (0.5 pt) Individual program,

$$V = max\ U(c_1, c_2, l) = \log(c_1 - l^{k+1}/(k+1)) + \theta \log(c_2),$$

subject to

$$c_1 + qc_2 \leq wl(1 - t_L) + R.$$

implies:

$$l = w^e(1 - t_L)^e$$

$$s = (\theta/(1+\theta))[(w(1 - t_L))^{1+e}/(1+e) + R]$$

$$c_2 = s/q$$

$$c_1 = (1/(1+\theta))[(1 + e(\theta+1))(w(1 - t_L))^{1+e}/(1+e) + R]$$

$$V = (1+\theta)\log[(w(1 - t_L))^{1+e}/(1+e) + R] - \theta \log(q) + ct$$

So the elasticity of savings with respect to q is zero.

b) (1 pt) Taxes collected in period 2 are equal to $t_K rs = (1+r)c_2(q-p)$. As we assume implicitly that the government can use debt, taxes collected in period 2 should be discounted at rate r. Thus the budget constraint is: $wlt_L + c_2(q-p) \geq g$.

[Note that when the government cannot use debt, taxes must be equal to spending on a cash flow basis, the capital stock is equal to savings, and the budget constraint should be $wlt_L + srt_K \geq g$.]

Lagrangian: $L = V + \lambda(wlt_L + c_2(q-p) - g)$

Write FOC with respect to t_L and q. Eliminating λ and using the budget constraint, we obtain,

$$t_K = (\theta + 1)((1 + r)/r)et_L/(1 - t_L)$$

$$t_L(1 - t_L)^e = (g/w^{1+e})(1 + e)/(1 + e(\theta + 1))$$

c) (1 pt) Numerical Simulation:

- ($\theta = 1$, $e = 1/4$), $t_L = 0.22$ and $t_K = 0.21$
- ($\theta = 1/5$, $e = 1/4$), $t_L = 0.26$ and $t_K = 0.16$
- ($\theta = 1$, $e = 1$), $t_L = 0.21$ and $t_K = 0.80$

As seen in class, relative tax rate on capital depends on structure of compensated elasticities. The compensated cross price elasticities σ_{2L} and σ_{L2} are zero, the compensated elasticity of labor σ_{LL} is equal e and the compensated elasticity of period 2 consumption is $\sigma_{22} = -1/(1 + \theta)$.

Decreasing θ, increases $|\sigma_{22}|$, and increases t_K relative to t_L.

Increasing e, increases σ_{LL}, and decreases t_L relative to t_K.

The most realistic case is probably the second one. People behave as it they had substantial discount rates.

t_K is not zero because σ_{LL} is non zero and thus both taxes t_L and t_K produce distortions.

4. Progressive Capital Income Taxation in the Infinite Horizon Model (2 pts)

a) (0.5 pts) $\dot{a}_t^i = w + ra_t^i - T(ra_t^i) - c_t^i$ is the standard asset accumulation equation: change in wealth is equal to wage income w plus after-tax interest income $ra_t^i - T(ra_t^i)$ minus consumption c_t^i.

b) (0.5 pt) Hamiltonian: $H = u(c) + \lambda(w + ra - T(ra) - c)$

Standard first order conditions (see e.g. Blanchard and Fischer, Chapter 2).

FOC in c: $H_c = 0$ implies $c^{-\sigma} = \lambda$

FOC in a: $\rho\lambda - \dot{\lambda} = H_a$

implies $\rho\lambda - \dot{\lambda} = \lambda r(1 - T'(ra))$.

Replacing $\lambda = c^{-\sigma}$ in this equation, one gets:

$$\frac{\dot{c}}{c} = \frac{1}{\sigma}(r(1 - T'(ra)) - \rho)$$

This is the Euler equation of the infinite horizon model.

c) (0.5 pt) In steady state, we must have $\dot{c} = 0$ and therefore $r(1 - T'(ra)) - \rho = 0$. As T' is strictly increasing, this defines a unique long-term wealth level a^*.

The idea is that if $a > a^*$ then $r(1 - T'(ra)) - \rho < 0$ and the individual reduces consumption (and thus wealth accumulation) up to point where $a = a^*$ where consumption (and wealth) become stable. If $a < a^*$ then $r(1 - T'(ra)) - \rho > 0$ and the individual increases consumption (and thus wealth accumulation) up to point where $a = a^*$ where consumption (and wealth) becomes stable.

d) (0.5 pts) This model predicts that progressive taxation will eventually eliminate completely wealth inequality because progressive taxation reduces the net-rate of return of the wealthy and thus reduces their rate of wealth accumulation relative to the poor.

It is plausible to think that indeed progressive taxation can reduce the concentration of wealth but obviously the implications of the dynastic model are too strong to be realistic. Nevertheless, this shows that progressive taxation might be a useful tool for redistribution in the context of the dynastic model.