Econ 230B
Spring 2011
Emmanuel Saez
Antonio Rosato rosato@econ.berkeley.edu

Problem Set 2

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1. Social Security Privatization

Consider an OLG model where individuals live for 2 periods, work only in period 1, earn a wage w, and have utility:

$u(c_1^t) + \delta u(c_2^t)$,

where the upper script t denotes the generation born in period t. Suppose that population grows at rate n ($N_{t+1}/N_t = 1+n$). The market rate of return on private savings is denoted by r. Assume that $r > n$. Assume that r and w are exogenously given and constant over time.

There is a pay-as-you system which imposes a tax at rate τ on earnings w of generation t equal to $d = \tau w$ in period t to pay benefits b_{t-1} to generation $t-1$ (old in period t).

a) Express b_t as a function of w, τ, and n. Write the life-time budget constraint of generation t and show that the pay-as-you go system (assuming no credit constraints) amounts to a tax at rate $\tau(r-n)/(1+r)$.

b) Derive the FOC for individual optimal savings (or borrowing) on top of the forced pay-as-you system. Denote by s private savings. Show that $-1 < \partial s/\partial d < 0$ (in steady state equilibrium where $d_t = d_{t+1}$, and that $\partial s/\partial d = -1$ if $r = n$.

The economy is in steady-state in the pay-as-you-go system and the government decides to switch from a pay-as-you go system to a fully funded system at period t_0. The young in period t_0 will invest a fraction τ of their earnings in a private account with market return r. On top of this forced funded retirement, individuals can also save or borrow at market rate r.

c) Suppose that the government finances the transition by reneging to pay benefits for generation $t_0 - 1$ (old in period t_0). Show that generation $t_0 - 1$ is hurt but that all future generations are better off.

d) Suppose instead that the government issues debt a_{t_0} per capita (at market rate r) to pay the promised benefits to generation $t_0 - 1$. Moving forward, the government will keep the debt per capita constant, and each generation will pay (in old age) the amount needed to keep the debt constant per capita.

Write down how much each generation has to pay per capita to keep the debt constant.

Show that in net, there is no change in welfare for ANY generation relative to the former pay-as-you-go system.

e) (General equilibrium) Suppose now that r and w are endogenous and equal to $r_t = f'(k_t)$, $w_t = f(k_t) - k_t f'(k_t)$ where k_t is the capital stock per capita, and that $k_{t+1} = s_t/(1+n)$ (see Blanchard and Fischer (1989), Section 3.2 if you are not familiar with this).

Show that the privatization scheme from question d) (starting from the pay-as-you-go steady state) has no effect on the capital stock, on wages, and on welfare of ANY generation.

There are two periods representing work and retirement and a single consumption good each period. We denote first and second period consumption by c and k, respectively. Individuals earn income y only in period one. We normalize the interest rate to zero so that the undistorted budget constraint of the individual is $c + k \leq y$.

Individuals are hyperbolic discounters. Self-0 utility (when the individual can plan without feeling the urge to consume in the current period) is:

$$U^0 = \theta u(c) + k,$$

where θ is a taste shock distributed according to $F(\theta)$ that is revealed only in period 1.

Self-1 utility (when the individual feels an urge to consume in the current period) is:

$$U^1 = \theta u(c) + \beta k,$$

with $\beta < 1$ representing the self-control problem. In what follows, we assume that self-0 utility is the legitimate utility that the individual (ex-ante) and the government want to maximize. Do not worry about $k < 0$ or corner solutions in what follows.

2. Optimal Retirement Savings Subsidies for Hyperbolic Individuals

a) Derive the consumption plan $(c^1(\theta), k^1(\theta))$ that self-1 chooses conditional on the realization θ of the taste shock.

b) Suppose that self-0 could commit to a consumption plan $c^0(\theta), k^0(\theta)$.

Derive the optimal consumption plan $(c^0(\theta), k^0(\theta))$ that self-0 would like to implement for each realization θ of the taste shock.

c) Suppose that the government implements a matching program for savings so that any amount s saved in period 1 translates into $(1 + m) * s$ available for consumption in period 2. The matching contributions $m * s$ are financed by a lumpsum tax T on earnings y. T is independent of θ and chosen so that the government breaks even.

Derive the budget constraint of the individual with the matching program and the consumption choices $(c^1(\theta), k^1(\theta))$ of self-1 under this program.

d) Derive the optimal match rate m (and corresponding lumpsum tax T) that maximizes the utility of self-0.

e) Show that the simple optimal match program from d) is actually the optimal contract $c(\theta), k(\theta)$ that maximizes expected utility of self-0:

$$\int [\theta u(c(\theta)) + k(\theta)] dF(\theta),$$

subject to the overall budget constraint

$$\int [c(\theta) + k(\theta)] dF(\theta) \leq y,$$

and the incentive compatibility constraint:

$$\theta u(c(\theta)) + \beta k(\theta) \geq \theta u(c(\theta')) + \beta k(\theta'),$$

for any θ, θ'.

3. Optimal labor and capital income taxation

Consider an overlapping generation economy with constant population and identical individuals. Each individual lives for two periods and has a utility function of the form,

$$U(c_1, c_2, l) = \log(c_1 - l^{k+1}/(k+1)) + \theta \log(c_2),$$

where c_1 is period 1 consumption, c_2 is period 2 consumption, and l is labor supply when young. The old do not work and finance their consumption with savings. Assume that the wage rate is w and the interest rate on savings is r. The government imposes a linear tax t_L on labor income and a linear tax on interest income t_K. Assume also that the government provides a lumpsum amount R to the young. To simplify computations, we normalize the price of c_1 to one and we note $p = 1/(1+r)$ the pre-tax price of c_2 and $q = 1/(1+r(1-t_K))$ the after-tax price of c_2.

a) Set up the individual budget constraint (using q) and solve for the optimal l, c_1, c_2 and savings s as a function of $w(1-t_L)$, q, R, θ, and the elasticity of labor supply $e = 1/k$. Derive the indirect utility function V. Compute the elasticity of savings with respect to the net-of-tax interest rate $r(1-t_K)$.

b) Assume that the government cannot use lumpsum taxation ($R = 0$) and sets t_L and q so as to raise a given amount for public spending g and maximize indirect utility V. Set-up the Ramsey static maximization problem for the government. Denote by λ the multiplier of the government budget constraint. Assume that w and r are exogenous.

Derive the first order conditions with respect to t_L and q. Combine these two equations to eliminate λ and express t_K in terms of t_L, r, e, and θ. Finally, use the budget constraint to obtain an equation relating t_L, g, w, e, and θ.

c) Numerical Simulation using the two equations obtained in b): assume that $w = 1$, $r = 2$ (a period represents a generation so $r = 2$ is reasonable), and $g = 1/4$.

Obtain the optimal t_L and t_K in three cases: ($\theta = 1$, $e = 1/4$), ($\theta = 1/5$, $e = 1/4$), and ($\theta = 1$, $e = 1$). Explain intuitively the pattern for the results. Which case is the most realistic? Why is the optimal t_K not zero?

4. Progressive Capital Income Taxation in the Infinite Horizon Model

Consider a standard infinite horizon model in continuous time where each individual supplies inelastically one unit of labor at each instant for an exogenous and constant (over time and across individuals) wage w. Assume that the pre-tax interest tax rate r is exogenous and constant. The government imposes a non-linear tax on capital income $T(.)$. Let us denote by a_t^i wealth holding of individual i at time t, and c_t^i consumption of individual i at time t. Taxes paid at time t are equal to $T(ra_t^i)$.

a) Find the differential equation describing the evolution of wealth a_t^i for a given path of consumption c_t^i.

b) Individual i maximizes the following utility function:

$$U^i = \int_0^\infty u(c_t^i) e^{-\rho t} dt$$

where ρ is the discount rate (the same for each individual) and instantaneous utility is $u(c) = c^{1-\sigma}/(1-\sigma)$. Individual i starts with initial wealth a_0^i.

Show that the optimal path of consumption of individual i satisfies (use a standard Hamiltonian technique):

$$\frac{\dot{c}_t^i}{c_t^i} = \frac{1}{\sigma}(r(1 - T'(ra_t^i) - \rho)$$

c) Assume that $r > \rho$. Assume that T is progressive so that T' is strictly increasing with $T'(0) = 0$, $T(0) = 0$, and $T'(\infty) = 1$. Show that, in the long-run, wealth holdings a_t^i is converging to a unique value a^*. Find an implicit expression for a^* is terms of r, ρ, and $T(.)$.

d) What does this model predict for the impact of progressive capital income taxation on wealth inequality? Is this a realistic outcome?