Econ 230B
Spring 2012
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Problem Set 1

Solution

1. Optimal Income Taxation (4 pts)

(a) (0.25 pts) Standard plot.

(b) (0.25 pts) FOC individual maximization implies that $l = (w(1-\tau))^{1/k}$, hence $z = w^{1+1/k}(1-\tau)^{1/k}$. Thus, there are no income effects and the compensated and uncompensated elasticities are equal to $e = 1/k$ for both skills.

(c) (0.25 pts) Denote by T total tax revenue: $T = \tau(\lambda_1 z_1 + \lambda_2 z_2) = \tau(1-\tau)^e(\lambda_1 w_1^{1+e} + \lambda_2 w_2^{1+e})$. FOC in τ implies that $\tau^* = 1/(1+e)$.

(d) (0.25 pts)

$$T = \lambda_1 \tau_1 (1-\tau_1)^e w_1^{1+e} + \lambda_2\{\tau_1 \bar{z} + \tau_2[(1-\tau_2)^e w_2^{1+e} - \bar{z}]\}$$

For (d), take the FOC wrt to τ_2 to get:

$$\tau_2/(1-\tau_2) = (1/e) \cdot [z_2 - \bar{z}]/z_2$$

(e) (0.5 pts) Take the FOC wrt to τ_1 to get:

$$\tau_1/(1-\tau_1) = (1/e) \cdot [1 + \lambda_2 \bar{z}/(\lambda_1 z_1)]$$

Explaining: $\tau_2^* < \tau^* < \tau_1^*$:

1) Increasing the flat tax rate τ creates a mechanical increase in revenue proportional to average earnings and creates a negative behavioral response proportional to average earnings as well.

2) Increasing the tax rate τ_2 in the top bracket creates a mechanical increase in revenue proportional to $(z_2 - \bar{z})$ but creates a negative behavioral response proportional to z_2.

3) Increasing the tax rate τ_1 in the bottom bracket creates a mechanical increase in revenue proportional to z_1 and creates a negative behavioral response proportional to z_1. However, the tax rate increase also raises more tax from high skilled worker with no negative behavioral response (inframarginal tax).

(f) (0.5 pts) The demogrant is higher under nonlinear tax because the nonlinear tax is a more powerful tool than linear tax (and hence will do at least as well).

A Rawlsian social welfare function will maximize tax revenue to redistribute to the disabled who are the worst off individuals in the economy with utility $u = R$.

(g) (0.25 pts) Putting no weight on high skilled workers does not affect τ_2 (which will remain at tax revenue maximizing level τ_2^*) but putting some weight on low skilled workers will put τ_1 below the tax revenue maximizing rate τ_1^*. As a result, the demogrant R will be lower.

(h) (1 pts) Disabled workers work iff utility when working is higher than utility when not working. Utility when not working is R. When working, $l = w_1^e(1-\tau_1)^e$ and hence $u = R + w_1^{1+e}(1-\tau_1)^{1+e}/(1+e)$. Thus, a disabled person will work iff:

$q \leq w_1^{1+e}(1-\tau_1)^{1+e}/(1+e) = \bar{q}$.

Hence the fraction working is $P(\bar{q})$. Note that $\bar{q}$ is decreasing in τ_1 and $dP/d\tau_1 = -P'(\bar{q})z_1$.

Under this scenario,

$$T = [\lambda_1 + \lambda_0 P(\bar{q})]\tau_1(1-\tau_1)^e w_1^{1+e} + \lambda_2\{\tau_1\bar{z} + \tau_2[(1-\tau_2)^e w_2^{1+e} - \bar{z}]\}$$

Taking the FOC in τ_1, one gets:

$$0 = dT/d\tau_1 = -\lambda_0 P'(\bar{q})z_1\tau_1 z_1 + [\lambda_1 + \lambda_0 P(\bar{q})]z_1[1 - e\tau_1/(1-\tau_1)] + \lambda_2\bar{z}$$

$$e \cdot \tau_1/(1-\tau_1) = 1 + \lambda_2\bar{z}/(\lambda z_1) - z_1\lambda_0 P' \cdot \tau_1 z_1/(\lambda\bar{z})$$

where $\lambda = \lambda_1 + \lambda_0 P(\bar{q})$.

There are two additional effects relative to (e):

1) There are more low skilled workers (relative to high skilled workers): $\lambda > \lambda_1$. This makes the tax τ_1 less desirable

2) There is another layer of response to τ_1 through $\bar{q}$ which makes the behavioral response to τ_1 larger.

Those two effects will make the new optimal τ_1 smaller than τ_1^*.

(i) (0.75 pts) Tax reform analysis.

The basic idea is to start with a simple difference estimate: compare top incomes in period 1 vs period 0. In regression terms,

$$\log z_{it} = \alpha + e \cdot \log(1-\tau_{it}) + \epsilon_{it}$$

This regression is run with OLS but limited to the sample of top x% earners in period 0 and top x% earners in period 1. $\hat{e}$ estimated e without bias if absent the reform, top incomes would have stayed constant on average.

This assumption can be checked by running the same placebo regression on low income earners (subject to tax rate τ_1).

If low incomes increased, then one can do a DD estimate which is unbiased, if absent the reform, low incomes would have grown at the same rate as high incomes (parallel trend assumption). The DD regression estimate is:

$$\log z_{it} = \alpha_t + group_i + e \cdot \log(1 - \tau_{it}) + \epsilon_{it}$$

where $group_i$ is a dummy for belonging to the high income group and α_t is a set of period dummies.

2. Bunching at kink points (3pts)

a) (0.5pt)

$$\max wh - T(wh) - \frac{h^{1+k}}{1+k}$$

FOC h: $w(1-T') = h^k$ hence $h = w^{1/k}(1-T')^{1/k}$ and $z = wh = w^{1+1/k}(1-T')^{1/k}$:

Hence, three cases depending on size of w:

+ if $w \leq \bar{z}^{k/(k+1)}$ then $z = w^{1+1/k}$. This is the first bracket.

+ if $\bar{z}^{k/(k+1)} \leq w \leq \bar{z}^{k/(k+1)}/(1-\tau)^{1/(k+1)}$ then $z = \bar{z}$. This is bunching at $\bar{z}$.

+ if $\bar{z}^{k/(k+1)}/(1-\tau)^{1/(k+1)} \leq w$ then $z = w^{1+1/k}(1-\tau)^{1/k}$. This is the second bracket.

b) (0.5pt) Elasticity is $1/k$.

Fraction bunching is $\int_{w_1}^{w_2} f(w)dw$ where $w_1 = \bar{z}^{k/(k+1)}$ and $w_2 = \bar{z}^{k/(k+1)}/(1-\tau)^{1/(k+1)}$

c) (0.5pt) Histogram attached (created with matlab).

Histogram shows bunching at \$10,000 which is $\bar{z}$.

d) (1.5pt) All individuals with w in (w_1, w_2) bunch at $\bar{z}$.

Absent the tax rate τ, those with wage w_1 would earn $\bar{z}$ and those with wage w_2 would earn $w_2^{1+1/k} = \bar{z}/(1-\tau)^{1/k}$.

Excess bunching is 193 individuals (with earnings exactly equal to \$10,000). There are also 193 individuals on the left of the kink with earnings between \$10,000-\$827 and \$10,000-\$1. So, absent the kink, those bunching taxpayers would have spread across a band of width \$827 approximately.

Hence \$827$= \bar{z}[1 - 1/(1-\tau)^{1/k}]$.

which translates into $e = 1/k = \log(1 - 827/10000)/\log(1-0.3) = 0.24$ which is very close to the 0.25 I have used to simulate the data.

3. Behavioral Responses to Taxation (3 pts)

(a) (0.5 pts) Use Roy's identity to get $l = (\partial v/\partial w)/(\partial v/\partial y)$ (no minus sign because labor supply is minus leisure).

$$\epsilon^u = (w/l)(\partial l/\partial w) = w/(8 + w - 2y/25)$$

Income effects: $\eta = w\partial l/\partial y = -w/25$

$$\epsilon^c = \epsilon^u - \eta = w/(8 + w - 2y/25) + w/25$$

(b) and (c) (0.5 pt) Draw the budget sets (in a (l, c) diagram) and find the number of hours of labor supply for individuals with a pre-tax wage n of 10, and for those with a pre-tax wage n of 20.

$$l(w, y) = 4 + \frac{w}{2} - \frac{y}{25}$$

$n = 10$, $w = 9$, $y = 0$, $l = 4 + 9/2 - 0 = 8.5$ and $Z = 85$, and $Tax = 8.5$

$n = 20$, $w = 10$, $y = 36$, $l = 4 + 10/2 - 36/25 = 9 - 36/25 = 7 + 14/25 = 7.56$ and $Z = 151.2$ and $Tax = 9 + 0.5(151.2 - 90) = 39.6$

(d) (0.5 pts) Labor supply: $n = 10$, $w = 10$, $y = 0$, $l = 4 + 10/2 - 0 = 9$ and $Z = 90$. As $Z = 90$ would push them on the taxable schedule, they will work 9 minus epsilon so as to stay on the non-taxable schedule.

(e) (0.5 pts) If they stay on the second bracket, the virtual income is unchanged. If they move to the second bracket, the virtual income becomes zero.

(f) (0.5 pts) The lumpsum tax has to be 8.5 to raise the same revenue. The individuals are clearly better off (pay the same tax with no distortion).

(g) (0.5 pts) Tax rate τ, net wage $20(1 - \tau)$, virtual income $90\tau - 9$ so labor supply is:

$l = 4 + 10(1 - \tau) - \frac{90\tau - 9}{25} = 14 + 9/25 - (10 + 18/5)\tau = 14.36 - 13.6\tau$

Government maximizes:

$\tau(Z - 90) = \tau[20l - 90] = \tau[20(14 + 9/25) - 20(10 + 18/5)\tau - 90] = \tau(280 + 36/5 - 90 - (200 + 72)\tau) = \tau(197.2 - 272\tau)$

FOC: $197.2 - 2 \cdot 272\tau = 0$ so $\tau = 98.6/272 = 29/80 = 0.3625$