Econ 230B
Spring 2012
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Problem Set 1

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1. Optimal Income Taxation

We consider an economy made up of individuals who have identical preferences defined over consumption c and labor l, but different wage rates. The utility function takes the simple form: $u(c,l) = c - l^{1+k}/(k+1)$ where $k > 0$ is a given fixed parameter. An individual with wage rate w supplying labor l, earns $z = wl$ and consumes $c = z - T(z)$ where $T(.)$ is the (possibly nonlinear) income tax.

We assume that there are 3 types of individuals: disabled individuals unable to work $w_0 = 0$, low skilled individuals with wage rate w_1, and skilled individuals with wage rate w_2. Obviously, we assume that $w_1 < w_2$. We assume that the fractions of disabled, low skilled, and high skilled in the population are $\lambda_0, \lambda_1, \lambda_2$ (and that $\lambda_0 + \lambda_1 + \lambda_2 = 1$).

We assume that the government imposes the following income tax: $T(z) = -R + \tau_1 \cdot z$ if $z \leq \bar{z}$ and $T(z) = -R + \tau_1 \cdot \bar{z} + \tau_2 \cdot (z - \bar{z})$ if $z > \bar{z}$. $R > 0$ is the demogrant.

(a) Plot the budget constraints on a diagram (l, c) for low skilled and high skilled workers. From now on, we assume that low skilled workers are always in the first bracket (with marginal tax rate τ_1) and that high skilled workers are always in the top bracket (with marginal tax rate τ_2).

(b) We denote by z_1 and z_2 the earnings of low skilled and high skilled workers respectively. Estimate the uncompensated and compensated elasticities of earnings z_i with respect to the net tax rate $1 - \tau_i$ for $i = 1, 2$.

(c) Suppose that the government uses a flat tax where $\tau_1 = \tau_2 = \tau$. Estimate the tax rate τ^* maximizing tax revenue as a function of k (taking R as given).

(d) Taking R, τ_1, and $\bar{z}$ as fixed, compute the tax rate τ_2^* that maximizes taxes collected from the high skilled. Express τ_2^* as a function of k, z_2, and $\bar{z}$.

(e) Taking R and $\bar{z}$ as fixed and assuming $\tau_2 = \tau_2^*$, compute the tax rate τ_1^* that maximizes total taxes collected. Express τ_1^* as a function of k, z_1, λ_1, and λ_2, and $\bar{z}$. Explain intuitively why $\tau_2^* < \tau^* < \tau_1^*$.

(f) Suppose that taxes collected using the tax rates τ_1 and τ_2 actually fund the demogrant R.

Will the demogrant be higher when the government uses the flat tax τ^* from (c) or the nonlinear tax τ_1^*, τ_2^* from (d) and (e)? What type of social welfare function would lead the government to maximize taxes collected on workers in order to maximize the size of the demogrant?

(g) Suppose now that the government puts some weight on the marginal consumption of low skilled workers (but no weight on the marginal consumption of high skilled workers). How would this affect the optimal choice of τ_1 and τ_2 relative to the tax maximization rates τ_1^* and τ_2^* derived in (d) and (e)? What will happen to the demogrant R.

(h) Suppose now that disabled workers face a cost of work q that is distributed according to a cumulated distribution $P(q)$ with density $p(q)$. When a disabled person pays the work cost q, she becomes like a low skilled worker with wage rate w_1 and utility function $u = c - l^{1+k}/(1+k) - q$. Compute the fraction of disabled workers who work as a function of w_1, τ_1, and the distribution $P(.)$.

Under this scenario, how does the tax rate τ_1 maximizing tax revenue compares with τ_1^* from (e) which was derived assuming no disabled person could work (explain the economic intuitions if you cannot do the full math).

(i) Suppose that the government increases τ_2 from 30% to 40% from period 0 to period 1 and that you have access to two cross sections of earnings data (one for period 0 and one for period 1). Explain how you would estimate the elasticity of earnings with respect to $1 - \tau_2$ using this reform. Make sure to be precise about the type of estimate and regression you would use. You should also discuss how you could test the robustness of your estimates using the data available.

2. Bunching at kink points

a) Consider a utility function based on consumption c and hours of work h of the form:

$$u(c, h) = c - \frac{h^{1+k}}{1+k}$$

Individuals have a pre-tax wage rate w, supply hours of work h, and earn $z = w \cdot h$.

The tax schedule depends on earnings $z = w \cdot h$ and takes the following form:

$T(z) = 0$ if $z \leq \bar{z}$

$T(z) = \tau \cdot (z - \bar{z})$ if $z > \bar{z}$,

where τ is the constant marginal tax rate in the top bracket. Draw the budget set for a given individual and solve for the optimal (c^*, z^*) choice as a function of w. Make sure to distinguish cases where the individual is on the first bracket, bunches at the kink, or is on the second bracket.

b) Derive the compensated elasticity of hours of work with respect to net of tax wages for this utility function.

Suppose that wages are distributed according to a density function $f(w)$ (with population normalized to one). Give a formula for the fraction of individuals bunching at the kink point.

c) I have created a data-set of 5,000 observations displaying earnings outcomes for a such a population of individuals assuming that $\tau = 0.3$, and that w is distributed according to some distribution $f(w)$. I have then graphed a histogram of earnings by \$250 bands (\$0-\$249, \$250-\$499, \$500-\$749, etc.). What is $\bar{z}$ and why?

d) Using the histogram from c) and your answers to a) and b), try to give an estimate of the compensated elasticity of hours of work with respect to net of tax wages. You do not need to provide standard errors, just a point estimate.

3. Behavioral Responses to Taxation

An economy is made up of individuals who have identical preferences defined over consumption c and labor l, but different wage rates. The utility function $u(c, l)$ is increasing in c and decreasing in l and such that the indirect utility function is:

$$v(w, y) = (y - \frac{25}{2} \cdot w - 16.5 \cdot 25) \cdot e^{-\frac{w}{25}}$$

where w denotes the per-hour after-tax wage rate, and y denotes virtual income. No one in the economy has any non-labor income. Half of the population has a pre-tax hourly wage rate n of 10, while the other half has a pre-tax hourly wage n of 20.

The economy has the following income tax schedule:

$T(Z) = .1 \cdot Z$ for $Z < 90$

$T(Z) = 9 + .5 \cdot (Z - 90)$ for $Z \geq 90$

where $Z = n \cdot l$ denotes earnings (pre-tax hourly wage times number of hours of work).

(a) Show that the labor supply function (hours of work) as a function of after-tax wage rate and virtual income is:

$$l(w, y) = 4 + \frac{w}{2} - \frac{y}{25}$$

Give an expression (as a function of w and y) for the uncompensated and compensated elasticities of labor supply with respect to the after tax wage rate w.

(b) Draw the budget sets (in a (l, c) diagram) and find the number of hours of labor supply for individuals with a pre-tax wage n of 10, and for those with a pre-tax wage n of 20.

(c) How much revenue does the government collect from the existing income tax system?

(d) Now imagine that in the name of fiscal stimulus, policymakers in this economy propose eliminating the lower bracket of the income tax, so that $T(Z) = 0$ if $Z < 90$ (the tax in the second bracket remains the same as above). Describe the budget set for individuals with wages of 10, and with wages of 20, after this change. What is the labor supply of individuals with wage 10 with the new tax system?

(e) Does elimination of the lower bracket in part (d) change the virtual income of individuals who were previously earning more than 90? Explain why or why not.

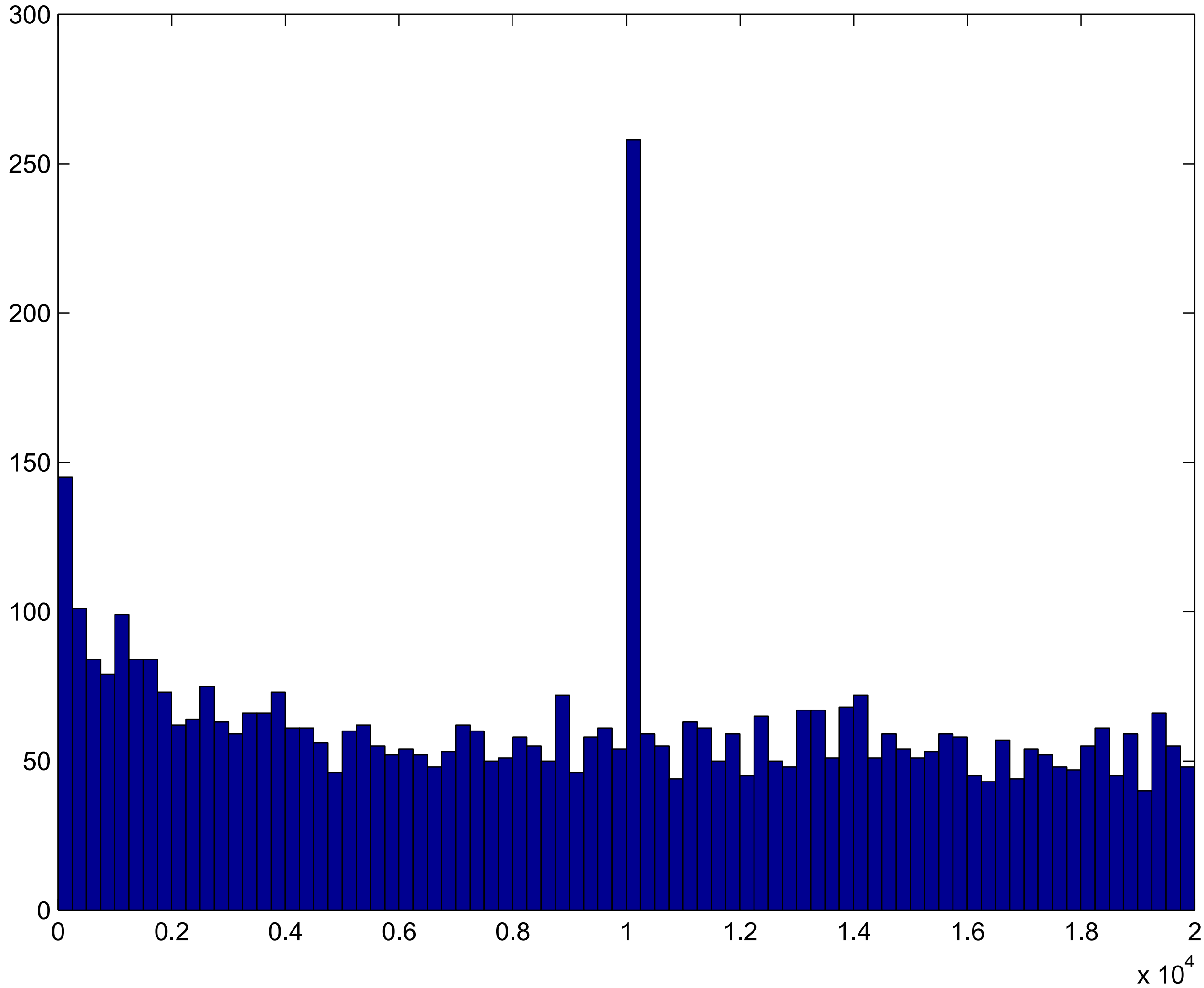
(f) Fiscal conservatives worry that the stimulus package has too large an effect on the government deficit. They propose raising the revenue that was previously collected by the lower bracket of the income tax with a lump sum tax (the tax in the second bracket remains the same as above). How large would the lump sum tax need to be to return the government's revenue to the level collected with the initial income tax system? Draw the new budget constraint and explain whether those with wage rate of 10 are better or worse off with this lumpsum tax than with the initial tax system.

(g) Liberals now decide, starting from the initial tax system described at the beginning of the problem, to increase (or decrease) the marginal tax rate in the second bracket from 0.5 to τ so as to maximize tax revenue collected from individuals with wage rate 20. Thus, tax schedule becomes:

$T(Z) = .1 \cdot Z$ for $Z < 90$

$T(Z) = 9 + \tau \cdot (Z - 90)$ for $Z \geq 90$

What is the τ you would recommend the government to choose? (The computations get a bit messy here, present the result in the form of a fraction $\tau = N_1/N_2$ where N_1 and N_2 are integers if you can. If you cannot, explain intuitively the trade-off involved in the determination of τ.)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