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Micro and macro elasticities in a life cycle model with taxes ☆

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Abstract

We build a life cycle model of labor supply that incorporates changes along both the intensive and extensive margin and use it to assess the consequences of changes in tax and transfer policies on equilibrium hours of work. We find that changes in taxes have large aggregate effects on hours of work. Moreover, we find that there is no inconsistency between this result and the empirical finding of small labor elasticities for prime age workers. In our model, micro and macro elasticities are effectively unrelated. Our model is also consistent with other cross-country patterns.

JEL classification: E24; J22; J64

Keywords: Labor supply; Life cycle; Taxes

1. Introduction

Time devoted to market work in continental Europe is currently only about 70% of the US level. Recent work by Prescott [26], Rogerson [29] and Ohanian et al. [24] argues that differences in tax and transfer policies can account for a large share of this difference. Following Lucas and Rapping [17], these papers all use a stand-in household model that abstracts from the distinction between employment and hours per employee, and assume that the stand-in household has a

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relatively high labor supply elasticity. One critique of these exercises is that the assumed labor supply elasticity of the stand-in household is much larger than that implied by most estimates based on micro data. Specifically, if the labor supply elasticity of the stand-in household was instead set to standard estimates based on micro data, then it is no longer the case that taxes account for a large share of differences in market work between the US and continental Europe.[1] In this paper we argue that this critique is misplaced.

To make this point, we develop an overlapping generations model that replicates the salient features of life cycle labor supply, and then use this model to analyze how tax and transfer policies affect hours of work in the steady state. In this framework we can carry out both standard micro data estimation exercises based on life cycle variation for prime aged workers, as well as standard macro estimation based on variation in aggregates across steady states. Our main findings are twofold. First, macro elasticities are virtually unrelated to micro elasticities, and second, macro elasticities are large. In particular, for micro elasticities that vary by a factor 25, ranging from .05 to more than 1.25, the corresponding macro elasticities are in the range of 2.25–3.0.

Our model builds on the earlier work of Prescott et al. [27] on lifetime labor supply by embedding it into a life cycle setting. Like them, we focus on the importance of nonlinearities in the mapping from time at work to labor services provided. This feature gives rise to equilibrium allocations in which workers choose time allocations in which both the extensive and intensive margins are operative, i.e., a worker chooses both what fraction of his or her life to devote to employment, and what fraction of his or her period time endowment to devote to work while employed. By embedding this analysis into a life cycle model we are able to generate standard life cycle profiles for hours of work—including the fact that hours drop discontinuously to zero at older ages. This allows us to reproduce micro estimates based on life cycle variation for prime aged workers.

In addition to reconciling micro and macro tax elasticities, our life cycle model delivers two additional predictions relative to earlier analyses based on stand-in household models, both of which are also corroborated by the cross-country data. First, our model predicts that increases in the size of tax and transfer policies imply less time devoted to work both in terms of employment to population ratios and hours of work per person in employment. Second, our model implies that differences in employment to population ratios are dominated by differences among young and old individuals.

The labor supply problem in our model is very similar to the one studied by French [6]. He considers an individual decision problem of lifetime labor supply in the presence of nonconvexities. A key implication of his model is that labor supply responses are much smaller for prime aged individuals than they are for older individuals, since the latter group will include the extensive margin associated with retirement. While French analyzes the decision problem of a given individual, we embed this problem into a general equilibrium model and focus on the relationship between individual preference parameters and aggregate responses.

Our results are also related to those in Chang and Kim [3]. Similar to us, they find that in their model, micro and macro elasticities need not be the same, and that macro elasticities can be significantly larger. While we view our study as complementary to theirs, there are several important differences that distinguish the two studies. First, we study a life cycle model and hence can explicitly connect to micro estimates based on life cycle variation. Second, our analysis allows for variation along both the intensive and extensive margin. Not only does this allow one

[1] Alesina et al. [2] is a recent example where this critique is put forward.

to better match the cross-country differences in hours of work, but we show that there is an important interaction between intensive and extensive margins: less adjustment on the extensive margin necessarily implies more adjustment on the intensive margin. Third, and related to this point, Chang and Kim argue that increases in heterogeneity in the steady state cross-sectional distribution of wages imply a large reduction in implied macro elasticities. We find that having an operative intensive margin reduces the quantitative impact of this effect.

An outline of the paper follows. Section 2 presents the model, and Section 3 considers the effects of tax and transfer programs on the equilibrium, and the relationship between micro and macro elasticities. Section 4 concludes.

2. Model and equilibrium

The model is purposefully specialized along several dimensions in order to best highlight those relationships that are the focus of our analysis. We consider a continuous time overlapping generations framework in which a unit mass of identical, finitely lived individuals is born at each instant of time t. Continuous time is convenient for our analysis because we study the endogenous determination of the length of the working life, and it allows us to model this as a continuous choice variable. We normalize the length of the lifetime of each agent to one and assume that each individual is endowed with one unit of time at each instant. Letting a denote age, individuals have preferences over paths for consumption $(c(a))$ and hours worked $(h(a))$ given by:

$$\int_0^1 U\big(c(a), 1 - h(a)\big)\, da \tag{2.1}$$

where U is twice continuously differentiable, strictly increasing in both arguments and strictly concave. Note that we assume that individuals do not discount future utility.[2]

Labor is the only factor of production, and the aggregate production function is written as:

$$Y(t) = L(t) \tag{2.2}$$

where $L(t)$ is aggregate input of labor services. A key feature of our model is the mapping from hours of work into labor services, which is described by two functions, $e(a)$ and $g(h)$. In particular, we assume that if an individual of age a devotes h units of time to market work then it will yield $l = e(a)g(h)$ units of labor services. The function $e(a)$ is standard in the life cycle labor supply literature—it represents exogenous life cycle variation in individual productivity. This feature will be the driving force behind the variation in hours worked during that part of the life cycle in which an individual is employed.[3] We assume that the function $e(a)$ is single peaked and twice continuously differentiable.

As in Prescott et al. [27], the function $g(h)$ plays a critical role in the analysis. A standard assumption in the literature is that g is linear with slope equal to one, so that for a worker of a given age, labor services are linear in hours of work. Prescott et al. [27] assumed that g

[2] This is done purely for convenience to allow us to focus on a zero interest rate steady state. From the perspective of hours worked, interest rates and discount factors serve primarily to tilt the life cycle profile.

[3] Rogerson and Wallenius [30] show that a model in which the life cycle driving force is changes in disutility for work produces virtually identical results.

was initially convex and then concave. While we could assume a general g function with these properties, for simplicity we assume that $g(h)$ takes the specific form:

$$g(h) = \max\{0, h - \bar{h}\}$$

where $\bar{h} > 0$. One justification for the initial convex region is fixed costs associated with getting set up for a job and costs associated with being supervised. One implication of such a g function is that hourly wage rates are lower for part time employees. Evidence in favor of this implication is presented in Moffitt [22], Keane and Wolpin [11] and Aaronson and French [1]. It is also consistent with the observation that firms do not consider part-time workers for many positions.[4]

It is important to note the significance of the convexity of the $g(h)$ function. In a static setting with homogeneous agents, this implies that it may be optimal to randomly select a fraction of workers to work positive hours and have the remaining workers work zero hours. Loosely speaking, this feature of technology can serve to endogenize the length of working time in a model of indivisible labor. Generalizing this homogeneous worker model to a dynamic setting with no discounting and no life cycle effects, Prescott et al. [27] showed that if time is continuous, optimal allocations take the form of a constant working time for employed workers, and a constant fraction of individuals employed at each instant. Importantly, such an allocation can be implemented as an equilibrium without lotteries, since individuals can choose the fraction of their lifetime that they work and use asset markets to smooth consumption in the face of an uneven income stream.[5]

In the next section we show that in our overlapping generations model with life cycle effects, the assumed $g(h)$ function gives rise to equilibria in which there is a well-defined notion of a working life—individuals will begin work at a particular age and work continuously until retirement. The events of entering and leaving the labor force are discontinuous events, in the sense that hours of work jump discontinuously at these two points. In particular, hours of work do not gradually decrease to zero prior to retirement. While our specification emphasizes nonlinearities in the mapping from hours worked to labor services provided, one could also allow for the possibility of nonlinearities in the mapping from leisure time to leisure services. Qualitatively, this would generate the same type of effects.

We also assume the presence of a government in this economy that runs a simple fiscal policy. Specifically, at each instant the government taxes all labor earnings at the proportional rate τ and uses the proceeds to fund a uniform lump-sum transfer to all living households whose magnitude is determined via a balanced budget rule.

2.1. Equilibrium

We consider the following market structure. We assume that at time zero there are markets for labor services and consumption at all future dates. Let $w(t)$ and $p(t)$ denote the paths for prices in these two markets. We assume competitive behavior in all markets. If a given individual is alive at two dates t and t', this market structure implicitly allows an individual to borrow or lend resources across these two dates at the gross interest rate $p(t)/p(t')$. Given that the aggregate production function is linear in labor services, competitive equilibrium necessarily implies that $w(t) = p(t)$ at each t.

[4] Of course, firms for which demand fluctuates over short periods of time may find it optimal to hire part-time workers even in the presence of a g function like the one we assume.

[5] An earlier result in this spirit was presented in Mulligan [23]. More recently, Krusell et al. [13] show that this holds in steady state for an infinitely lived agent model with discrete time and discounting.

Our analysis will focus on steady state equilibria associated with this market structure. As is well known, overlapping generations models can give rise to multiple steady state equilibria. For our economy, there is always a steady state equilibrium in which $p(t)$ is constant (i.e., a zero interest rate steady state), though it may require some government action regarding debt issuance. In the analysis that follows we will assume that if necessary, the government follows a policy that results in this steady state equilibrium being reached, and will therefore focus on the zero interest rate steady state equilibrium.[6]

Given that we focus on the steady state equilibrium with constant $p(t)$, we can normalize this price to one, which by our earlier remark implies that $w(t)$ will also be one for all dates. The lifetime utility maximization problem for a newborn individual in the steady state equilibrium can then be written as:

$$\max_{c(a),h(a)} \int_0^1 U\big(c(a), 1-h(a)\big)\,da \quad \text{s.t.} \quad \int_0^1 c(a)\,da = \int_0^1 e(a)g\big(h(a)\big)\,da.$$

Consider first the special case studied by Prescott et al. [27], in which $e(a)$ is constant over an individual's life. They show that the solution for $h(a)$ can take one of two forms. One possibility is that $h(a)$ is positive for all a, in which case the solution for $h(a)$ is unique and has $h(a)$ constant for all a. The other possibility is that $h(a)$ is equal to zero for some a (in a set with positive measure). In this case there is a continuum of solutions for $h(a)$, but each is characterized by the same two values: f, the fraction of the individual's life spent in employment, and h, the time devoted to work at any instant in which the individual is employed. That is, the solution pins down hours of work when employed and total hours supplied over the lifetime, but the timing of work is indeterminate.[7] Of course, in steady state equilibrium, it is necessary that the pattern of hours worked across individuals be such as to yield constant aggregate hours at each point in time.

Now consider the case in which $e(a)$ is not constant. The next proposition states a very simple property of the optimal labor supply solution to this problem.

Proposition 1. *The optimal solution $h^*(a)$ has a reservation property. In particular, there exists a value e^* such that $h^*(a) > 0$ if $e(a) > e^*$ and $h^*(a) = 0$ if $e(a) < e^*$.*[8]

Proof. Suppose not. Then there are ages a_1 and a_2 such that $h^*(a_1) > 0$, $h^*(a_2) = 0$ and $e(a_2) > e(a_1)$. Consider the alternative solution in which the individual switches the hours of work and consumptions at these two ages. Lifetime utility is identical under these two scenarios, but the alternative generates higher lifetime income, implying that consumption can be increased, thereby leading to higher lifetime utility. □

[6] Alternatively, one could avoid the issues associated with the overlapping generations model by following Laitner [14, 15] and assume two sided altruism. In this framework one obtains the standard result from infinitely lived agent models in which the steady state interest rate is uniquely determined by the discount factor.

[7] The idea that theory predicts only the total time spent in employment and not the timing of employment was first noted by Mincer [21] in his study of labor supply by married women.

[8] Formally, this result can be violated on a set of measure zero. For simplicity, we will abstract from this issue in both the statement of propositions and our proofs.

Relative to the case in which $e(a)$ is constant, allowing this function to vary over the life cycle serves to eliminate the indeterminacy concerning the timing of employment.[9] Intuitively, allowing individual productivity to vary over time breaks the indeterminacy regarding the timing of labor supply, since the individual prefers to work when productivity is high. The above result does not rule out the possibility that $e^* = 0$, in which case all individuals will work positive hours in the market at all points during their lives. But independently of whether hours are always positive, this result coupled with our single peaked assumption on the profile $e(a)$ implies that there are unique starting and stopping ages for employment, though one or both of these could still be at a corner.

The same logic that implies that the individual should work when productivity is highest also implies that conditional on working, hours of work should be increasing in $e(a)$. In particular, we have the following proposition.

Proposition 2. *Let $h^*(a)$ be the optimal solution for hours of work over the life cycle. Let a_1 and a_2 be distinct ages for which $h^*(a) > 0$. Then $e(a_1) > e(a_2)$ implies $h(a_1) \geqslant h(a_2)$.*

Proof. Assume by way of contradiction that $e(a_1) < e(a_2)$ and $h^*(a_1) \geqslant h^*(a_2)$. The assumed profiles cannot be utility maximizing, since by switching the values for both c and h at ages a_1 and a_2, lifetime utility and expenditure are unchanged, while income increases, thereby allowing for higher utility. □

3. Tax policies and labor supply elasticities

In this section we consider the quantitative effects of changes in tax and transfer policies on the equilibrium hours worked profiles for individuals.[10]

3.1. Calibration

For these calculations we adopt the following functional forms:

$$U(c, 1-h) = \log(c) - \alpha \frac{h^{1+\gamma}}{1+\gamma}, \tag{3.1}$$

$$g(h) = (h - \bar{h}) \quad \text{for } h \geqslant \bar{h},\ 0 \text{ otherwise},$$

$$e(a) = e_0 - e_1 \left| .5 - a \right|. \tag{3.2}$$

Preferences are assumed to be separable and to be consistent with balanced growth, thereby dictating the $\log(c)$ term. The functional form choice for disutility from working is standard and is convenient since the parameter γ determines the elasticity of hours with respect to the tax rate in a standard labor supply model in which $g(h) = h$. The assumed functional form for $e(a)$ implies a piecewise linear productivity profile that is symmetric around mid-life. While the data suggests that linearity is not necessarily a good assumption for this profile, we adopt it because it permits a parsimonious way to investigate the importance of the slope of this profile in affecting how hours

[9] Mulligan [23] notes this same property in a model with indivisible labor.

[10] Rogerson and Wallenius [30] show analytically for the case of separable preferences that an increase in τ will necessarily lead to less time spent in employment and fewer hours of work when employed.

and employment respond to changes in taxes.[11] Given our assumption on preferences, specifically that utility from consumption is $\log(c)$, the solution for $h(a)$ is unaffected by a proportional shift in the $e(a)$ profile, so that we can normalize e_0 to one with no loss in generality.

Given these functional forms, we investigate how the parameter γ matters for the life cycle profile of hours and how it responds to changes in tax and transfer policies. For each value of γ we choose values for the three parameters α, $\bar{h}$, and e_1 so as to match three target values. The first target is the fraction of life spent in employment, which we denote by λ. If we interpret our model as representing an adult life span of 60 years, then a working life of approximately 40 years implies a target value for λ of .67. The second target is peak hours of work over the life cycle, which we denote by $h^{\max}$. If the peak workweek for employed workers over the life cycle is around 45 hours per week and individuals have roughly 100 hours of discretionary time per week, then recalling that we normalized the time endowment to one at each instant, the target value for $h^{\max}$ is .45. The third target is the variation in hourly earnings over the life cycle. Given a target value for λ, the value of e_1 will influence the range of productivities over the life cycle, and hence the range of hourly wages. We choose a value for e_1 so that hourly wages at their peak are twice as large as hourly wages at their lowest point. Because the results that we report below are very robust to changes in these values, we do not focus on justifying these exact values.

We note that the calibrated value of $\bar{h}$ is increasing in γ. This is intuitive. To see why, note that the greater the value of $\bar{h}$ the greater is the nonconvexity in the mapping from hours to labor services, and that it is this nonconvexity that induces retirement in the model. The higher the value of γ, the less the individual likes to have hours change over the life cycle. Because retirement is necessarily associated with a large change in hours of work, it follows that it requires a greater nonconvexity to induce retirement for higher values of γ. Further discussion of this issue is postponed until the next subsection.

Some care needs to be taken in matching up wages in the model with wages in the data. In the steady state equilibrium, the wage per unit of labor services, which we denoted by w, is equal to one at all points in time. But wages in the data are measured as labor earnings per hour of work, and so we compute this same measure in our model. We denote this wage rate as w^h, where the superscript h denotes that we are measuring wages per hour of work. If the function g were the identity function then earnings per hour of work in the model would be exactly equal to $e(a)$ and hence the range of wages over the life cycle would be exactly equal to $e(.5)/e(a^{\max})$, where $a^{\max}$ is the age that has the lowest productivity and positive work hours, which given our calibration is equal to $.5 + .5\lambda$.[12] But since the function g is nonlinear, this no longer holds. The range of wages over the life cycle in our calibrated model is given by:

$$\frac{w^h(.5)}{w^h(a^{\max})} = \frac{e(.5)g(h(.5))/h(.5)}{e(a^{\max})g(h(a^{\max}))/h(a^{\max})}. \tag{3.3}$$

While the hourly wage ratio is influenced by $e(.5)/e(a^{\max})$, these values are no longer identical. Nonetheless, one can choose e_1 such that this ratio is equal to 2 in the benchmark calibration.

In calibrating the model, we also assume a tax rate of .3, which corresponds to the average effective tax on labor income in the US in recent years. Having calibrated the model, we next examine what happens to equilibrium hours if the tax rate were increased to .5, which corresponds

[11] What really matters for the solution is the distribution of productivity values over the life cycle, and not the particular age associated with a given productivity value. In this sense the symmetry assumption implicit in our $e(a)$ function is not an essential feature of the specification.

[12] The same productivity level also obtains at age $.5 - .5\lambda$.

Table 1
Estimated values of b_1.

$\gamma = .5$	$\gamma = 1$	$\gamma = 2$	$\gamma = 10$
1.29	.59	.28	.05

to the average effective tax on labor income in several economies in continental Europe in recent years.[13]

3.2. Micro elasticities

Before reporting the results of the change in tax and transfer policies, it is of interest to examine some features of the calibrated benchmark economies. Given a value of γ and the calibration procedure just described, the model will generate a life cycle profile for hours worked, $h(a)$, and hourly wages, $w^h(a)$. We generate a panel life cycle data set for hourly wages and hours worked by choosing 67 equally spaced values of a over the portion of the life cycle with positive hours worked and evaluating the two functions $h(a)$ and $w^h(a)$ at these points. Note that all of the data points in the sample are times at which individuals are employed. As is standard in the labor supply literature, we take this data and run the regression:

$$\log(h_t) = b_0 + b_1 \log\left(w_t^h\right) + \varepsilon_t \tag{3.4}$$

where we use t to index the 67 data points for a given individual. The resulting parameter estimate b_1 is the so-called micro Frisch labor supply elasticity.

Table 1 shows the estimated values of b_1 for our benchmark calibrated model for four different values of γ: .5, 1, 2, and 10.

The table shows that higher values of γ are associated with lower Frisch elasticities, though note that the nonlinearity of the g function implies that the Frisch elasticity is not equal to $1/\gamma$. In particular, the nonlinearity of g implies that higher hours imply higher hourly wage rates, thereby lowering the estimated elasticity relative to a standard model. The "bias" induced by the nonlinearity of the g is substantial. The values of γ are only about 50–60% of the values that one would infer based on a linear specification.

There is a voluminous literature that has estimated Frisch elasticities using variation in hours and wages over the life cycle. Early examples include Ghez and Becker [7], MaCurdy [18], and Heckman and MaCurdy [9]. The early literature found relatively small estimates for males, on the order of .3 or less, but much larger values for women. Subsequent work, including recent papers by Kimball and Shapiro [12], Pistaferri [25] and Domeij and Floden [5] have refined these estimates in various ways, and found larger estimates, in the range of .7–1.0 for males. (See Hall [8] for a critical survey of the recent literature.)

Before proceeding, we revisit the issue of the size of the value of nonconvexity necessary to induce retirement in each of the specifications. The calibrated values of $\bar{h}$ for the cases of $\gamma = .5$, 1, and 10 are equal to .06, .15, and .39 respectively. In interpreting these values for $\bar{h}$ it is important to keep in mind that these calculations assume a single source of nonconvexity. As noted earlier, it is reasonable to also consider nonconvexities in the mapping from leisure time to

[13] Several authors have produced estimates of effective tax rates for various countries, including Mendoza et al. [20], Prescott [26] and McDaniel [19]. While there are small differences in methodology across studies, the 20% differences between the US and countries such as Belgium, France, Germany and Italy is a robust finding.

Table 2
Relative outcomes for $\tau = .5$.

γ	H	λ	$h^{\max}$
.50	.777	.857	.856
1.00	.784	.825	.918
2.00	.788	.808	.956
10.00	.790	.794	.991

leisure services, and if one included this factor the required values of $\bar{h}$ would obviously be lower. With this in mind, we would suggest that for values of γ less than 1 the required nonconvexities do not seem unreasonably large. In contrast, the values of $\bar{h}$ required when γ takes on a value such as 10 seem unreasonable. Given that the recent micro estimates summarized above suggest values of γ that are less than 1, one could simply view this as an additional piece of evidence that such high values of γ are hard to reconcile with the data. In particular, any argument in support of values of γ that are as high as 10 would have to face the challenge of explaining how such a value is consistent with observed retirement patterns. Having noted this qualification regarding the cases of high γ values, in what follows we will report results for the full range of values considered above.

3.3. Changes in tax and transfer policies

We now turn to the evaluation of tax and transfer policies. For each of the four different calibrated economies (one for each of the four values for γ in Table 1), we consider what happens to the steady state hours profile if we increase the tax rate on labor income from .3 to .5, assuming that the proceeds fund a uniform lump-sum transfer to all individuals subject to a balanced budget constraint at each point in time. With our functional forms, one can show that such a tax causes a proportional shift in the hours profile, conditional on being employed.[14] It follows that one can summarize the shift in the hours profile by simply reporting the shift in $h^{\max}$. For each economy we compute the values of aggregate hours (H), fraction of life spent in employment (λ), and peak hours worked over the life cycle ($h^{\max}$), all relative to the values in the benchmark calibrated economy with $\tau = .3$. Table 2 reports the results.[15]

Several features are worth noting. First, note that the implied change in aggregate hours worked is large in all four cases—more than 20%. Second, despite the dramatic differences in estimated Frisch elasticities in the four economies—a factor 25 difference between the highest and lowest—the changes in aggregate hours worked are essentially constant across the four different economies. Third, although the value of γ has virtually no effect on the change in aggregate hours worked, it has very significant effects on how the change in aggregate hours is broken down into changes in length of working life versus changes in hours worked while employed. In analyzing this decomposition, it is important to note that the relative change in $h^{\max}$ is a measure of the change in total hours due to changes in the h profile holding λ constant, since as noted earlier, the h profile shifts proportionately, and for a given λ, a proportionate shift in the profile shifts aggregate hours by the same amount. However, it is not true that a shift in λ leads to a

[14] See Rogerson and Wallenius [30] for this result.

[15] While Table 2 contrasts outcomes for just two different tax rates for a range of values of γ, we note that the effects are very close to linear in the tax rate.

proportionate shift in aggregate hours, since as λ decreases the marginal employment episodes that are lost represent fewer hours of work. In any case, when $\gamma = .50$ the downward shift in the hours profile accounts for over 60% of the total decrease in hours, while when $\gamma = 10$ this downward shift accounts for less than 5% of the shift.

A key finding of the above analysis is that tax rate differences of the magnitude found between the US and many countries of continental Europe lead to large differences in hours of work, independently of the value of γ. The model also has two other predictions that are very relevant in this context. First, consistent with the data, it predicts that differences in aggregate hours are accounted for by sizable differences along both the intensive and extensive margins. Second, our model predicts that all of the differences in employment rates are accounted for by differences among young and old workers. Rogerson [28] documents that this property is found in the data.

One additional implication of the model is also of interest. Although changes in taxes do not affect technology in our model, they can affect productivity measures such as output per hour because of the difference between labor services and time devoted to market work. Note that there are two opposing effects of higher taxes on productivity per hour in our model. On the one hand, the decrease in hours is concentrated among lower productivity workers since this is where the extensive margin is operative, leading to higher output per hour in the high tax economy. On the other hand, higher taxes shift the hours profile down, thereby lowering the ratio of labor services to hours for employed individuals, leading to lower productivity. The importance of these two effects is influenced by the relative size of adjustment along the intensive and extensive margin, and hence by the value of γ. However, it turns out that these effects are relatively small in our numerical simulations. For all four economies the increase in taxes is associated with a drop in output per hour, but the decrease is less than 1%, and ranges between .9% and .6% as γ is varied from 10 to .5.

We close this subsection with an example that attempts to provide some insight into the finding that the aggregate response is roughly independent of the curvature parameter γ. To do this we consider a continuous time indivisible labor version of the model with no life cycle effects, and consider how exogenous responses along the intensive margin will influence the aggregate response to a change in taxes. Instantaneous utility is now given by:

$$\log c(t) - \frac{\alpha}{1+\gamma} h(t)^{1+\gamma}$$

and there is a fixed workweek $\bar{h}$. We assume that each unit of time supplied as labor now yields one unit of labor services. We consider a tax and transfer policy as before, i.e., labor earnings are taxed at constant rate τ, and all individuals receive a lump-sum transfer T.

We again focus on the zero interest rate steady state. As before, individuals will use their income to finance a constant stream of consumption. Because there are no life cycle effects and labor is indivisible, the individual optimization problem reduces to deciding what fraction of one's life to spend in employment. Denoting this fraction by e, the equilibrium condition for fraction of life spent in employment is given by:

$$ev(\bar{h}) = (1-\tau).$$

Substituting for $v(\bar{h})$ and taking logs gives:

$$\log(e) + (1+\gamma)\log\bar{h} + \log\frac{\alpha}{1+\gamma} = \log(1-\tau).$$

Since all generations solve the same problem, we can use this to yield the following expression:

$$\log H + \gamma \log \bar{h} + \log \frac{\alpha}{1+\gamma} = \log(1-\tau)$$

where $H = e\bar{h}$ is aggregate hours.

Now consider a change in τ and assume that $\bar{h}$ changes exogenously as τ changes. We then have:

$$\Delta \log H + \gamma \Delta \log \bar{h} = \Delta \log(1-\tau).$$

Holding $\Delta \log \bar{h}$ fixed, it follows that the response in H will depend on γ. But if

$$\Delta \log \bar{h} = B \times \frac{1}{\gamma} \Delta \log(1-\tau)$$

for some constant B, then we have:

$$\Delta \log H = (1-B) \times \Delta \log(1-\tau).$$

These last two expressions say that if the elasticity of the intensive margin h with respect to changes in after tax returns is proportional to $1/\gamma$ then the elasticity of aggregate hours with respect to after tax returns will be independent of γ. If one examines Table 2 one sees that the elasticity of the intensive margin with respect to after tax returns is approximately proportional to $1/\gamma$.

3.4. Comparison with a stand-in household economy

The model economy that we have studied is not a single agent economy, in the sense that at any point in time there are many different types of individuals alive. However, it is interesting to ask what one might infer about labor supply if one were to interpret the outcomes generated by the tax changes in our model by using a standard static stand-in household model. In particular, consider a static economy with a single agent, with preferences given by

$$\log(c) - \mu \frac{h^{1+\theta}}{1+\theta} \tag{3.5}$$

and a linear technology that can turn one unit of time into one unit of consumption:

$$c = h. \tag{3.6}$$

There is a government that taxes labor at the constant proportional rate of τ and uses the proceeds to fund a lump-sum transfer to the representative agent.

Faced with the information in Table 2, we ask what an economist using this model to interpret the hours differences would conclude about the parameter θ that dictates the labor supply elasticity for the stand-in household in this economy. Standard calculations lead to the following expression for hours of work in terms of taxes:

$$h = \left[\frac{1-\tau}{\mu}\right]^{1/(\theta+1)}. \tag{3.7}$$

If we let h_i denote the hours that correspond to a country with tax rate τ_i, for $i = 1, 2$, then using the above expression to interpret data on taxes and hours of work leads to the following expression for θ:

$$\theta = \frac{\log(1-\tau_1) - \log(1-\tau_2)}{\log(h_1) - \log(h_2)} - 1. \tag{3.8}$$

Table 3
Implied values for θ.

$\gamma = .5$	$\gamma = 1$	$\gamma = 2$	$\gamma = 10$
.33	.38	.41	.43

Applying this expression to the four calibrated economies, we obtain the results shown in Table 3.

The associated Frisch elasticities, given by $1/\theta$, range from 2.3 to 3, despite the fact that the Frisch elasticities inferred from micro data range from .05 to 1.25.[16]

The above calculation shows that a static stand-in household model with a fairly high labor supply elasticity can reproduce the steady state effects of taxes on aggregate hours found in the life cycle model studied earlier. It is also of interest to ask whether the welfare implications of tax changes are similar across the two specifications. Our measure of welfare is the percent increase in lifetime consumption required to make households living in the high tax economy indifferent to living in the low tax economy. It turns out that the answers are remarkably similar in the life cycle and stand-in household models. For example, in the $\gamma = 1$ economy the welfare cost of the higher tax system is 10.7% of consumption, while in the corresponding stand-in household model the welfare cost is 10.4% of consumption.

3.5. The role of $g(h)$

The above results indicate that in our life cycle economy, micro labor supply elasticities are not particularly relevant in predicting the aggregate effects of permanent changes in taxes. It is important to emphasize the feature of the economy that is responsible for this result. In particular, the mere fact that our economy is an overlapping generations model is not important in generating this result. Rather, the key feature of our economy is the nonlinear mapping from time spent working to labor services, and the fact that this feature generates a life cycle profile for hours worked with hours equal to zero for some parts of the life cycle. To understand this, consider an economy that is identical to the one that we have studied except assume that the function g is identically equal to one. Fig. 1 illustrates how this will influence the findings.

In this figure, the top line shows the life cycle productivity profile. The two solid lines indicate the life cycle profile for hours worked in the case of linear and nonlinear g. As the picture shows, if g is nonlinear then we can generate outcomes in which hours worked are concentrated in the period of life in which productivity is highest. In particular, hours worked are not continuous in productivity. In contrast, if g is linear, it is optimal for the individual to smooth hours worked across time, although hours of work will be higher when productivity is higher. But in this case hours vary continuously with productivity. The two dashed lines indicate the effects of higher taxes on hours of work in the two cases. If g is nonlinear, then the hours worked profile shifts down and the reservation productivity level shifts up, while in the case of a linear g function, the only effect is a downward shift in the hours profile. In both cases the extent of the downward shift of the hours profile is very strongly related to the micro labor supply elasticity. Because this downward shift is the only effect when g is linear, it turns out that there is a strong relationship between micro and macro elasticities in this case.

[16] One can show that our steady state equilibrium corresponds to the allocation that maximizes an equal weighted sum of individual utilities. It follows that one can derive an analytic expression for preferences of the stand-in household.

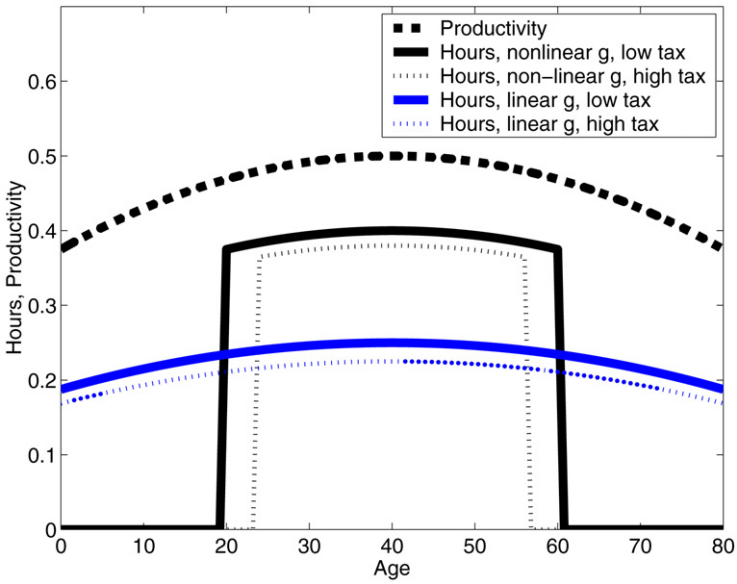

Fig. 1. Hours and productivity over the life cycle.

But the issue is more severe than simply being that the micro elasticity only captures one piece of the aggregate adjustment in hours in the case of a convex $g(h)$ function. This is because our simulations show that the smaller is the part that the micro elasticity captures, the larger is the part that it does not capture, i.e., the higher the value of γ, the larger is the response on the extensive margin.

3.6. The importance of heterogeneity

Previous work on the implications of labor indivisibilities for aggregate labor supply elasticities has stressed that heterogeneity may have a large influence on the implied aggregate labor supply elasticity. This argument appears in different contexts in both Mulligan [23] and Chang and Kim [3,4]. Given these findings, it is of interest to examine the importance of heterogeneity for the aggregate response of hours to taxes in our model. The key dimension of heterogeneity in the cross-section of our steady state is the distribution of individual productivities across consumers. This heterogeneity in the cross-section is exactly the same as the heterogeneity that a given individual faces over their lifetime, and hence is characterized by the function $e(a)$. In our calibrated examples considered above, we chose the value of e_1 so as to achieve a given degree of heterogeneity in wages both in the cross-section and over the life cycle. In fact, the variation in e_1 across the four different cases was quite small, ranging from .46 for $\gamma = .5$ to .522 for $\gamma = 10$. To explore the importance of heterogeneity for our results, we continue to choose values for $\bar{h}$ and α so as to match values for λ and $h^{\max}$, but will no longer calibrate the value of e_1 to target a particular range of wages. Instead, we simply consider a range of values for e_1 and report the results for the effect of an increase in taxes from .3 to .5 in each case. We also report the implications for the amount of cross-sectional heterogeneity across employed workers.

Table 4
Heterogeneity and relative outcomes ($\gamma = 2$).

e_1	H	λ	$h^{\max}$	$w(.5)/w(a^{\max})$	b_1
.90	.728	.732	.991	.875	.26
.70	.758	.770	.972	.664	.27
.50	.789	.810	.956	.493	.28
.30	.820	.852	.941	.351	.30
.10	.849	.893	.928	.238	.32

Table 4 shows the implied effects of differences in e_1 from .1 to .9 for the case of $\gamma = 2$.[17]

Note that although we are holding γ constant in this exercise, it does not follow that the estimate of the Frisch elasticity from the life cycle profile is necessarily constant. The last column of Table 4 reports the estimated Frisch elasticities for the different values of e_1. Although the values are influenced by e_1, the range of estimates is not very large. Turning to the results, the second through fourth columns report the same information that we have focused on before—the relative values for aggregate hours, time spent in employment and peak hours of work. Consistent with the findings of previous researchers, this increase in heterogeneity does reduce the aggregate response for a given increase in tax rates and a given value of γ. However, while the effect is significant, it should be emphasized that even if we were to consider a factor 5 increase in the range of e values in the cross-section (or a doubling of the range of cross-sectional hourly wages), the aggregate consequences are still very large—a 20% increase in taxes still leads to a decrease in hours of work of more than 15%.

It is important to note that the other studies that we referred to were based on indivisible labor models, i.e., they assumed that hours of work conditional upon employment were exogenous and equal for all workers. In our model, hours of work are endogenous and respond to changes in the environment. This is significant, since a comparison of the results for relative values of λ and $h^{\max}$ shows that as e_1 decreases, the drop in λ becomes smaller, but the drop in $h^{\max}$ actually increases. When $e_1 = .9$, the drop in $h^{\max}$ is basically one percent, while when $e_1 = .1$ the drop in $h^{\max}$ exceeds seven percent. Precisely because of this opposing effect on $h^{\max}$, a comparison of the second and third columns indicates that changes in e_1 have a much larger effect on λ than on H. It follows that an indivisible labor model may preclude an important margin of adjustment in some contexts.

There is one additional point of interest to note concerning heterogeneity. In our model we assumed that there is no heterogeneity within a cohort. One may conjecture that adding heterogeneity within a cohort will further diminish our aggregate effects. However, at least one form of within cohort heterogeneity will have no impact on our findings. Specifically, assume that within each cohort there is a distribution of permanent productivities, represented as proportional shifts of the productivity profile $e(a)$. As noted earlier, with $u(c) = \log(c)$, proportional shifts of the productivity profile have no impact on the lifetime hours profile, and so this form of heterogeneity would have no impact on our findings.

[17] Recall that a proportional shift in the $e(a)$ profile has no effect on hours of work in our model, so that it is the ratio e_1/e_0 that matters and not $e_0 - e_1$.

4. Conclusion

In this paper we develop a general equilibrium life cycle model of labor supply that incorporates both intensive and extensive margins of labor supply. In the equilibrium of our model, individuals have well-defined working lives, in the sense that they enter the workforce at some point in their life and then work continuously until some later point, at which time they withdraw from employment and do not work again. We then use this model to analyze the implications for observed differences in tax and transfer programs between the US and several countries in continental Europe. In the context of this exercise we can use our model to compute micro labor elasticities using life cycle variation in hours and wages for prime age workers, as well as macro labor elasticities using variation in aggregate hours across economies. Our analysis produces five main findings. First, macro elasticities and micro elasticities are virtually unrelated: a factor 25 difference in micro elasticities is associated with only a thirty percent change in the associated macro elasticities. Second, macro elasticities are large-in the range of 2.3–3.0. Third, in our model with variation in either productivity or disutility of work over the life cycle, tax and transfer programs necessarily imply that higher taxes lead to less work on both the extensive and intensive margin. Fourth, the employment differences generated by differences in tax and transfer programs are necessarily concentrated among young and old workers. Fifth, the assumed nonlinearity of labor services in work hours implies a significant bias in the mapping from estimated Frisch elasticities to the preference parameter governing curvature over disutility of work.

There are many natural extensions of interest. In terms of understanding life cycle variation in wages and hours of work it is of interest to consider alternatives which stress endogenous accumulation of human capital.[18] In terms of assessing the implications of tax and transfer programs for hours of work it is of interest to consider a richer description of how tax and transfer programs interact with age and productivity. Lastly, while our analysis has addressed the issue of reconciling micro and macro elasticities in the context of permanent differences in tax and transfer programs, there is a related issue of reconciling micro and macro elasticities in the context of business cycle fluctuations which our analysis does not specifically address.[19]

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[18] Two recent examples are Imai and Keane [10] and Ljungqvist and Sargent [16]. The latter also examines the implications for tax and transfer programs.

[19] Chang and Kim [3] pursue this issue in a model with incomplete markets, idiosyncratic shocks and indivisible labor.

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