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MIGRATION AND OPTIMAL INCOME TAXES*

J.A. MIRRLEES**

Nuffield College, Oxford OX1 1NF, UK

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The issue addressed in this paper is the optimal taxation of incomes earned in the home economy, and of incomes earned abroad, when people can migrate. As a preliminary, the optimal taxation of home incomes when there is migration and no taxation of foreign incomes, is discussed. Then in a more general setting, we deal with optimal taxation of different kinds of labour when another kind of labour is not taxable, and show how this bears on the taxation of foreign incomes. The last sections of the paper analyse a simple model in which people choose between taxable labour at home, taxable labour abroad, and untaxable labour. A condition is found implying that the optimal tax on foreign income is higher than on the home income of a person of equal utility.

1. Migration

High tax rates encourage emigration. The resulting loss of tax revenue is widely believed to be an important reason for keeping taxes down. If, as Bhagwati (1980) has proposed, the emigrants' foreign incomes were taxed, there would be two advantages to the domestic government: emigrants would contribute to tax revenues, and tax rates could be higher. There are also implications for other taxes and subsidies. In particular, education would become a better investment for the State, and should therefore be subsidised to a greater extent. Bhagwati and Hamada (1982) have shown that, in a simple model, foreign and domestic incomes should be taxed at the same rate, namely (nearly) 100 percent. The assumptions of that model lead perhaps too directly to the conclusion. Income is the outcome of education,

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in the same way that a firm's profits are the outcome of its investments, consumers wishing to maximize discounted net income less education costs. Thus, nearly full taxation of the return, and nearly full subsidisation of the capital cost, induces individuals to do what the State would like, with essentially all costs and benefits accruing to the State. Fundamental reasons for less than full taxation are absent from the model, and one wonders whether the conclusion that the same tax rates should be levied on domestic and foreign incomes would hold in a more realistic model.

The model also ignores the possibility of emigrants severing themselves completely from their country's tax system. One might perhaps think it obvious that this is a reason for reducing the foreign-income tax rate relative to domestic tax rates, a point possibly too obvious to be worth exploring formally.

This paper examines the optimal taxation of foreign incomes by LDCs in models that may be a little more realistic than the Bhagwati–Hamada model. Although only special cases are solved explicitly, the results for these cases tend to support the case for high taxation of foreign earnings. It should be emphasised that the arguments apply only to LDCs with governments whose expenditures benefit the generality of the population. As a preliminary, section 3 is devoted to the theory of optimal taxation when foreign earnings are not taxed.

It should be emphasised at the outset that the income taxes and subsidies appearing in the models correspond to all taxes on incomes *and expenditures* in the real world. If it were desirable to tax foreign earnings at the same rate as domestic earnings, the foreign-income rate should include an element corresponding to such taxes as sales tax, value-added tax, and import duties.

A second simplification I have allowed myself is to ignore all intertemporal considerations, and even to pretent that individuals either earn income at home or abroad but not both. These are not satisfactory assumptions, but models allowing migration to be temporary or permanent, and to take place part of the way through the working life, seem to become complicated very quickly. In section 4, where the discussion is conducted heuristically, it is possible to allow for partial migration.

2. Criteria

Three different criteria occur to people thinking about migration, depending on the group whose welfare is to count. The first criterion attempts to restrict the group to those who do not migrate. This is hardly satisfactory. Some of those who do not migrate may have wished to do so; some who do migrate may do a great deal for their country of origin, such as sending home remittances. It is therefore hard to see how a loyalty criterion could be implemented. The fact that many countries do so little to tax emigrants, temporary or permanent, suggests that governments are not guided by a loyal-citizen criterion. Since it is morally unattactive, we should be pleased to be able to reject it. An alternative interpretation of the criterion is that the government should restrict taxation (and benefits) to voters. That opens the question: Who should be voters? But, whether the voters are all nationals, or one colonel, we should be prepared to argue that they ought to vote in the interests not only of one another, but of others, if that seems to be right.

The second criterion defines the relevant group as that of nationals, whether working in the country or not. 'Nationals' had better be understood in a nonlegal sense, since the group is otherwise endogenous, the extent to which people change nationality being influenced by economic variables. Perhaps the best definition is nationality at birth, though even that is not always well-defined. This criterion, like the first, is unsure what to do about immigrants. Strictly speaking, they are excluded; but I hope few of us believe that is morally right. Yet the alternative of including all who would, or might, like to immigrate is not consistent with the spirit of the criterion.

The third main possibility is to include all humans. Criteria that do so are surely morally defensible, but they may be thought not to be what an adviser to a democratic State is expected to be guided by. The network of double-tax agreements, and the allowances for foreign-income tax provided by many countries, suggest that the conclusion is too hasty. Totally to neglect the welfare of citizens of other countries is not acceptable as explicit policy motivation.

In the case of LDCs, one might reduce possible conflict between the national and world welfare functions by insisting that marginal income for those living in the countries to which emigrants go have a negligible weight in the world welfare function because of their high incomes. Specifically, it would then be permissible to neglect the effect on their incomes of changes in their governments' tax revenue, brought about by changes in emigration. These effects are in principle substantial, and can be neglected only if there is some reason to regard the welfare change as negligible. One can also often neglect immigration as negligible, simply because incomes are too low to encourage it. In the present paper, effects on foreigners are neglected on these grounds. But it is not always permissible to do so. Doctors and engineers may migrate from one poor country to another. There are LDCs where the use to which government revenues are put is not such as to make one assign them more weight than government revenue in a typical industrial country.

3. No tax on foreign incomes

The easiest theory of optimal income taxation is that for an economic model where each individual's productivity — here identified with his wage

— is identifiable and fixed, though his inclination to migrate is unknown. An individual of productivity n receiving after-tax income $x(n)$ has utility

$$v(n)=u(x(n),n). \tag{1}$$

The number of such individuals who remain in the economy is $f(v(n),n)$, an increasing function of v. A small change in $x(n)$, δx, induces a few people more or less to emigrate, but they are almost indifferent between staying and going. Assuming that this indifference correctly reflects what they will or would experience, the impact on total utility is

$$u_x \delta x \cdot f(v(n),n). \tag{2}$$

Notice that this argument neglects the effects upon those living in the foreign economies of resulting changes in tax revenue there.

The impact of δx on tax revenue in the domestic economy is, since by assumption marginal productivities do not change, to reduce it by

$$\delta x \cdot f(v(n),n)+(x-n)f_v \cdot u_x \cdot \delta x. \tag{3}$$

For optimality, (2) and (3) must be in constant proportion as n varies. Thus,

$$u_x f=\lambda f+\lambda(x-n)f_v u_x \tag{4}$$

for some constant λ. The constant is to be determined by the economy's budget constraint. Information about propensities to migrate is conveniently expressed by the *elasticity of numbers with respect to after-tax income:*

$$\eta=\frac{xf_v u_x}{f}. \tag{5}$$

With this notation, (4) can be rewritten as

$$\frac{n-x}{x}=\frac{1}{\eta}\left(1-\frac{u_x}{\lambda}\right). \tag{6}$$

The left-hand side is tax as a proportion of after-tax income.

In general η is a function of n as well as x, so that (6) is not an explicit formula for the optimal tax rates. When η is constant, it is easily solved. For example, if

$$\eta=0.5, \qquad u=t(n)-\frac{1}{x}, \tag{7}$$

then (6) becomes

$$n = 3x - \frac{2}{\lambda x}. \tag{8}$$

Thus, writing $\lambda = 12/a$

$$x(n) = \tfrac{1}{6}[n + (a + n^3)^{1/2}]$$

and

$$\frac{x}{n} \to \tfrac{1}{3} \qquad (n \to \infty).$$

In this case x is a convex function of n. There is a minimum consumption level, depending on the resource constraint, and the marginal tax rate falls from 5/6 on the lowest incomes to 2/3 on the highest. This example suggests that rather high tax rates are justifiable even if the propensity to migrate is quite large. Of course other sources of labour supply elasticity have been neglected.

To help intuition about η, consider the following situation. Denote foreign earnings, net of foreign tax, by m, and suppose that m, n are jointly distributed in the population with density $g(m, n)$. (One might well suppose that a nonzero proportion of people with home wages n have no foreign opportunities, i.e. $m = 0$; but it is simpler to neglect that here, for it makes no essential difference to the analysis.) Suppose, furthermore, that working abroad involves the same disutility of labour for anyone as working at home and is equivalent to multiplying after-tax income by $\gamma < 1$: i.e. an (m, n)-person who works abroad has utility $u(\gamma m, n)$.

Then the number of n-people who decide not to migrate is

$$f(v, n) = \int_0^M g(m, n)\,\mathrm{d}m, \tag{9}$$

where $M = M(v, n)$ is defined by

$$u(\gamma M, n) = v. \tag{10}$$

From (9) we have

$$f_v = g(M, n)\, M_v$$

$$= \frac{g(M, n)}{\gamma u_x(\gamma M, n)}.$$

In formula (5), η is defined in terms of $u_x(x,n)$, where x is after-tax income of an n-person, satisfying $u(x,n)=v$. By (10), $\gamma M = x$. Consequently,

$$\eta = \frac{x f_v u_x}{f} = \frac{xg}{\gamma f}$$

$$= Mg(M,n) \Big/ \int_0^M g(m,n)\,\mathrm{d}m. \tag{11}$$

It is to be expected that under any tax schedule, and in particular the optimum, η will vary to a substantial extent with n. To explore this, it is worth analysing another specific example. Let

$$u(x,n) = u_1(x) + t(n). \tag{12}$$

Let $\log m$ and $\log n$ be distributed according to a binormal distribution with means zero and variances σ_m^2 and σ_n^2 and correlation coefficient ρ, so that $g(m,n)$ is proportional to

$$\frac{1}{mn}\exp\left[-\frac{\mu^2 - 2\rho\mu\nu + \nu^2}{2(1-\rho^2)}\right],$$

where

$$\mu = \frac{\log m}{\sigma_m}; \qquad \nu = \frac{\log n}{\sigma_n}.$$

The restriction to zero means is no real restriction: different means can be accommodated by varying the parameter γ.

With a little manipulation, we find, using (11), that

$$\eta = \frac{1}{\alpha}\frac{1}{\psi(\zeta)}; \qquad \alpha = \sigma_m \sqrt{(1-\rho^2)}, \tag{13}$$

where we define

$$\psi(\zeta) = \mathrm{e}^{(1/2)\zeta^2} \int_{-\infty}^{\zeta} \mathrm{e}^{-(1/2)z^2}\,\mathrm{d}z \tag{14}$$

$$\zeta = \frac{\mu - \rho\nu}{\sqrt{(1-\rho^2)}} = \frac{1}{\sqrt{(1-\rho^2)}}\left[\frac{\log M}{\sigma_m} - \frac{\rho \log n}{\sigma_n}\right]. \tag{15}$$

ψ is an increasing positive function, approximately $1/(1-\zeta)$ for $\zeta < -3$ ($\psi(-3) = 0.305$), and approximately $\sqrt{(2\pi)}\,\mathrm{e}^{(1/2)\zeta^2}$ for large ζ.

The optimal-tax formula (6) for this case is,

$$\frac{n}{x}-1=\alpha\psi(\zeta)\cdot(1-u_1'(x)/\gamma). \tag{16}$$

Recollect that by (10), $M=x/\gamma$, and this should be substituted in (15):

$$\zeta=\frac{1}{\alpha}\log(xn^{-\tau}\gamma); \qquad \tau=\rho\sigma_m/\sigma_n. \tag{17}$$

To gain some qualitative insight, we shall analyse the implications of (16) for small and large n in turn. But notice first that, if we define x_0 by

$$u_1'(x_0)=\gamma, \tag{18}$$

eq. (16) is satisfied when $x=x_0$ and $n=x_0$. Thus,

$$x(x_0)=x_0. \tag{19}$$

By concavity of u, $x<n$ for $n>x_0$ (income taxation) and $x>n$ for $n<x_0$ (income subsidisation).

As $n\to 0$, one expects that x tends to a positive limit. I can show that it does, provided that $\lim_{x\to 0}u_1=-\infty$. This seems a reasonable assumption to make, and it will be assumed. If $\lim x$ is positive as $n\to 0$, the left-hand side of (16) tends to -1. Also, by (17), $\zeta\to\infty$; and ψ therefore tends to infinity. It follows from (16) that $u_1'/\gamma\to 1$ as $n\to 0$, i.e.

$$x(0)=x_0. \tag{20}$$

Before further comment on the joint significance of (19) and (20), consider n large. To avoid a lengthy analysis, assume that ζ tends to a limit, possibly $\pm\infty$, and consider the three possibilities:

(i) $\zeta\to-\infty$. Then $\psi\sim-1/\zeta$, and (16) implies that

$$-\alpha\log\frac{xn^{-\tau}}{\gamma}\cdot\left(\frac{n}{x}-1\right)\to 1$$

if $x\to\infty$, or is bounded above in any case. Since $xn^{-\tau}\to 0$, $n/x\to 1$. These two statements can be consistent only if $\tau>1$, and then we have

$$x\sim n-\frac{1}{\alpha(\tau-1)\log n}. \tag{21}$$

(ii) $\zeta \to \bar{\zeta}$. Then $x \sim \gamma e^{\alpha\bar{\zeta}} n^{\tau} \to \infty$, and from (16),

$$\frac{x}{n} \to [1 + \alpha\psi(\bar{\zeta})]^{-1}. \tag{22}$$

These statements can be consistent only if $\tau = 1$, and then we have

$$[1 + \alpha\psi(\bar{\zeta})]\gamma e^{\alpha\bar{\zeta}} = 1. \tag{23}$$

(iii) $\zeta \to \infty$. Again $x \to \infty$, and (16) implies that $n/x \to \infty$. With $xn^{-\tau} \to \infty$, this requires $\tau < 1$. We have $\psi \sim \sqrt{(2\pi)}\, e^{(1/2)\zeta^2}$ as $\zeta \to \infty$. Therefore

$$\sqrt{(2\pi)}\, \alpha \frac{x}{n} \exp\left[\frac{1}{2\alpha^2}\left\{\log\left(\frac{xn^{-\tau}}{\gamma}\right)\right\}^2\right] \to 1.$$

Taking logarithms,

$$\frac{1}{2\alpha^2}\left\{\log\left(\frac{xn^{-\tau}}{\gamma}\right)\right\}^2 + \log\left(\frac{x}{n}\right) \to -\log\{\alpha\sqrt{(2\pi)}\}.$$

Since $\log(x/n) = \log(xn^{-\tau}) - (1-\tau)\log n$, and $xn^{-\tau} \to \infty$, we deduce, on dividing by $\{\log(xn^{-\tau}/\gamma)\}^2$, that

$$\frac{(1-\tau)\log n}{\{\log(xn^{-\tau}/\gamma)\}^2} \to \frac{1}{2\alpha^2}.$$

Taking square roots, we obtain:

$$x \sim \gamma n^{\tau} \exp[\alpha\sqrt{(2(1-\tau)\log n)}]. \tag{24}$$

In summary, we have shown that $x/n \to 1$, and the marginal tax rate therefore tends to zero, when $\rho\sigma_m/\sigma_n = \tau \geqq 1$; but that $x/n \to 0$, and the marginal tax rate tends to one when $\rho\sigma_m/\sigma_n < 1$. The latter case is perhaps the most realistic. In the lower range of n, where incomes are subsidised, we have found that $x = x_0$ both at $n = 0$ and at the zero-tax level. Thus, x is a *decreasing* function of n near $n = 0$. In the setting of the problem it was supposed, unreasonably, that it would be possible, if desirable, to have after-tax income a decreasing function of before-tax income. Since we have found that it is optimal to exploit this freedom in a model with no elasticity of labour supply other than through migration, we should really modify the problem at least by requiring x to be a nondecreasing function of n. If we do so, it is optimal to have x constant for an initial range of n. In this model it is optimal to have a marginal tax rate of 100 percent on the lowest range of incomes.

4. The foreign-income tax: General considerations

From the point of view of the worker, domestic labour and foreign labour are substitutes. Therefore if one is taxed, both should be. From the general theory of nonlinear taxation [see, for example, Mirrlees (1976)] we know that the marginal rate of tax on one commodity should be greater than the marginal rate on another if the marginal rate of substitution of the first for the second increases with ability. The result is independent of the distribution of ability, but it does depend on the assumptions (among others) that ability can be characterized one-dimensionally, and that individual consumers use some of each of the commodities. We therefore cannot apply the theorem automatically. It is plausible that more able people find it easier to substitute a dollar of foreign earnings for a dollar of home earnings, and therefore plausible that foreign income should be taxed at a higher rate than domestic income. But this is not a strict implication of the theorem. In particular, one may well wonder whether the presence of opportunities for earning untaxed foreign income may not so affect the marginal rate of substitution between taxed foreign income and home income as to greatly weaken the result.

This issue is worth exploring formally, despite the highly restrictive assumption that abilities in the population can be characterised by a single real variable. Consider, then, a model in which a typical consumer has utility function

$$u(x, y, y', z, n),$$

where

x = income after tax,
y = foreign earnings net of tax, subject to domestic tax,
y' = foreign earnings net of tax, not subject to domestic tax, and
z = domestic earnings before tax.

Recollect that in this kind of model one identifies earnings, before deduction of the taxes that are to be determined, with labour supplied by the consumer. Foreign tax rates being fixed throughout the analysis, we can use variables for foreign income that are net of tax collected by foreign governments. n is the consumer's 'ability'.

The tax policy of the domestic government makes x a function of y, y', and z of the form

$$x = c(y, z) + y'.$$

We know, from the theorem alluded to above, that in a model where there is no untaxable commodity, the difference between the marginal tax rates on two income sources (as a proportion of before-tax income from the source) has the opposite sign to the partial derivative with respect to n of the ratio of the marginal utilities of the two income sources. In the present model that means that foreign income is taxed at a higher marginal rate than domestic income under the optimal system if

$$\frac{\partial}{\partial n}\frac{u_y(x,y,z,n)}{u_z(x,y,z,n)}<0. \tag{25}$$

This is the correct result when there is no untaxed commodity. We can deduce the corresponding result when the untaxed income source y' is introduced. The consumer chooses y' to maximize

$$u(c(y,z)+y',y,y',z,n).$$

Denoting the maximized utility by $\bar{u}(c,y,z,n)$, the above result now applies to the utility function $\bar{u}$. By the envelope theorem,

$$\bar{u}_y=u_y(c+y',y,y',z,n).$$

$$\bar{u}_z=u_z(c+y',y,y',z,n),$$

where y' is the function of c,y,z,n defined by the fact that it maximizes u. Define

$$s(c,y,y',z,n)=u_y/u_z.$$

We have to consider the partial derivative of s with respect to n, taking account of the dependence of y' on n. Thus, foreign income should be taxed more highly at the margin than domestic income if

$$s_n+s_{y'}\frac{\partial y'}{\partial n}<0. \tag{26}$$

It seems likely, as remarked above, that s, being willingness to substitute home for foreign earnings, would have a negative partial derivative with respect to n: $s_n<0$. It also seems plausible that y' would increase with n, given y and z and c. But it does seem reasonable that y' should have the opposite effect on s from n, i.e. that $s_{y'}>0$. In this case the presence of an untaxed source of income does seem to be a good reason for having a lower marginal tax rate on the source for which it is a closer substitute.

The case for supposing that the partial derivative of s with respect to untaxable foreign income y' is positive is by no means overwhelming. One way of thinking about this question is to consider the special case

$$u = u_0(x) + u_1(z/n) + u_2(y/n) + u_3(y'/n), \tag{27}$$

where u_1, u_2, and u_3 might be thought of as utility arising from labour activity in successive subperiods of the consumer's life. In this particular case it is evident that s, being the ratio of the derivatives of u_1 and u_2, is independent of y'. Consequently, by (26), the condition for higher tax on foreign income is simply that s_n be negative, a condition that, as we have remarked, seems quite plausible. The form (27) may not seem particularly plausible, with consumption separated from labour and allocated over the lifetime independently of labour. An additively separable utility function for consumption and labour is quite commonly used, and is at least not evidently absurd. The implicit assumption of a rather perfect capital market is much more unrealistic, but there is no reason to think that a more detailed treatment of intertemporal consumption would affect the presumption about $s_{y'}$ one way or the other. One influence neglected by the additive form (27) is the way that experience of working abroad may make the transition to complete independence from the home country, severing the tax link, more palatable. Like all intertemporally additive utility functions, it supposes that the influence of recent circumstances is no different from the influence of earlier experiences. The best case for supposing that $s_{y'}$ is positive is that working abroad may tend to make the home country, its needs, and the obligations it imposes, less vivid and compelling.

For the more general case, where consumers differ in more than one dimension of ability, where for example their earning capacity may not be highly correlated with their earning capacity at home, no result as conveniently applicable as (25) is available. A simple generalisation of the Atkinson–Stiglitz theorem [Atkinson and Stiglitz (1976)] tells us that foreign and home income should be taxed at the same rate if the consumer's utility function takes the form

$$u(\phi(x, y, z), x, n_1, n_2, \ldots). \tag{28}$$

In this case the marginal rate of substitution between y and z is the same for everyone who has the same x, y, and z. Unfortunately, (28) is not a particularly plausible form in our context. That does not imply that the two sources of income should be taxed at different rates. It does not seem to be worth pursuing the impact of a nontaxable income source y' on the Atkinson–Stiglitz result in the present context, interesting though the question is, more generally.

From a heuristic discussion like this, one should not draw firm conclusions. But I think it shows that the 'common-sense' belief that taxes on foreign income ought to be low because it is easy to change citizenship, or cheat, is not very well founded. There may be other offsetting arguments for taxing foreign income at a higher rate. In any case, escape to nontaxed status, whether legally or illegally, may often be as easily available to the home earner as the foreign-income earner. Escape routes do often provide a case for lower tax rates. One must assess their bearing on different kinds of earnings rather carefully before concluding that they provide a case for low foreign-income tax rates.

5. A model of foreign-income taxation

The rather general model indicated in the previous section is, it seems, hard to get detailed results from. Qualitative results are quite interesting, but quantitative ones are a better basis for policy discussion, however preliminary. In this section I extend the model used in section 3 to the case where foreign income can, sometimes, be taxed by the home country. From the previous analysis we found that it might be optimal, under special circumstances, to have a tax system under which after-tax domestic earnings decrease with earnings, while foreign after-tax earnings increase. That is a mildly interesting curiosity, but hardly realistic. The model omitted labour-supply elasticity which surely exists. One simple way of bringing it in is to introduce the possibility of untaxed labour, as in section 4. The model is therefore generalized as follows.

Imagine a country whose citizens can choose (i) to stay in the country as income-tax payers, (ii) to work abroad, report their incomes to the home government, and pay income tax at a different rate, or (iii) not to pay income taxes to the home government. They vary in their abilities to earn income in these three categories, and in their willingness to sever legal connections with the home country. This conception is expressed by supposing that people of given income-earning ability would choose the one of the two taxable possibilities that provides greatest utility; but that a proportion depending on the utility thus available will prefer nontaxable status. We then have to identify the tax policies that maximize total utility, subject to the home government's budget constraint.

The next few pages are devoted to the mathematical analysis of this problem. Conditions characterising the optimum for nicely-behaved situations are stated as the Solution at the end of the section. These take the form of a pair of differential equations and corresponding initial conditions. The equations are not as readily interpreted as eq. (6) above, although a sympathetic eye would see a resemblance to the previous equation [particularly in the specialized form (16)]. Numerical solution would be possible, though there are difficulties which will be alluded to below. Rather than pursue that approach, we turn in the final section to the analysis of special cases for which substantial information may be obtained.

In the present section, notation is first established, then a Lagrangean for the problem set up. The Lagrangean is a double integral, in the space of home and foreign incomes. First-order conditions are obtained by considering the subpopulation with a particular home income, and choosing the utility level for those who choose to be taxable at home with due regard to the effects on migration. This procedure yields an equation for the derivative with respect to utility of the taxable foreign income of someone indifferent about migrating. A similar procedure yields another equation for the derivative of home income with respect to utility. These conditions provide the two differential equations. The initial conditions are obtained by careful attention to those who have the lowest foreign-income earning ability.

An individual who works at home in an occupation that attracts tax has productivity n. If he worked abroad with taxable status, his income net of foreign tax would be m. Taxes exist — and are to be chosen optimally — that provide him with utility $u_h(n)$ if he stays at home, $u_f(m)$ if he works abroad. This formulation embodies the assumption that the government cannot know what an individual's income would have been had he gone abroad instead of staying at home, or vice versa. An individual of type (m,n) who chooses to remain a taxpayer gets utility

$$v=\max\{u_h(n),u_f(m)\}. \tag{29}$$

In equilibrium, $f(m,n,v)$ of such people choose to remain taxable, the others emigrating for good and severing their tax liability to the home government, or indulging in other untaxable activities, where they have greater utility. To be more precise, f is a density function for m and n.

The after-tax earnings that people require to achieve specified levels of utility are given by convex increasing functions:

$$\begin{aligned} y(m)&=Y(u_f(m)),\\ z(n)&=Z(u_h(n)), \end{aligned} \tag{30}$$

where y refers to foreign earnings, after deduction of home tax, and z to home earnings, net of tax. Notice that this notation differs from that used in the previous section, where y and z referred to earnings before deduction of home tax.

Let H be the set of (m,n) for which $u_h\geqq u_f$, and F the set for which $u_h<u_f$.

Then the State's tax revenue is

$$T=\iint_H [n-z(n)]f(m,n,u_h(n))\,dm\,dn$$
$$+\iint_F [m-y(m)]f(m,n,u_f(m))\,dm\,dn. \tag{31}$$

This formulation assumes that the indifferent stay at home. One would suppose that they form a set of measure zero, so that the convention is of no significance.

The total utility of (m,n)-people who leave the tax system is

$$\int_v^{\infty} wf_w(m,n,w)\,dw=\Omega(m,n,v) \tag{32}$$

as a definition. Welfare will be measured by total utility. The welfare of the whole relevant population is

$$W=\iint_H [\Omega(m,n,u_h)+u_h f(m,n,u_h)]\,dm\,dn$$
$$+\iint_F [\Omega(m,n,u_f)+u_f f(m,n,u_f)]\,dm\,dn. \tag{33}$$

Our problem is to find how to maximize W subject to the government budget constraint, for which a multiplier λ is introduced. Thus, we seek to maximize $W+\lambda T$ by choice of the functions u_h and u_f. It is convenient to write $W+\lambda T$ in the form

$$L=\iint_H \phi(m,n,u_h(n))\,dm\,dn+\iint_F \psi(m,n,u_f(m))\,dm\,dn, \tag{34}$$

where

$$\phi(m,n,v)=\Omega(m,n,v)+[v+\lambda(n-Z(v))]f(m,n,v),$$
$$\psi(m,n,v)=\Omega(m,n,v)+[v+\lambda(m-Y(v))]f(m,n,v). \tag{35}$$

The inverse utility functions Z and Y are central to the analysis. It is assumed that, offered the same after-tax income at home and abroad, anyone would choose to remain in his own country, which is equivalent to

$$Y(v)>Z(v), \quad \text{for all } v. \tag{36}$$

A further reasonable assumption is that

$$Y'(v)\geqq Z'(v), \quad \text{for all } v. \tag{37}$$

The problem to be solved is not immediately of standard type, since the region of integration is divided up in a rather inconvenient way. Necessary conditions for maximization will be found by first considering the choice of the function u_h, given that u_f has already been chosen, and afterwards reversing the role of the two functions. Before doing that we shall make one further modification in the problem. When considering a model with taxation only of domestic incomes, we found that, under optimal taxes, it might be the case that utilities decrease with productivity over certain ranges. We noted that such an arrangement is not likely to be feasible. In the model we are now considering, it would also be somewhat intractable to allow utility to decrease with income. Let us therefore constrain u_f and u_h to be nondecreasing functions. Let us also make the reasonable (and justifiable) assumption that they are continuous functions.

Define a function M inverse to u_f. Specifically:

$$M(v) = \max\{m : u_f(m) \leq v\}. \tag{38}$$

Like u_f, M has to be a nondecreasing function. The region of (m, n)-space that we call H, where people would choose to work in the home country, can now be defined by the inequality

$$m \leq M(u_h(n)). \tag{39}$$

It follows that we can write the integral L as $\int L(n)\,dn$ with

$$L(n) = \int_0^{M(u_h(n))} \phi(m, n, u_h(n)\,dm$$

$$+ \int_{M(u_h(n))}^{\infty} \psi(m, n, u_f(m))\,dm. \tag{40}$$

We must choose the function u_h as the nondecreasing function that maximizes L. It would be nice if we could, for each n, choose $u_h(n)$ so as to maximize $L(n)$, given in (40), and then find that the resulting u_h is a nondecreasing function of n. It turns out that our model is simple enough for this to be true. We shall not take the space to prove this rigorously. Another point is worth making rigorously. We can show that

$$u_h(0) > u_f(0). \tag{41}$$

To prove this we simply have to show that it is never desirable to have

$M(u_h(n))=0$. We see from the definitions (35) that

$$\phi(m,n,u)-\psi(m,n,u)=\lambda f(m,n,u)(Y-Z-m+n) \tag{42}$$

is positive when $m=0$. Therefore small positive M in (40) always yields a larger value of $L(n)$ than having $M=0$. This proves (41). We may therefore define the minimum utility level by

$$\omega=u_h(0). \tag{43}$$

People with m less than $M(\omega)$ never work abroad and pay tax. We define

$$m_0=M(\omega). \tag{44}$$

Below this level of m the choice of u_f has no effect, and we can therefore restrict attention to the choice of the function for $m\geqq m_0$.

The first-order condition for the choice of $u_h(n)$, when M is differentiable at that value of u, is obtained from (40) by differentiation:

$$\begin{aligned}&[\phi(M(u),n,u)-\psi(M(u),n,u)]M'(u)\\&=-\int_0^{M(u)}\phi_u(m,n,u)\,\mathrm{d}m.\end{aligned} \tag{45}$$

Notice that we have used the fact that $u_f(M(u))=u$. The statement that (45) holds at $u=u_h(n)$ can be expressed equivalently as the statement that it holds when $n=N(u)$. In an exactly similar way, we get the first-order condition for $u_f(m)$ when the function N inverse u_h is differentiable at $u=u_f(m)$, and u_f is strictly increasing in m:

$$\begin{aligned}&[\phi(m,N(u),u)-\psi(m,N(u),u)]N'(u)\\&=\int_0^{N(u)}\psi_u(m,n,u)\,\mathrm{d}n.\end{aligned} \tag{46}$$

This holds when $m=M(u)>m_0$. As in the previous case, it turns out that u_f is strictly increasing in the relevant range $m\geqq m_0$, and the constraint that it be nondecreasing is therefore satisfied.

Finding conditions that determine the numbers ω and m_0 is the familiar task of finding terminal conditions in calculus of variations problems. It is most straightforward if ϕ and ψ do not vanish when $n=0$, nor do their derivatives and difference. A heuristic derivation is as follows.

Consider small changes in the u_f function near m_0. The effect is to move a few marginal people with $n=0$ into or out of the domestic economy: the change in L is proportional to $\phi(m_0, 0, \omega) - \psi(m_0, 0, \omega)$. At the optimum, this expression must be zero. Using (42), this implies

$$m_0 = Y(\omega) - Z(\omega). \tag{47}$$

With a little trouble, it can be shown that this is also a valid condition when f tends to zero as n tends to zero.

Now consider the effect of changing $u_h(0)$ by a small amount, while leaving other utility levels the same (except, necessarily, for n very close to zero). The effect on L is proportional to

$$\int_0^{m_0} \phi_u(m, 0, \omega)\, \mathrm{d}m, \tag{48}$$

which therefore must vanish. It would vanish automatically if ϕ_u were zero when $n=0$, but it is then possible to get an essentially similar condition, which will be noted below.

It only remains to replace the functions ϕ and ψ by their definitions (35). We have

$$\phi_u = [1 - Z'(u)]f + \lambda[m - Z(u)]f_u \tag{49}$$

and a similar expression for ψ_u. Using these, we have the Solution.

Solution

The functions M and N are given by

$$(N - M - Z + Y)f\, M'(u) = -\{\rho - Z'(u)\} \int_0^M f\, \mathrm{d}m$$

$$-(N - Z) \int_0^M f_u\, \mathrm{d}m, \tag{50}$$

$$(N - M - Z + Y)f\, N'(u) = \{\rho - Y'(u)\} \int_0^N f\, \mathrm{d}m$$

$$+(M - Y) \int_0^N f_u\, \mathrm{d}n, \tag{51}$$

which hold for $u > \omega$, with

$$M(\omega) = Y(\omega) - Z(\omega), \text{ called } m_0,$$

$$N(\omega) = 0,$$

$$\frac{\rho - Z'(\omega)}{Z(\omega)} = \lim_{n \to 0} \left\{ \int_0^{m_0} f_u(m, n, \omega)\,\mathrm{d}m \Big/ \int_0^{m_0} f(m, n, \omega)\,\mathrm{d}m \right\}. \tag{53}$$

The optimal relationships between income before and after taxes, $z(n)$ and $y(m)$, are deduced from the equations

$$z(N(u)) = Z(u); \qquad y(M(u)) = Y(u).$$

Eqs. (50) and (51) are translations of (45) and (46), using (42) and (49). The number ρ is $1/\lambda$, and is determined by the resource constraint. Eq. (53) is given in the more general form that is valid when f and f_u vanish at $n=0$. (Essentially, this comes from the principle that the multipliers in a calculus-of-variations problem of Pontrjagin type should tend to zero as rapidly as possible.)

Notice that, because of the initial conditions, eqs. (50) and (51) yield expressions of the form '0/0' for $M'(\omega)$ and $N'(\omega)$. These initial derivatives have to be found by l'Hopital's rule. This makes general analysis of the solution difficult. But there is a class of examples that is more amenable, and which seems to be general enough to be interesting.

6. A special case

Define

$$F(M, W, u) = \int_0^M \int_0^N f(m, n, u)\,\mathrm{d}m\,\mathrm{d}n. \tag{54}$$

Unfortunately this function has no very natural economic interpretation: it is the number with earning ability $(m, n) \leqq (M, N)$ who would choose to remain taxable if all of them were offered the same utility u. Using the definition, we have

$$F_M = \int_0^N f\,\mathrm{d}n, \qquad F_N = \int_0^M f\,\mathrm{d}m,$$

$$F_{Mu} = \int_0^N f\,\mathrm{d}n, \qquad F_{Nu} = \int_0^M f_u\,\mathrm{d}m,$$

$$F_{MN} = f,$$

which allow us to write the basic equations, (50) and (51), a little more briefly.

The real advantage of this new function emerges if it takes the special form:

$$F(M,N,u)=F(G(M,N),u). \tag{55}$$

Using the notation F for both functions should occasion no confusion. The new F may be taken to be increasing in G, which is increasing in M and N. Eqs. (50) and (51) can now be written:

$$(N-M-Z+Y)fM'=-[(\rho-Z')F_G+(N-Z)F_{Gu}]\,G_N, \tag{56}$$

$$(N-M-Z+Y)fN'=[(\rho-Y')F_G+(M-Y)F_{Gu}]\,G_M. \tag{57}$$

From these it follows that

$$(N-M-Z+Y)f\frac{\mathrm{d}G}{\mathrm{d}u}=-[(Y'-Z')F_G$$

$$+(N-M-Z+Y)F_{Gu}]G_M\,G_N,$$

which can be rewritten:

$$(N-M-Z+Y)\left(f\frac{\mathrm{d}G}{\mathrm{d}u}+F_{Gu}\,G_M\,G_N\right)=-(Y'-Z')\,F_G\,G_M\,G_N. \tag{58}$$

We know that G is an increasing function of u, and that G_M, G_N, F_G and F_{Gu} are all positive. (Note that $F_{Gu}\,G_M=F_{Mu}>0$.) Furthermore, $Y'\geqq Z'$. Eq. (58) therefore implies that

$$N-M-Z+Y\leqq 0,$$

i.e.

$$N(u)-Z(u)\leqq M(u)-Y(u). \tag{59}$$

This proves the following:

Proposition. *If F can be written in the separable form (55), the tax optimally paid by a person who earns taxed income at home is not greater than that paid by a person of equal utility who earns taxed income abroad.*

The point of the separability condition is that it is general enough to allow a greater propensity to leave the tax system for people with high foreign earning power m, compared to those with relatively high home earning power n. This can be done by making G more sensitive to variations in M then in N; but we are then forced to make possibly unacceptable assumptions about the underlying distribution of m and n. The condition is however a strongly sufficient one.

A further result is obtained by going back to (56) and (57). It is convenient to define

$$\gamma = F_G/F_{Gu}. \tag{60}$$

Corollary. Under the above assumptions,

$$N \geqq Z + \gamma(Z' - \rho), \tag{61}$$

$$M \geqq Y + \gamma(Y' - \rho). \tag{62}$$

These conditions place lower bounds on the optimal tax rates if we are prepared to estimate the magnitudes of ρ and γ.

It will be evident from the above discussion that in one very special case it is easy to calculate optimal tax schedules, namely when

$$Z(u) = Y(u) - K \tag{63}$$

for some constant, K. Eq. (63) means that migration is equivalent to a constant income-loss K, independent of income-levels. This assumption presumably seriously overstates the relative willingness of the rich to migrate. It implies that $Z' = Y'$. Then the inequalities (59), (61) and (62) just derived become equalities, and the optimal taxes are defined by

$$N = Z + \gamma(Z' - \rho), \tag{64}$$

$$M = Y + \gamma(Y' - \rho) = N + K, \quad \text{by (63)}. \tag{65}$$

From these equations we can deduce that a home earner should pay more than a foreign earner with the same income. Since at equal utility $M > N$, equal income implies that the home earner is better off: $u_h > u_f$. Therefore, by concavity of utility which is equivalent to convexity of Z and Y, $Z'(u_h) > Z'(u_f) = Y'(u_f)$. With $N(u_h) = M(u_f)$ in (64) and (65), we then have

$$Z(u_h) < Y(u_f), \tag{66}$$

showing that the home earner pays more tax at all levels of income. It will be appreciated that this result depends on the extreme assumption that $Y(u)-Z(u)$ is constant. If, instead, it is an increasing function of u, the tax on the foreign earner can be greater even at equal incomes.

As an illustrative example suppose that

$$U_{\mathrm{h}}(x)=-\frac{1}{x}; \qquad U_{\mathrm{f}}(x)=-\frac{1}{x-K},$$
$$f(m,n,u)=h(m,n)(-u)^{-\eta}. \tag{66}$$

Easy calculations yield:

$$\gamma=(-u)/\eta,$$

$$Z(u)=1/(-u),$$

$$Y(u)=1/(-u)+K.$$

From these equations, we have

$$Z'(u)=Z^2; \qquad \gamma=1/(\eta Z).$$

Substitution in (64) then tells us that $z(n)$ is given by solving

$$n=\frac{1+\eta}{\eta}z-\frac{\rho}{\eta}\frac{1}{z}. \tag{67}$$

Disposable income of foreign earners is given by

$$y(m)=z(m-K)+K. \tag{68}$$

To determine m_0 and ω, we use the auxiliary conditions in the Solution above. From (53),

$$\frac{\rho-1/\omega^2}{1/(-\omega)}=\frac{\eta}{-\omega},$$

which simplifies to

$$\omega=\left(\frac{1+\eta}{\rho}\right)^{1/2}. \tag{69}$$

The other condition yields:

$$m_0 = Y(\omega) - Z(\omega) = K. \tag{70}$$

These results are consistent with (67) and (68), with

$$z(0) = y(m_0) = \left(\frac{\rho}{1+\eta}\right)^{1/2}. \tag{71}$$

From (67) it can be seen that the marginal tax rate

$$1 - z'(n) \to \frac{1}{1+\eta} \tag{72}$$

as $n \to \infty$. The same result holds for foreign incomes. Notice that the details of income distribution incorporated in the function h affect results only through ρ.

A numerical solution for the case

$$K = 1, \qquad \eta = \tfrac{1}{2}, \qquad \rho = \tfrac{3}{2}$$

is shown in fig. 1.

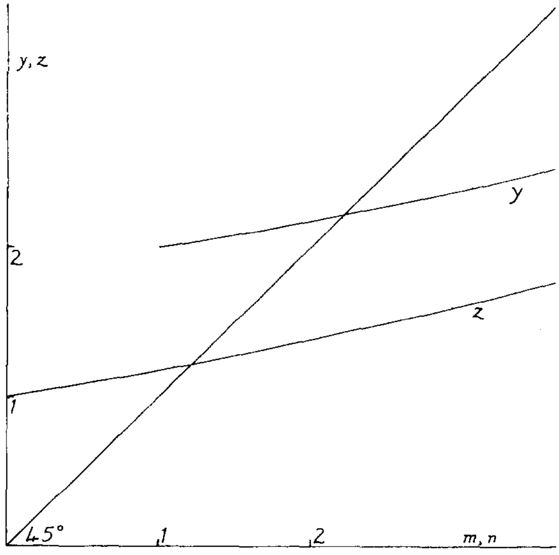

Fig. 1

7. Conclusions

The final numerical example made several assumptions. One, very favourable to home taxes being higher than taxes on foreign income, was that the cost of working abroad be equivalent to a reduction of income independent of income actually enjoyed. Another, which is probably favourable to the opposite conclusion, was that the distribution of incomes be described by a density function of the multiplicative form $h(m,n)q(u)$. This means that, among people enjoying the same utility, the propensity to migrate is the same for those with relatively high n as for those with relatively high m. The basic structure of the model used tends to have the latter work abroad, the former at home; and it may seem that a change in prospective utility would be more likely to induce those working abroad to give up citizenship, either through taste or opportunity. Yet we have seen that a more general assumption works in much the same way, yielding the result that those working abroad pay higher taxes than people provided with equal utility working at home.

Taking these results with the general ideas presented in the opening section of the paper, that an income tax on foreign earnings should include an element corresponding to expenditure taxes at home; and that the relative magnitude of the optimal taxes depends on the degree of substitutibility of home and foreign taxable earnings with untaxed alternatives; it seems that it may well be desirable to institute substantial income taxes on foreign earnings, if only the narrowly economic considerations incorporated in our model are relevant.

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