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VAT, Tariffs, and Withholding: Border Taxes and Informality in Developing Countries

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Abstract

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This paper explores the implications of a distinctive feature of the value added tax (VAT) that is stressed by practitioners but essentially ignored by theorists: that it functions, in part, as a tax on the purchases of informal operators from formal sector businesses and, not least, on their imports. It stresses too the potential importance of the creditable withholding taxes that are levied by many developing countries—which have also been ignored. If both of these instruments are optimally deployed, it is shown, then the usual prescription that a small economy should not deploy tariffs remains valid even in the presence of an informal sector; and indeed a simple strategy is established—generalizing the standard prescription developed in models without informality—for deploying these instruments so as to preserve government revenue and increase welfare in the face of efficiency-improving tariff cuts. Conditions are established under which a VAT alone is fully optimal, precisely because it is in part a tax on informal sector production. But they are restrictive: more generally, an efficient tax structure requires deploying both a VAT and withholding taxes.

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I. INTRODUCTION

This paper addresses two of the key challenges for tax design that currently face almost all developing countries. The first is that of dealing with a large informal sector. Hard to measure, but recently put by Schneider (2002) at an average of around 40 percent of gross national income in these countries, informality² represents both a costly narrowing of the tax base and a potentially serious distortion of economic activity. Long a concern to policy makers—and, of course, an issue too, albeit a less fundamental one, in many developed countries—the implications of informality for tax policy design have recently attracted increased theoretical interest, notably from Piggott and Whalley (2001), Emran and Stiglitz (2005), Munk (2004), and, taking a somewhat different perspective, Gordon and Li (2005). The second challenge is that of determining the proper degree of reliance on trade taxes. These still account for 20 percent or (often) more of all tax revenue in many developing countries, so that continuing pressures towards further trade liberalization, combined with pressing revenue needs, raise the question of how reduced trade tax revenue can be replaced from domestic sources. What is troubling here is the emerging evidence which suggests that many low income countries (though by no means all) have experienced difficulty achieving such replacement in the past (Baunsgaard and Keen, 2005).

At the juncture of these two issues, and in some respects overarching both, are questions as to the proper role and design of the value added tax (VAT). Now adopted by more than 130 countries, including many of the poorest—around three-quarters of all countries in sub-Saharan Africa now have a VAT, for example—this has been, and remains, the centerpiece of tax reform in many developing countries. And prominent among the tasks it is commonly allocated in developing countries is a lead role in replacing revenue lost as a consequence of trade reform.³ Recent work, however, questions the effectiveness of the VAT in this context. Empirically, Baunsgaard and Keen (2005) find no evidence that countries with a VAT have recovered revenue significantly more fully than have those without (although the case studies in IMF (2005) stress that the design of the VAT, not merely its existence, is likely to be critical). Theoretically, Emran and Stiglitz (2005) derive a series of results establishing conditions under which a shift from tariffs to a VAT—modeled as a tax on final sales by formal sector firms—is welfare-worsening.

Unraveling all this requires understanding what a VAT does, can, and cannot do. And a fundamental point here is that the VAT is not, in practice, simply a tax on final consumption, or even on formal sector sales. The essence of a VAT is that it is charged on all imports, and on all domestic sales by registered firms, with full credit or refund to registered taxpayers of

² The precise definition of informality raises issues that need not be addressed here, other than for the purposes of the later formal modeling. Schneider (2002) appears to have in mind a broad definition that includes illegal as well as unreported legal activities.

³ In practice, an important role is also played in such replacement strategies by excises—taxes levied at particularly high rates—on a few key commodities. Since production of these is commonly tightly monitored, the noncompliance issues that excises raise are very different from those associated with the VAT, which are the focus here.

the VAT that they have themselves been charged on their purchases.⁴ Only if this chain of crediting and refunding is unbroken is the VAT equivalent to a tax on final consumption. And there are many reasons why the chain is broken in practice, including in particular (though not only) the existence of traders who fail to register for the tax even if legally obliged to do so. But while such traders will not remit VAT on their sales, they will pay VAT—without benefit of any credit or refund—on both their imports and their purchases from VAT-compliant firms. The VAT, in such cases, functions as an unrecovered input tax. It is, thus, not even simply a tax on formal sector sales: for informal sector operators, it acts as a tariff on their imports and a tax on their purchases from the formal sector. This indeed is precisely one of the structural characteristics that proponents of the VAT emphasize (and welcome): while informal operators may be able to completely escape income tax, for example, the VAT may well reach them on their inputs. But this central design feature of the VAT has been ignored or set aside in the theoretical literature. It is clearly one of some importance, not least, in interpreting the formal results of Emran and Stiglitz (2005).

A primary purpose of this paper is, thus, to explore this fundamental but formally neglected feature of the VAT: that when the chain of crediting is broken, it functions in part as an input tax. The issue, indeed, is central to a wider assessment of the value added tax, long overdue for a tax that now raises around one-quarter of the world's tax revenue. As a general point, it has been known since Newbery (1986) that it may be desirable to tax inputs when the ability to tax final sales is constrained. But it remains an open question whether the VAT as it operates in practice, with its distinctive combination of full crediting on some transactions and input taxation on others, does this in the best possible way. Motivated by the particular concerns set out above, the analysis that follows focuses on one particularly pressing aspect of this wider question: the appropriate roles in developing countries of the VAT—complete with its partial taxation of informal sector inputs—and of tariffs.

Each of these instruments will be seen to have its limitations: those of tariffs are well-known, but that which will emerge below for the VAT seems not to have been appreciated. This limitation of the VAT then leads to a second major purpose of this paper. In practice, developing countries very commonly levy taxes on imports and sales by the formal sector, often at quite high rates, in addition to the normal VAT and tariffs, structuring these precisely so as to bear differentially on informal operators. Income tax withholding on imports, in particular—creditable for compliant taxpayers, akin to a tariff for noncompliant—will be shown below to be widespread. The VAT itself is also sometimes applied at differentially higher rates on imports. These instruments, however, also appear to have been entirely neglected in the theoretical literature (and in much of the policy-oriented literature too). This, no doubt, is largely because they are rarely found in developed countries. Whatever the reason for this neglect, the important point is that developing countries have evidently found it useful to deploy additional instruments, beyond the VAT and tariffs, in response to informality, and these should not be ignored. Thus a central aim in what follows is to argue

⁴ Attention is confined throughout the discussion and analysis to the invoice-credit method VAT, with the destination principle implemented by zero-rating. With the sole exception of Japan, all national-level VATs are now of this kind.

for, and begin, the incorporation of such taxes into the analysis of the tax policy design problem facing developing countries.

The next section of the paper develops further and illustrates some of these general points. The formal analysis begins in Section III, which sets out a simple framework for considering the appropriate balance between the three tax instruments on which the paper focuses—the VAT, tariffs, and withholding taxes—and characterizes their (quite different) effects on output levels and the degree of informality. This paves the way for a characterization of optimal tax structures in Section IV. The implications for dealing with the revenue consequences of trade liberalization are addressed in Section V. Section VI concludes.

II. TAXES AND INFORMALITY

The importance of the informal sector for optimal tax design, and for the VAT in particular, has recently been stressed by Piggott and Whalley (2001), Stiglitz (2003), Munk (2004) and Emran and Stiglitz (2005). As stressed above, however, a common feature of these analyses is that while the discussion is couched in terms of the “VAT,” the formalities do not capture one of the core features of the VAT in practice: that firms which are for any reason ‘exempt’ from the VAT⁵—whether because they are honest but below the threshold level of size at which registration for the VAT is compulsory or (the case particularly in mind here) because they dishonestly conceal themselves from the tax authorities—will nevertheless be charged VAT on their inputs that are either imported or purchased from VAT-compliant formal sector firms.

The point is especially important in comparing the relative effects and merits of the VAT and import tariffs. VAT, recall, is levied on imports, by essentially the same formalities as apply to import tariffs.⁶ And it is charged even on imports by firms that are not registered for the VAT. But whereas firms that are VAT-registered will be able to claim a credit or refund of that import VAT, informal operators will not: for them VAT at import stage is precisely equivalent to a tariff. Quantitatively too, the point is critically important. As shown in Table 1, VAT collected at the border commonly accounts for more than half of gross VAT collections in developing countries. No information is available on the extent to which these revenues are, in practice, credited or refunded to VAT-compliant taxpayers. The important point for present purposes, however, is that the collection of VAT at national borders, from the informal sector as well as the formal, is in practice crucial to the functioning and effectiveness of the VAT. Any practical advantage that tariffs may have as an administratively convenient way of taxing informal firms through their imports, thus, applies with equal force to the VAT.

⁵ “Exemption,” recall, means that no VAT is charged on sales but nor can any VAT paid on inputs be credited or refunded: the Australian terminology, which instead uses the term “input-taxed,” is more telling.

⁶ Except under deferred payment schemes, whose use outside the EU is extremely limited.

Table 1: VAT Revenue from, and Income Tax Withholding on, Imports in Selected Countries

Country	VAT Revenue from Imports (In percent of gross VAT) 1/	Income Tax Withholding on Imports
Algeria	64.0	No
Armenia	64.3	No
Argentina	33.3	Yes – 3 percent
Benin	62	Yes – 3 percent
Burkina Faso	55	Yes – 1 percent (5 percent if no TIN) 2/
Cameroon	39.0	No
CAR	63.8	Yes – 10 percent
Egypt	60.5	Yes – 1 percent
Ethiopia	62.2	Yes – 3 percent
Gabon	43.3	Yes – 2.5 percent
Georgia	58.7	No
Guinea	83	Yes – 5 percent
Jordan	42.3	Yes – 2 percent (if importer is not a certified compliant)
Kenya	39.3	No
Kyrgyz	81.1	No
Lebanon	69.0	No
Madagascar	61.0	No
Malawi	53.0	No
Mali	n.a.	3 percent (7.5 percent if no TIN) 2/
Mauritania	73	Yes – 4 percent
Mauritius	48.5	No
Morocco	55.3	No
Niger	59	Yes – 4 percent (7 percent if no TIN) 2/
Pakistan	52.0	Yes – 6 percent
Rwanda	46.1	Yes – 5 percent
Senegal	54	No
South Africa	46.0	No
Sudan	48.1	Yes – 3 percent
Tanzania	60.0	No
Togo	60	Yes – 1 percent (5 percent if no TIN)
Trinidad and Tobago	51.0	No
Tunisia	46.5	Yes – 10 percent
Uganda	56.2	Yes – 6 percent
Zambia	35.5	No

Source: IMF staff.

1/ For most recent year available, usually 2004 or 2005.

2/ Taxpayer Identification Number.

This tariff-like aspect of the VAT, as it affects the informal sector, is important to bear in mind, for example, in interpreting the results of Emran and Stiglitz (2005) on conditions under which a revenue neutral shift from tariffs and towards a ‘VAT’ may be welfare-reducing: for ‘VAT’ is in effect defined there, as the author’s note in concluding,⁷ as only that part of any actual VAT that is not collected on imports. Thus, a one-for-one shift from tariffs to a ‘VAT’ on the same good, as the latter is defined there, would reduce the total charge on imports purchased by informal operators. In practice, however, under a real VAT, it would not: such a shift would leave the total tax on informal imports unchanged, and so be likely to have quite different effects. While this, of course, has no bearing on the formal results, it does affect their interpretation and call for great care in drawing practical policy conclusions about the VAT. In this respect, the results of Emran and Stiglitz (2005) are much less damaging to conventional advice than they may appear. It is hard to imagine any practitioner recommending adoption of a VAT that was not to be collected on imports.

It could, of course, still be the case, nevertheless, that the presence of an informal sector invalidates the conventional prescription (which will be spelled out later) that any revenue lost as a consequence of trade reform be recouped by shifting towards (excises and) a real VAT. As Emran and Stiglitz (2005) stress, existing results, such as those of Keen and Ligthart (2002), miss a potentially crucial consideration in assuming away the existence of informal activities. And the recent empirical literature also casts some doubt on the extent to which such strategies have indeed succeeded in recouping lost trade tax revenue: as noted above, Baunsgaard and Keen (2005) find not only that many low-income countries have failed to recover lost trade tax revenue from domestic sources, but that the degree of recovery has not been significantly greater in those with a VAT. That may plausibly reflect weaknesses in the design and implementation of the VAT, perhaps with multiple rates or excessively low thresholds complicating compliance and administration. But it may also be that informality exposes inherent structural weaknesses of the VAT that dilute its effectiveness in replacing trade tax revenues.

For there is one important respect in which the structure of the VAT may make it a less than perfect instrument for revenue replacement, despite the way in which, as just stressed, it neatly acts as a surrogate tariff on the informal sector. This is that the VAT taxes at the same proportionate rate both imports by the informal sector and sales by the formal sector.⁸ It is far from obvious, however, that it is optimal to set these two rates at the same level: intuitively, one might want to levy a higher rate on informal sector inputs in order to levy an implicit tax

⁷ See also their footnote 14.

⁸ The discussion here and throughout the paper takes as benchmark a VAT levied at a single positive ad valorem rate. In practice, of course, the VAT can be, and often is, levied at multiple positive rates. But the most common reasons for such differentiation (to deal with equity concerns, or exploit, for efficiency gains, aspects of preference structure) are not at issue here. Moreover, the single rate benchmark serves to capture the conventional prescription of practitioners; and indeed most VATs introduced in recent years have been single rate. One interpretation of the results below, as will be seen, is precisely as establishing a theoretical case for a form of VAT rate differentiation: not, as is usual, by the nature of the product but rather by the likely compliance status of its purchaser.

on the value added that cannot be taxed directly. This in turn suggests that there is scope for deploying a third tax instrument, in addition to the VAT and tariffs: a tax that is levied only on informal sector imports. Stiglitz (2003), indeed, hints at the potential usefulness of what would be, in effect, a differentiated tariff of this kind.

The potential usefulness of such an instrument is suggested, moreover, by the fact that many developing countries already deploy taxes of precisely this kind. In particular, it is very common (though by no means universal) for developing countries to levy on imports a withholding tax that is creditable against income tax liability. The final column of Table 1 above documents this for a wide range of countries, and shows that the rate is in some cases quite high. Pakistan, for example, currently imposes an advance income tax at the rate of 6 percent on the value of imports, this being fully creditable against final liability. And there are other types of withholding taxes on imports not reflected in the table. Peru, for instance, charges—as well as the usual import VAT—additional VAT at rates varying between 2 and 10 percent of import values, this being regarded as prepayment of VAT due on subsequent sales.

These instruments share the attractive feature that they impose no additional charge on taxpayers in the formal sector, so long as the amount withheld (whether under the income tax or the VAT) is fully creditable and, if it exceeds final liability, refundable, but at the same time do impose an additional charge on informal operators, for whom the tax functions as a tariff. Such withholding taxes on imports, it should also be noted, appear to be fully WTO-consistent⁹ so long as they, indeed, ultimately levy no additional charge on compliant importers, in practice as well as in principle: they are then simply devices to help enforce domestic taxes.

Two further points on the use of withholding taxes in developing countries should be stressed. First, while the tax treatment of imports is currently a prominent concern given the prospective revenue replacement issue that continued trade liberalization poses for many developing countries—and so will be a focus in the analysis that follows—broadly the same considerations apply to all inputs purchased by informal operators from the domestic formal sector. On these purchases too they will bear unrecovered VAT, just as they do on imports. And it is also fairly common for additional withholding taxes, analogous to those on imports discussed above—taking the form of income tax prepayment or higher VAT rates—to be

⁹ While Article III.2 of the GATT prohibits the imposition on imports of internal taxes in excess of those applied to like domestic products, Article XX provides an exception to that extent that such measures may be necessary to enforce other laws or regulations not otherwise contrary to the GATT—which can reasonably be supposed to include compliance with the domestic tax system. The only dispute relating to withholding taxes on imports seems to be that brought by the European Communities against Argentina, which levied both income tax withholding and a differentially higher VAT rate on imports (WTO (2000), WT/DS155/R). While the panel ruled against both, a primary consideration in this decision was the view that it would have been administratively feasible for Argentina to have paid interest to compensate for the earlier/higher payment on imports. Though further clarification will need to await further disputes, the implication, thus, appears to be that withholding schemes of this kind are permissible so long as all reasonable administrative steps (and precisely what that means may well be country-specific) are taken to neutralize their ultimate impact on the differential effective charge faced by compliant importers.

charged on domestic sales.¹⁰ Many of the considerations that are analyzed formally below in the context of imports thus apply more generally to purchases by informal operators from formal ones. The important border in thinking about the VAT and tax design is, thus, not simply the physical border between home and abroad: it is that between the formal and informal sectors, since it is at this point that the VAT becomes an input tax.

Second, there is very little evidence on the extent to which withholding tax payments are, in practice, ultimately credited or refunded to compliant taxpayers. The limited administrative capacity in many developing countries suggests, however, that the implementation of the crediting arrangements is often imperfect (at least for firms other than the largest firms, which may be subject to special arrangements). Clearly there is a risk that these taxes become *de facto* tariffs even for formal sector firms. The question addressed here, however, is the prior one of whether such instruments, assuming that they could indeed be implemented, might serve any useful purpose.

III. ANALYZING TAXES AND INFORMALITY

This section sets out a simple model for analyzing the issues raised above, and derives the key comparative statics effects of the VAT, tariffs and withholding in the presence of an informal sector. Issues of optimal design are addressed in the next section.

A. A Model of Informality

The framework is a simple one of an economy comprising a formal and an informal sector,¹¹ competing in the production of a single homogenous consumer good, produced using a single, imported intermediate good. (As noted shortly, the framework can also be interpreted in terms of a domestically produced intermediate, so capturing the effects of VAT on informal sector purchases from the formal sector along the lines discussed above.) Production in the formal sector is subject to constant returns, whereas that in the informal sector is subject to decreasing returns: this provides a simple device¹² whereby the two sectors can co-exist and change in size in response to tax changes.

To address the range of design issues raised above, allowance is made for three types of taxes:

¹⁰ One such device is VAT withholding, by which some firms in the formal sector—perhaps all government agencies, or enterprises included in the large taxpayer unit—remit to the government an amount corresponding to (part of) the VAT that is payable on their purchases, with this amount then available to the seller as a credit against the VAT due.

¹¹ By “formal” is meant here operators who are fully tax compliant, and by “informal” those who pay only taxes levied on their purchases. In practice, of course, the distinction is not so sharp: traders may register for the VAT in order to ensure creditability of their input VAT, for example, but fail to charge tax on all of their sales.

¹² Piggott and Whalley (2001) and Stiglitz (2003) use essentially the same structure.

- Value added tax at the (ad valorem, tax-inclusive) rate T_V , chargeable, in principle, on both final sales to consumers and on the imported intermediate. Formal sector firms do indeed charge VAT on their sales, and receive full credit/refund for the VAT they themselves are charged on their imports; informal firms, however, do not charge tax on their final sales and, by the same token, receive no credit or refund of the VAT paid on their imports.
- Import duty at the (specific) rate T_M , paid by both formal and informal firms. As is normal practice, this is included in the base of the VAT levied on imports.
- An additional (specific) withholding tax T_W levied only on the imports of informal operators (and also included in the base of the VAT¹³). This captures the kind of instrument—advance payment of income tax, or a higher VAT rate—described in the preceding section. Since this tax is creditable (refundable, if need be) for compliant firms, it is formally equivalent to a differential tariff on informal sector imports. It bears stressing, however, that the underlying instrument differentiates not by the country of production but by compliance with domestic taxation.

Denoting the consumer price by Q , perfect competition and constant returns in the formal sector imply that

$$(1 - T_V)Q = c(P + T_M) \quad (1)$$

where $c(\cdot)$ denotes the unit cost function in the formal sector (which is referred to by the use of lower case letters) and P the world price of the imported intermediate, which is taken as fixed. That neither the input VAT nor the withholding tax appear in (1) reflects the assumption that the taxes are fully credited and so ultimately impose no additional liability.¹⁴ Informal producers are able to sell at the same price Q as do formal, but do not remit any output tax; they do, however, bear unrelieved VAT on their intermediate purchases. They thus earn profits of

$$\pi = QY - C(\rho, Y), \quad (2)$$

where Y denotes the aggregate output of the informal sector, C its cost function, and

$$\rho \equiv \left(\frac{P + T_M + T_W}{1 - T_V} \right) \quad (3)$$

¹³ This is for analytical simplicity rather than realism.

¹⁴ The operation of the VAT, more precisely, is that tax is charged on imports at the border, but a formal sector firm is then able to credit this against VAT due on final sales, implying an overall VAT payment of $T_V Q$.

the tax-inclusive import price faced by informal producers.¹⁵ Profit-maximization thus implies that

$$Q = C_Y(\rho, Y), \quad (4)$$

with the subscript (or prime, for functions of a single variable) indicating differentiation, and the second order condition satisfied as a consequence of diminishing returns, $C_{YY} > 0$.

There is, in the background, some other input to formal and informal production, taken to be untaxed and whose price is taken as fixed (and suppressed) throughout. Thus c and C will be strictly concave in their price variable to the extent that there is substitution in production between this and the imported input on which the analysis focuses. There is taken to be a single representative consumer, with preferences defined over the taxed good (which is not traded internationally) and the untaxed numeraire, with all income effects concentrated on the latter. Indirect utility, ignoring any unchanging lump sum income, is thus simply $V(Q) + \delta R$, where R indicates provision of some public good and δ the constant¹⁶ marginal valuation that the consumer places on it. Normalizing the marginal utility of income at unity, consumer demand is $X(Q) = -V'(Q)$, with equality of demand and supply requiring

$$X(Q) = y + Y \quad (5)$$

where y denotes formal sector output.

This model is necessarily somewhat special, there being no general model of informality to use. In one respect, however, it is more flexible than it may appear. For (taking $T_M = 0$) one can alternatively think of the intermediate good as being purchased from a domestic formal sector, which charges VAT and perhaps some additional withholding tax of the kind discussed in the preceding section. The model thus serves to capture the key feature of the VAT that, as noted above, is stressed by practitioners but essentially ignored in the theoretical literature: that it reaches the informal operators in so far as they import their inputs or purchase them from the formal sector.¹⁷

¹⁵ To see this, note that since T_V is the tax-inclusive VAT rate while the world price P is tax-exclusive, the tax-inclusive price is given by $\rho = P + T_V \rho + T_M + T_W$, which implies (3).

¹⁶ The assumption that δ is fixed is inessential for present purposes: the valuation of the public good plays no role in the comparative statics, and in the necessary conditions for the design problem δ can be interpreted as the derivative of some more general valuation function.

¹⁷ The model does not feature any sales by the informal sector to the formal. This is less of a limitation than it may seem, because traders making such sales have an incentive to register for the VAT voluntarily, since they can then reclaim the VAT paid on their inputs while their customer, being VAT compliant, can reclaim the VAT that they themselves must then charge. Noncompliance with the VAT may nevertheless be attractive for such operators as part of a wider concealment strategy. In particular, revealing themselves to the tax authorities

(continued...)

B. Taxes and Informality

The three types of tax have quite different effects on both the overall level of final output and the extent of informal activity. These are summarized in:

PROPOSITION 1:

(a) *An increase in the VAT rate reduces both aggregate output and the output of the formal sector, but increases informal sector output.*

(b) *An increase in the import tariff reduces aggregate output. Informal sector output rises, however, iff $c' > C_{\rho Y}$.*

(c) *An increase in the withholding tax on informal sector imports reduces informal sector output and increases formal sector output by exactly the same amount, leaving total output unchanged.*

Proof: Appendix I.

That a higher VAT rate reduces formal sector output is as one would expect, since the increased output price reduces demand while the full recovery of input VAT by such firms means that there is no impact on input costs. What is striking in part (a) of the result, however, is that informal sector does not merely take an increased share of a smaller market: it becomes absolutely larger. The reason, however, is straightforward: the increase in the consumer price needed for formal sector firms to break even, at which informal operators sell, but without remitting any output tax, increases their margin by a larger amount than the increased input costs they face (as a result of higher unrecovered input VAT) reduce it. This higher margin then induces an expansion of informal sector output.

Increasing the import tariff raises the final consumer price, and so reduces total output, through its impact on formal sector input costs. Informal sector output, however, may nevertheless increase: this will be the case iff the tariff-induced increase in marginal cost in the formal sector, c' , exceeds that in the informal sector, $C_{\rho Y}$ (which in turn is the condition that the formal sector make greater use of the imported intermediate at the margin). For when this is the case, the increase in the final consumer price required for the formal sector to break even is more than enough to cover the increased input costs faced by informal operators (from both the tariff itself and the VAT levied on it), which again leads the informal sector to produce more.

Part (c) shows that the withholding tax is neatly targeted to controlling the shares of formal and informal operators within a fixed market size. Intuitively, any tax on imports payable only by the informal sector has no direct impact on the formal sector, so that the final price—

for VAT purposes is likely to reveal them for income tax purposes too. Hence the potential role for VAT withholding of the kind described in footnote 9.

and hence total demand and output—remains unchanged: the reduced output of the informal sector is thus exactly offset by increased formal sector output.

IV. OPTIMAL TAX AND TARIFF DESIGN IN THE PRESENCE OF INFORMALITY

With the three types of taxes described above—VAT, customs, and the withholding tax on inputs to the informal sector—having quite distinct effects, this section considers the key questions for policy design: What is the appropriate role and level of each instrument? Are they all needed in a well-designed tax system?

A. Preliminaries

To explore these issues, we suppose the object of policy to be the maximization of

$$W \equiv V(Q) + \lambda\pi + \delta R \quad (6)$$

where λ denotes some fixed¹⁸ weight attached to the profits π earned in the informal sector, and public spending, equal to tax revenue, is given by:

$$R = T_V Qy + T_M c'(P + T_M)y + \left(\frac{T_V P + T_M + T_W}{1 - T_V} \right) C_\rho(\rho, Y), \quad (7)$$

the first term on the right being revenue raised from the VAT on final sales (paid by the formal sector only), the second the tariff revenue from formal sector imports, and the third the payment of unrecovered VAT, tariffs and unrecovered withholding tax on imports by informal operators.

It will be assumed throughout that $\delta > 1$, establishing a need to deploy distorting taxes, with $\delta = 1$ corresponding to the case in which, implicitly, lump sum taxes are available. The appropriate value to assume for λ , which proves to play a key role in the analysis, is less obvious. Taking $\lambda = 1$ would be consistent with the representative consumer framework, informal sector profits then appearing simply as lump sum income entering the indirect utility function. To the extent, however, that many informal operators, especially perhaps in developing countries, tend to be among the less well-off, higher values of λ might be entertained as a rough way of incorporating some distributional concern: as something of a limiting case, if $\lambda = \delta$ then the government attaches the same value to profits earned in the informal sector as it does to tax revenue. It thus seems reasonable to suppose that $\lambda \in [1, \delta]$, and alternative possibilities within this range will be considered below.

¹⁸ As noted above in relation to δ , treating λ as fixed loses no generality for present purposes.

To simplify the analytics, which prove quite involved, it will be assumed—only for the remainder of this section—that the cost function of the informal sector takes the fixed coefficient form

$$C(\rho, Y) = (a_0 + a_1\rho)F(Y), \quad (8)$$

where $F(Y)$ is increasing and (ensuring decreasing returns) convex. The substantive restriction this implies is an absence of substitution effects in informal production (though they remain in the formal sector), which means that none of the three taxes induces production inefficiency within the informal sector. That loses little generality in delineating and comparing their respective roles: since all three would induce such inefficiency if there were substitution in informal production, this simplification does not inherently distort the policy comparison between them. (The assumption does matter, however, in terms of the proper level at which such taxes should be set, a point returned to later). Given (8), the production inefficiencies that potentially remain are those that the tariff induces in the input choices of the formal sector and that in the allocation of output between formal and informal sectors.

B. Optimal Tax Structures

To characterize optimal tax structures, some further notation is useful. Denoting by $E = -QX'/X$ the elasticity of final demand (defined as a positive number), by $s \equiv Y/(y + Y)$ the output share of the informal sector, by $\alpha(\rho) \equiv a_1\rho/(a_0 + a_1\rho)$ the share of the imported intermediate in informal sector costs, and by $\Omega(Y) \equiv F(Y)/YF'(Y) < 1$ the inverse of the output elasticity of informal sector costs, fully optimal policy is characterized in:

PROPOSITION 2: *If there is no substitution in informal sector production, then any optimal tax structure is characterized by:*

$$T_V = \left(\frac{\delta - 1}{\delta} \right) \left(\frac{1}{E(Q)} \right) \left(1 - \left(\frac{\delta - \lambda}{\delta - 1} \right) s(1 - \Omega(Y)) \right) \quad (9)$$

$$\frac{T_W}{\rho} = T_V \left(\frac{1 - \alpha(\rho)}{\alpha(\rho)} \right) + \left(\frac{\delta - \lambda}{\delta - 1} \right) \frac{F(Y)F''(Y)}{\alpha(\rho)(F'(Y))^2} \quad (10)$$

$$T_M = 0 \quad (11)$$

Proof: See Appendix II.

The most immediately sharp result is from the last part of the proposition: there is no need to use an import tariff, so long as both a VAT and a withholding tax can be optimally deployed.

With the withholding tax providing an instrument targeted to controlling the input costs of informal operators and the VAT another to controlling the final consumer price, the only distinct purpose that a tariff can serve is to raise revenue at the cost of inducing production inefficiency in the formal sector. And the thrust of this aspect of Proposition 2 is to show that this would serve no useful purpose if the other instruments are optimally deployed.

The general characterizations of the optimal VAT and withholding tax rates in Proposition 2 are somewhat complex. To understand them, it is useful to begin with the case in which $\lambda = \delta$. Though this attaches perhaps implausibly high weight to the welfare of informal sector operators, optimal policy in this case takes an instructively simple form, with (9) and (10) becoming

$$T_V = \left(\frac{\delta - 1}{\delta} \right) \frac{1}{E(Q)} \quad (12)$$

$$\frac{T_W}{\rho} = T_V \left(\frac{1 - \alpha(\rho)}{\alpha(\rho)} \right) . \quad (13)$$

Thus the VAT rate itself is in this case optimally set according to the usual Ramsey rule: it is set, that is, as if the VAT were simply a tax on all final consumption, without any amendment for the existence of an informal sector either in terms of the final sales that escape the tax (E , recall, is the elasticity of all final demand, not only of taxed consumption) or the unrecovered VAT that is borne on informal sector inputs. And the withholding tax is set as a mark-up on the VAT rate, by a factor equal to the ratio of the informal sector inputs that do not bear VAT to those that do.

Before turning to the intuition behind this solution to the design problem, one obvious implication bears emphasis:

***COROLLARY:** In the circumstances of Proposition 2, if profits in the informal sector have the same social value as tax revenue, and if the informal sector is subject to VAT on all its material inputs ($a_1 = 0$), then tariffs and withholding taxes should not be used: a VAT alone, at a rate characterized by the Ramsey rule, is fully optimal.*

The particular interest of this corollary is that it seems to be the only available result establishing identifying benchmark circumstances in which a ‘real’ VAT is fully optimal, precisely because it is not a tax on final consumption but, in part, an input tax on unregistered firms. And what is of interest is not only the existence of such conditions but that they are strong, including in particular the requirement that all informal sector inputs be subjected to the VAT. Outside that case, Proposition 2 shows, there is a distinct role for an additional withholding tax on informal intermediates.

This leads back to the intuition underlying the design in (12)–(13), which is straightforward. So long as informal sector operators import at least some of their inputs, the government has a hook by which, with a sufficiently high rate of withholding on imports, in effect all informal sector inputs can be fully taxed. (If informal operators import all their inputs, of course, the VAT alone is enough for this.) By deploying the withholding tax in this way, the government can shift production between formal and informal sectors, along the lines of Proposition 1(c), and with fixed coefficients in informal production (by the temporary assumption in (8)) it can do so without inducing any production inefficiency within the informal sector. Indeed it can be shown¹⁹ that at an optimum

$$C_Y(P, Y) - c(P) = -(\delta - \lambda) \frac{C_\rho C_{YY}}{\mathcal{C}_{\rho Y}}, \quad (14)$$

so that when $\delta = \lambda$ the marginal social costs of formal and informal production (which depend on world prices, not tax-inclusive ones) are optimally equated. With the distortion in the inter-sectoral allocation of output removed, the VAT rate itself is then set as if there were no informality, and so is optimally characterized by the usual Ramsey rule.

Indeed there is a sense in which, in this special case, the opportunities available to the government are so complete that it can ultimately render informality irrelevant. The inefficiency in the sectoral allocation of output is eliminated, as just noted, and although tax revenue is optimally lower than it would be if the VAT were applied to all consumption, whether produced in formal or informal sectors,²⁰ the impact of this on aggregate welfare W is offset by higher informal sector profits, since, in this case, they receive the same social weight.

Returning to the more general case of Proposition 2 in which $\delta > \lambda$, the optimal tax structure becomes considerably more complex, reflecting an additional consideration that now comes into play: with tax revenue valued more highly than informal sector profits—and with the government lacking the ability to levy a lump sum tax on pure profits in the informal sector—the VAT and withholding tax are optimally deployed in part to shift rents from the informal sector to the government. To this end, informal operators are now more tightly squeezed, on both their sales and their purchases. The rate of VAT, which becomes sensitive to the existence of the informal sector, is now optimally set below²¹ the level implied by the Ramsey formula (decreasing returns, recall, implying that $\Omega < 1$), by an amount that is greater the larger is the informal sector and (unless $\lambda = 1$) the more intense is the need for revenue. This serves to reduce the implicit subsidy that, as noted above, the VAT provides to

¹⁹ See Appendix III.

²⁰ At an optimum, total tax paid on informal sector output, $(T_V \rho + T_W) C_\rho$, becomes, using (13) and (8), $T_V (a_0 + a_1 \rho) F(Y)$. By (8) and (4), this in turn equals $T_V QY \Omega(Y)$; which, since $\Omega(Y) < 1$, is strictly less than would be paid if all informal sector sales were subject to VAT.

²¹ The optimal tax characterizations are not closed forms, of course, so that this and similar statements to follow are to be interpreted loosely.

informal operators. The optimal withholding tax, on the other hand, is now set above the level implied by (13), again by an amount that (again unless $\lambda = 1$) increases with the need for revenue. These two aspects interact, since a reduction in the VAT rate also reduces the unrecovered input VAT of the informal sector, tending to undo the effect of the increased withholding tax. Optimal policy requires balancing these two considerations.

In this more general case, it should be noted, the consequences of informality are no longer eliminated: from (14), it is now the case that $C_Y(P, Y) < c(P)$ at an optimum, so that there optimally remains an inefficiency in the intersectoral allocation of output. The reason for this is that input taxes are being asked to serve the two distinct functions of correcting the inefficiency of sectoral outputs and taxing pure profits, and cannot serve both perfectly. Optimality actually requires in this case an inefficiently low output share of the informal sector: intuitively, once the withholding tax has been set so as to eliminate the inter-sectoral inefficiency, a small further increase generates a first-order welfare gain in the form of additional revenue but no first-order inter-sectoral misallocation.

Proposition 2 and the special cases just discussed provide a sharp illustration of the potential value of the input-tax aspect of the VAT and withholding taxes. But there are clearly many caveats to be considered in drawing implications for tax design in practice. While administrative concerns are being set aside in the analysis here, it is clear that the implied withholding tax rates could, in many cases, be quite high and so increase the likely importance of providing substantial credits and refunds to formal sector operators if the economic purpose of the tax is not to be deeply compromised; and refunding in particular is something that, experience with the VAT clearly shows, many tax administrations have difficulty with. The implied optimal withholding tax rates, moreover, would become sector- and indeed firm-specific if allowance were made for some heterogeneity of informal sector operators. And relaxing the fixed coefficients assumption in (8) would introduce the further consideration that VAT and withholding then induce production inefficiency in the informal sector; which one would expect to point to reduced reliance upon them. The point remains, nonetheless, that allowing for input-VAT and withholding, which are important features of reality, profoundly affects the tax design problem faced in developing countries.

For completeness, one further result is of interest. Suppose that, for some reason—perhaps a strong commitment to trade liberalization combined with a fear that proper crediting of a withholding tax would prove impracticable—the government chooses to deploy only a VAT. The rate at which it should do so is then given in:

PROPOSITION 3: *Conditional on $T_C = T_W = 0$, optimality requires a VAT of:*

$$T_V = \left(\frac{\delta - 1}{\delta} \right) \left(\frac{1}{E_Y} \right) \left(\frac{1 - (\delta - \lambda)s / (\delta - 1)}{1 - s} \right) \quad (15)$$

where $E_Y \equiv \partial \ln y / \partial \ln(1 - T_V) > 0$ denotes the elasticity of formal sector output with respect to $1 - T_V$.

Proof: See Appendix IV.

The optimal stand-alone VAT rate is thus characterized by a straightforward modification of the Ramsey rule, in two respects.²² First, the price elasticity of final consumption is replaced by the elasticity of formal sector sales with respect to unity minus the VAT rate. Since the latter can be shown to exceed the former,²³ this points to an optimally lower rate of VAT than under the usual Ramsey rule. If $\lambda = 1$, so that informal sector profits have the same social value as consumer surplus, this is the only modification of the usual rule that is needed. At higher values of λ , however, a second adjustment comes into play, with the final multiplicative term in (15) pointing to a higher VAT rate than would otherwise be the case (to an extent that is greater the smaller is the informal sector): for with a higher social weight attached to informal sector profits, the feature of the VAT noted above—that is in part a subsidy to informality—points towards its heavier use.

V. REPLACING TRADE TAX REVENUE

Standard policy advice for countries facing a loss of trade tax revenue as a consequence of trade liberalization emphasizes increasing domestic consumption taxes, both excises and (the particular concern here) the VAT. More specifically, a conventional prescription is to respond to a cut in the tariff on some good by increasing the consumption tax on the same item by the same amount (or slightly less). This evidently preserves the gain in production efficiency from the tariff cut itself; for a small economy, it leaves the consumer price unchanged (or slightly lower), so that consumer welfare increases; and, since tax is now levied on domestic production as well as imports, government revenue increases. This simple strategy thus enables a welfare gain to be realized from trade liberalization without any reduction in aggregate tax revenue.²⁴ Note too that this strategy is equivalent, in the simplest case, to one of combining a tariff cut with whatever increase in the consumption tax is needed to keep the consumer price constant: it will prove more instructive below to focus on this alternative formulation.

In sharp contrast to these neat theoretical prescriptions, however, there is emerging evidence, as noted earlier, that at least some low-income countries have not in fact fully recovered lost trade tax revenues from domestic sources, including some with a VAT in place. And it has been argued explicitly by Glenday (2006), for example, and is implicit in Emran and

²² To reconcile this with the corollary above, note first that routine manipulation of the second part of (A1.3) in Appendix I gives $E_y = (E - sE_Y)/(1 - s)$, where E_Y is defined analogously to E_y . If $\alpha = 1$, as is supposed in the corollary, then it follows from the first part of (A1.3) that $E_Y = 0$. Hence $(1 - s)E_y = E$, so with $\lambda = \delta$ equations (15) and (12) coincide.

²³ Since $E_y < 0$, this follows from the relationship between the elasticities shown in the preceding footnote.

²⁴ The basic argument is developed, and some of the qualifications addressed, in Keen and Ligthart (2002), (2005). For infinitesimal reforms, see also Anderson (1999) and Hatzipanayotou, Michael, and Miller (1994).

Stiglitz (2005), that this may reflect the presence of a large informal sector. To fully explore this possibility and its implications, however, account must again be taken of the fact that VAT itself will be levied on the imports, and perhaps also other purchases, of the informal sector, so that increases in the VAT rate to maintain the final consumer price will, to that extent, also increase the revenue raised from the informal sector.

Suppose then that, in the present framework (dispensing once more with the assumption of fixed coefficients in the informal sector), the government were to match some tariff cut $dT_M < 0$ by such an increase in the VAT rate as to maintain the consumer price unchanged. From (1), this requires that

$$dT_V = -\left(\frac{c'}{Q}\right)dT_M > 0. \quad (16)$$

To see the consequences of this, suppose first that there is no informal sector. From (5) and (7), revenue is then

$$R = T_V QX(Q) + T_M c'(P + T_C)X(Q) . \quad (17)$$

Perturbing this and using (16), it is easily seen that $dR = T_M c'' X dT_M > 0$. Thus the presence of the tariffed intermediate does not in itself undermine the beneficial effects of the simple offsetting strategy.²⁵

But now suppose that there is an informal sector. Clearly the VAT change in (16) will leave the consumer price unchanged, as before. Now, however, the impact on tax revenue becomes unclear. This is for two reasons. The first is that the tax paid on each unit of informal import will fall (the increased unrecovered VAT less than fully offsetting the reduced tariff),²⁶ so that revenue would on this account fall even if the size of the informal sector were unchanged. The second is that production will, in general, shift between formal and informal sectors (the direction of this shift being, in general, ambiguous), which pay different amounts of tax per unit output. Setting the possible revenue losses from these two sources against higher tariff revenue from the increase in the demand for the intermediate for each unit of formal sector output—the source of the revenue gain in the absence of the informal sector—

²⁵ This result is consistent with Proposition 4 of Keen and Ligthart (2002), which establishes fairly mild conditions under which the strategy of maintaining an unchanged consumer price increases revenue and welfare if—as here—the tariffed intermediates are used only to produce nontraded goods.

²⁶ Strictly, this statement requires that the initial withholding tax not be “too large,” since otherwise the VAT increase itself may generate such a large increase in the VAT-inclusive withholding tax rate as to more than offset the impact of the tariff cut on the tax-inclusive import price faced by the informal sector. (The assumption that the withholding tax is included in the VAT base, recall, to simplify other aspects of the analysis rather than for realism).

there are few simple results on how matching the tariff cut by increasing the VAT alone affects total tax revenue.²⁷

These ambiguities, and particularly the revenue loss from reduced taxation of informal inputs, point to a potential role for the withholding tax T_w levied upon them. And indeed, bringing this additional instrument into play, the following establishes a clear strategy for responding to tariff reductions in a way that once again maintains the consumer price and informal sector profits unchanged, and increases tax revenue:

PROPOSITION 4: *Combining a tariff cut $dT_M < 0$ with an increase in the VAT rate as in (16) above and an increase in the (VAT-inclusive)²⁸ withholding tax on informal imports of*

$$d\left(\frac{T_w}{1-T_v}\right) = -\left(1 - \frac{(P+T_M)c'}{c}\right)dT_M \geq 0 \quad (18)$$

leaves both the consumer price and informal sector profits unchanged, and increases total tax revenue.

Proof: See Appendix V.

The essence of the replacement strategy set out in Proposition 4 is to leave unchanged not only the final consumer price, Q (through an offsetting increase in the VAT rate)—the essential element in results that ignore informality—but also the price of the informal intermediate, ρ (by an offsetting increase in the withholding tax). Thus not only does aggregate output remain unchanged, so too do the scale of formal and informal sectors and both the profits earned and the tax paid by the latter. The only impact on tax revenue that then remains is the increased duty paid by the formal sector as a consequence of its increased demand for the imported intermediate input.

Proposition 4 does not rule out the possibility of the VAT alone being enough to replace lost trade tax revenues without undermining any potential efficiency gain from the tariff cut itself. It does, however, provide a strong warning that it may not be. More positively, it reinforces again the potential usefulness of levying withholding taxes on imports.²⁹

²⁷ Nor, it seems, are there any simple results on the consequences of an alternative strategy of matching a tariff cut by such a change in the VAT rate as to leave unchanged ρ , the tax-inclusive input price faced by informal operators.

²⁸ Since the increase in the VAT rate itself tends to increase the VAT-inclusive withholding tax, it is conceivable that this will require T_w itself—the withholding tax in specific terms—to fall.

²⁹ The caveats discussed after Proposition 2 again apply, relating, for instance to the likelihood that the extensive crediting and refunding likely to be implied by high withholding taxes may be difficult to implement.

VI. CONCLUSIONS

This paper has explored the implications of a distinctive feature of the VAT that is stressed by practitioners but has been ignored or dismissed by theorists: that it functions, in part, as tax on the purchases that informal operators makes from formal sector businesses and, not least, on their imports. Recognizing this is key, for example, to understanding the comparative effects and roles of the VAT and import tariffs, and so also to addressing the difficulties that developing countries may face in replacing the revenue loss from prospective trade liberalization.

The limitation of the VAT, viewed in this light, is that it taxes informal inputs at the same ad valorem rate as it taxes final sales—which may not be fully optimal. The wide range of withholding taxes that are levied by developing countries can be seen as in part filling this gap, by differentially taxing the inputs of firms that are not tax compliant. And part of the purpose here has indeed been to draw attention to, and consider the potential importance of, such taxes—which seem to have been entirely neglected in the formal literature, and often receive scant attention from practitioners—as an addition to the armory of instruments available to policymakers.

An immediate implication of recognizing the nature and availability of these two tax instruments is that they leave, in the model analyzed here, no useful role for import tariffs: if both are optimally deployed, tariffs do no more than induce an unnecessary production inefficiency, and so should be avoided. And indeed there exists a simple strategy, generalizing the standard prescription developed in models without an informal sector, for using the VAT and withholding taxes so as to preserve revenue and increase welfare in the face of efficiency-improving tariff cuts. It has been shown too that there exist circumstances in which a VAT alone is fully optimal, precisely because it is a tax on informal sector production. But the conditions are restrictive: they include the requirement, in particular, that informal operators be charged VAT on all their inputs. More generally, withholding taxes—but not tariffs—are a potentially important part of an efficient tax structure.

The model underlying these results is highly stylized, so that the formal results are essentially illustrative. There are, for example, other ways in which informality might be characterized, with a fuller treatment needing also to consider the design of audit and penalty strategies for its deterrence, and to recognize the potential importance of noncompliance with taxes other than the VAT. And, although hard information is scarce, the withholding taxes that developing countries use in practice may not be the precisely targeted instrument formalized above. More generally, the analysis has excluded administration and compliance issues that, as stressed by Munk (2004), may be important to instrument choice and design;³⁰ not the least of these (though often ignored) are the potential challenges in crediting or refunding to tax compliant firms withholding taxes charged at the border. Nevertheless, the key design lessons that emerge seem to have a wider resonance for coherent tax design in the

³⁰ Here too there are subtleties to be faced: one of the arguments practitioners make for the VAT, for instance, is that it can serve as a catalyst for wider administrative improvement throughout the wider tax system.

challenging circumstances faced by developing countries. Not the least of these is the importance of recognizing that what matters in dealing with informality is not simply the border with the rest of the world, but rather the borders between the formal and informal sectors more generally.

Appendix I. Proof of Proposition 1

The comparative statics are derived by first combining (1) and (4) to give

$$c(P + T_M) = \left(\frac{1}{1 - T_V} \right) C_Y \left(\frac{P + T_M + T_W}{1 - T_V}, Y \right) \quad (\text{A1.1})$$

which defines informal sector output as a function $Y(T_V, T_M, T_W)$ of the three tax parameters. Using (1) in (5), the impact on formal sector output is found from

$$y = X \left(\frac{c(P + T_M)}{1 - T_V} \right) - Y(T_V, T_M, T_W) \quad . \quad (\text{A1.2})$$

Proceeding in this way first for the VAT gives

$$\frac{\partial Y}{\partial T_V} = \frac{1}{(1 - T_V) C_{YY}} (C_Y - \rho C_{\rho Y}) \quad ; \quad \frac{\partial y}{\partial T_V} = \left(\frac{1}{1 - T_V} \right)^2 X' c - \frac{\partial Y}{\partial T_V} \quad (\text{A1.3})$$

That $\partial Y / \partial T_V > 0$ then follows on noting that the presence of inputs other than the taxed intermediate on which we focus implies, by linear homogeneity of the cost function, that $C_Y > \rho C_{\rho Y}$.

The results for the tariff follow from

$$\frac{\partial Y}{\partial T_M} = \frac{1}{(1 - T_V) C_{YY}} (c' - C_{\rho Y}) \quad ; \quad \frac{\partial y}{\partial T_M} = \frac{c' X'}{1 - T_V} - \frac{\partial Y}{\partial T_M} \quad (\text{A1.4})$$

and those for the withholding tax from

$$\frac{\partial Y}{\partial T_W} = \frac{C_{\rho Y}}{(1 - T_V) C_{YY}} > 0 \quad ; \quad \frac{\partial y}{\partial T_W} = - \frac{\partial Y}{\partial T_I} \quad . \quad (\text{A1.5})$$

Appendix II. Proof of Proposition 2

It is convenient to take as the objects of choice T_V , T_M and ρ . For this, we now take (1) to implicitly define $Q(T_V, T_M)$, with

$$\frac{\partial Q}{\partial T_V} = \frac{Q}{1-T_V} \quad ; \quad \frac{\partial Q}{\partial T_M} = \frac{c'}{1-T_V}, \quad (\text{A2.1})$$

and informal output $Y(\rho, T_V, T_M)$ then implicitly defined, from (4), by $Q(T_V, T_M) = C_Y(\rho, Y)$, with

$$\frac{\partial Y}{\partial \rho} = -\frac{C_{\rho Y}}{C_{YY}} \quad ; \quad \frac{\partial Y}{\partial T_V} = \frac{Q}{C_{YY}(1-T_V)} \quad ; \quad \frac{\partial Y}{\partial T_M} = \frac{c'}{C_{YY}(1-T_V)} \quad (\text{A2.2})$$

Using also (5) to write the maxim and (6) as

$$\begin{aligned} W = & V[Q(T_V, T_M)] + \lambda \{ Q(T_V, T_M)Y(\rho, T_V, T_M) - C(\rho, Y(\rho, T_V, T_M)) \} \\ & + \delta \left\{ \left(\frac{T_V c(P+T_M)}{1-T_V} + T_M c'(P+T_M) \right) (X[Q(T_V, T_M)] - Y(\rho, T_V, T_M)) \right. \\ & \left. + (\rho - P)C_\rho[\rho, Y(\rho, T_V, T_M)] \right\}, \end{aligned} \quad (\text{A2.3})$$

the necessary condition on T_V is, using Roy's identity and (1),

$$\begin{aligned} 0 = & \frac{\partial Q}{\partial T_V} (-X + \lambda Y + \delta(T_V Q + T_M c')X') + \frac{\partial Y}{\partial T_V} (\lambda(Q - C_Y) - \delta\{T_V Q + T_M c' - (\rho - P)C_{\rho Y}\}) \\ & + \delta \left(\frac{1}{1-T_V} \right)^2 c y \end{aligned} \quad (\text{A2.4})$$

$$= -X + \lambda Y + \delta \left\{ (T_V Q + T_M c') \left(X' - \frac{1}{C_{YY}} \right) + (\rho - P) \left(\frac{C_{\rho Y}}{C_{YY}} \right) + y \right\}, \quad (\text{A2.5})$$

the second equality following on using (4), the relevant parts of (A2.1) and (A2.2), and dividing by $Q/(1-T_V)$. Proceeding similarly for T_M gives

$$0 = -X + \lambda Y + \delta \left\{ (T_V Q + T_M c') \left(X' - \frac{1}{C_{YY}} \right) + (\rho - P) \left(\frac{C_{\rho Y}}{C_{YY}} \right) + y + \frac{T_M (1-T_V) c''}{c'} \right\}. \quad (\text{A2.6})$$

Combining (A2.6) with (A2.5), it follows from $c'' < 0$ that $T_M = 0$, as claimed. With this simplification, proceeding similarly for ρ gives the final necessary condition

$$0 = (\delta - \lambda)C_\rho + \frac{\delta C_{\rho Y}}{C_{YY}}(T_V Q - (\rho - P)C_{\rho Y}) . \quad (\text{A2.7})$$

To derive the expressions for T_V and T_W in the proposition, note first that (A2.7) implies

$$\delta(\rho - P)\frac{C_{\rho Y}}{C_{YY}} = (\delta - \lambda)\frac{C_\rho}{C_{\rho Y}} + \frac{\delta T_V Q}{C_{YY}} . \quad (\text{A2.8})$$

Using this, and setting $T_M = 0$, in (A2.5), dividing by X , recalling (5), noting that $\Omega(Y) = C_\rho / Y C_{\rho Y}$, and rearranging gives (9). For T_W , multiply (A2.8) by $C_{YY} / Q = C_{YY} / C_Y$ recall (3), and rearrange to find

$$\delta \left(\frac{T_W}{1 - T_V} \right) \frac{C_{\rho Y}}{C_Y} = (\delta - \lambda) \frac{FF''}{(F')^2} + \delta T_V \left(1 - \frac{P}{1 - T_V} \left(\frac{C_{\rho Y}}{C_Y} \right) \right) , \quad (\text{A2.9})$$

use having also been made of the implication of (8) that $C_\rho C_{YY} / C_{\rho Y} C_Y = FF'' / (F')^2$. Adding and subtracting $\delta T_W C_{\rho Y} / (1 - T_V) C_Y$ on the right of (A2.9), and recalling that $\rho C_{\rho Y} / C_Y = \alpha$, rearranging gives equation (10).

Appendix III. Derivation of (14)

Note first that for the cost function in (8), $C_Y(\rho, Y) = (a_0 + a_1\rho)F'(Y)$. Adding and subtracting $a_1PF'(Y)$ gives

$$C_Y(\rho, Y) = (a_0 + a_1P)F'(Y) + a_1(\rho - P)F' \quad (\text{A3.1})$$

and hence, from (4) and (8),

$$Q = C_Y(P, Y) + (\rho - P)C_{\rho Y}, \quad (\text{A3.2})$$

the second equality following on again using (8). Using (A3.2) to substitute for $(\rho - P)C_{\rho Y}$ in (A2.7), (14) follows on recalling (1) and that $T_M = 0$.

Appendix IV. Proof of Proposition 3

Setting $T_M = T_W = 0$, the necessary condition on T_V in (A2.5) becomes, using (3) and (4),

$$0 = -X + \lambda Y + \delta y + \delta T_V \left\{ QX' + \frac{1}{C_{YY}} (\rho C_{\rho Y} - C_Y) \right\}. \quad (\text{A4.1})$$

Using in this the implication of (A1.3) that

$$\frac{1}{C_{YY}} (\rho C_{\rho Y} - C_Y) = -(1 - T_V) \frac{\partial Y}{\partial T_V}, \quad (\text{A4.2})$$

the conclusion follows on dividing by X and rearranging.

Appendix V. Proof of Proposition 4

With $dQ = 0$, $V(Q)$ and aggregate demand $X(Q)$ are evidently unchanged. And given too that $d\rho = 0$, it then follows from (4) that so too are y and Y . Perturbing revenue

$$R = (T_V Q + T_M c'(P + T_M))y + (\rho - P)Y \quad (\text{A5.1})$$

and using $dT_V = -(c'/Q)dT_M$ then gives $dR = T_M c'' dT_M > 0$. Equation (18) follows on perturbing (3) to find that $d\rho = 0$ requires

$$d\left(\frac{T_W}{1-T_V}\right) = -\left(\frac{1}{1-T_V}\right)^2 [dT_M + (P + T_M)dT_V], \quad (\text{A5.2})$$

with the result following on using (1) and (16), and noting that general properties of the cost function $c(P + T_M)$ imply that $c \geq (P + T_M)c'$.

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