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INCOME TAXATION WITH FIXED HOURS OF WORK

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When all jobs are full time and workers cannot substitute alternative jobs, the skills of workers are observable. Nevertheless, the optimum may involve a benefit for those who choose not to work (making their skills unobservable). When the tax structure and benefit for nonworkers are chosen to maximize welfare, they tradeoff the social marginal utility of consumption against the needed incentive to work. In contrast to more conventional models, the optimal tax schedule may have discontinuities and may involve subsidization of the work of low earners.

1. Introduction

There are many margins where individuals may adjust their behavior in response to income taxation. E. Sheshinski (1971) has analyzed optimal taxation where the only margin is the level of investment in education. J. Mirrlees (1971, 1976) has considered the situation where individual hours of work can be continuously varied.[1] Of course, these models can be interpreted as describing other margins. For example, in the Mirrlees model individuals might be choosing the difficulty of work rather than its length. Some individuals find that particular margins are not adjustable for them individually (even though they may be collectively adjustable). Here we shall consider a situation where an individual's only decision is whether to work. Hours of work, difficulty of labor, effort put forth are all taken to be given. Apart from this assumption the model is a generalization of the Mirrlees model in that individuals differ in the disutility of labor as well as in skill. The model is particularly simple since skill levels of workers are observable (given the fixed level of hours) and the tax can be set differently for each skill level. (Since workers differ in the disutility of work and the skills of nonworkers are not observable, achievement of a full optimum is generally not possible.) By looking across skill classes one has an overall income tax

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[1]Together with J. Mirrlees (1978), I have analyzed the situation of a variable length of working life, with fixed hours of work in any time period when the individual works. This note grew out of an attempt to understand how that model differs from previously analyzed cases.

structure which relates tax to income.[2] If the marginal distributions of disutility of work are the same for all skill levels, the marginal tax rates do not exceed one. Sufficient conditions are found for marginal tax rates to be positive for those paying positive taxes. Marginal tax rates may be negative for some of those whose work is subsidized.

2. General model

It is assumed that all individuals have the same concave utility function U defined over consumption, and the disutility of labor, my, where y is hours of work and m is a disutility factor. An individual of type $m-n$ has a disutility factor m and a marginal product per period worked of n. All jobs in the economy require the hours of labor, y, to equal one. Thus, for an $m-n$ individual who works, income equals n. The government selects the benefit, b, given to nonworkers and consumption of workers as a function of income, $c(n)$, to maximize the integral of utility. If the government calculated the optimum subject to the constraint that all individuals choose to work, people of the same skill would have identical consumption, and average marginal utility of consumption would be the same across skill levels.[3] In such an equilibrium, those of sufficiently low skill or high disutility of work might find their contribution to output of less social value than their disutility of work. Then, there might exist another allocation where the benefit to nonworkers is such that many of these individuals choose not to work. If more productive workers are not easily discouraged from working, such a change will raise social welfare. For the analysis which follows we assume that the distribution of skills and disutilities is such that the optimum occurs with some individuals choosing not to work.

We assume that m and n have the joint density $f(m,n)$, with n varying from $\underline{n}$ to $\bar{n}$ and, for each n, m varying from $\underline{m}(n)$ to $\bar{m}(n)(m(n)>0)$ with $f>0$ within this set. For each n there is a value $m^*(n)=m^*(c(n),b)$ such that those with lower disutility factors work $(U(c(n),m)\geqq U(b,0))$ while those with higher disutility do not work. When some people at skill n work, while others do not, the determination of who chooses to work is implicitly defined by

$$U(b,0)-U(c(n),m^*(n))=0. \tag{1}$$

[2]This is similar in method to E. Sadka's (1976) analysis of income taxation in the full optimum.

[3]If utility were additive, we would have equal consumption for everyone.

We can express social welfare maximization as

$$\max_{b,c(n)} \int_{\underline{n}}^{\bar{n}} \left[\int_{\underline{m}(n)}^{m^*(c(n),b)} U(c(n),m)f(m,n)\mathrm{d}m + \int_{m^*(c(n),b)}^{\overline{m}(n)} U(b,0)f(m,n)\mathrm{d}m \right] \mathrm{d}n$$

subject to

$$\int_{\underline{n}}^{\bar{n}} \left[\int_{\underline{m}(n)}^{m^*(c(n),b)} (c(n)-n)f(m,n)\mathrm{d}m + \int_{m^*(c(n),b)}^{\overline{m}(n)} bf(m,n)\mathrm{d}m \right] \mathrm{d}n = A, \tag{2}$$

where the net resources, A, are available for the provision of consumption.

We assume that an individual of skill n can only work at a job with marginal product n; he cannot take on a job requiring a lower skill level. This assumption does not make sense for many job differences. It does make sense in a few circumstances. Some skill differences (e.g. dexterity–strength, violinists–singers) do not involve one skill encompassing another. Rather the different skills go with jobs which produce outputs which have different social evaluations (values of the marginal product). The social evaluations then define which individuals have higher skills (higher marginal products). Another example arises where individuals have held particular jobs for long times, with retirement being the next best alternative to continuing in the same job. Without this strong assumption the model would become more complicated.

By assumption $c(n)$ affects only those of skill n. Thus, forming a Lagrangian, L, with multiplier λ, we can differentiate with respect to $c(n)$. We only consider those n for which there is an internal solution to $m^*(n)$ rather than having everyone or no one of a particular skill class working:

$$\frac{\partial L}{\partial c(n)} = \int_{\underline{m}(n)}^{m^*(n)} (U_1(c(n),m)-\lambda)f(m,n)\mathrm{d}m - \lambda(c(n)-n-b)\frac{\partial m^*(n)}{\partial c} f(m^*(n),n) = 0. \tag{3}$$

The first term of (3) is the net social gain from allocating more consumptionto workers of skill n. The second term is the cost in net resources of the induced increase in the number of workers as a result of increasing the consumption of workers. From (3) we see that the return to working $(c-b)$ is less than the output produced, n, if and only if U_1 averages less than λ for the workers of skill n. For given values of λ and b, eq. (3) is a necessary condition for $c(n)$. It need not be a sufficient condition, however since the problem is not generally well behaved. Nevertheless, the optimal level of $c(n)$ depends on aspects of the rest of the economy only through b and λ. That is, two economies with different available resources, A, and different distributions of skills but the same conditional distributions of disutility factors will have the same tax structure if they choose the same benefit for nonworkers and have the same shadow price on resources.

Differentiating with respect to b, we have the further condition

$$(U_1(b,0)-\lambda)\int_{\underline{n}}^{\bar{n}}\int_{m^*(n)}^{\bar{m}(n)} f(m,n)\mathrm{d}m\mathrm{d}n$$
$$=\lambda\int_{\underline{n}}^{\bar{n}}(c(n)-n-b)\frac{\partial m^*(n)}{\partial b} f(m^*(n),n)\mathrm{d}n. \tag{4}$$

The left-hand side of (4) is the social gain from allocating more resources to the consumption of those who do not work, while the right-hand side is the social cost of the net change in resources from the decline in work at all skill levels where there are internal solutions. We will only consider situations where the marginal utility of nonworkers exceeds the shadow price of resources and so consumption of nonworkers is limited by the disincentive effect of discouraging work.

The equations determining the pattern of consumption relative to income depend on the joint distribution of disutility and skill. In general there is no reason for consumption to necessarily increase with income. For example, if professors have high marginal products and low disutilities of labor, it is not necessary to pay them very much to induce work. There is a straightforward argument that if two skill groups of the same size have the same distribution of disutilities, the group with the lower skill does not have higher consumption. Assume this is false. Reversing consumption between two groups with the same disutility distribution has no effect on the integral of utility or on the number of individuals working. Now, however, output goes up since we have more higher skilled and fewer lower skilled individuals working. This is a contradiction to the presumed optimality of the first allocation.

3. Additive utility and independence of skill and disutility

Now let us consider the special case where the disutility distribution is the same for all skills and utility is additive:

$$f(m,n)=g(m)h(n), \qquad U_{12}=0. \tag{5}$$

We denote the cumulative distributions by G and H. Note that this assumption is not just that preferences can be expressed in an additive form, but that this is the form which is relevant for the social welfare function. We assume that the optimum does involve some nonworkers. Then the optimal allocation has two skill values n_1 and n_2 such that no one works for $n<n_1$

and everyone works for $n>n_2$. Consumption is increasing in n for $n_1<n<n_2$ and constant for $n>n_2$. If G/g is nondecreasing there is at most one value of n other than n_1 at which $c(n)$ equals $n+b$.

To obtain these results, let us consider the first-order conditions for this special case. We write marginal utilities as functions of the relevant variable only. Eqs. (3) and (4) become

$$\frac{\partial L}{\partial c(n)}=(U_1(c(n))-\lambda)G(m^*(n))-\lambda(c(n)-n-b)$$
$$\times\frac{\partial m^*(n)}{\partial c}g(m^*(n))h(n)=0, \tag{6}$$

$$(U_1(b)-\lambda)\int_{\underline{n}}^{\bar{n}}(1-G(m^*(n)))h(n)\mathrm{d}n$$
$$=\lambda\int_{\underline{n}}^{\bar{n}}(c(n)-n-b)\frac{\partial m^*(n)}{\partial b}g(m^*(n))h(n)\mathrm{d}n. \tag{7}$$

From the definition of m^*, (7) can be written as

$$\left(1-\frac{\lambda}{U_1(b)}\right)\int_{\underline{n}}^{\bar{n}}(1-G(m^*(n)))h(n)\mathrm{d}n$$
$$=-\int_{\underline{n}}^{\bar{n}}\left(1-\frac{\lambda}{U_1(c(n))}\right)G(m^*(n))h(n)\mathrm{d}n. \tag{8}$$

Considering the equation for $\partial L/\partial c(n)$, we note that n enters explicitly in only two ways – the multiplicative factor $h(n)$ and the term λn. Thus, for $n'>n''$ and any value of c we have

$$h^{-1}(n')\left.\frac{\partial L}{\partial c(n')}\right|_{c(n')=c}\geqq h^{-1}(n'')\left.\frac{\partial L}{\partial c(n'')}\right|_{c(n'')=c}, \tag{9}$$

with a strict inequality when $m^*(n'')>\underline{m}(n'')$.

This implies that the optimal consumption levels satisfy $c(n')\geqq c(n'')$ with a strict inequality if either $m^*(n')$ or $m^*(n'')$ is an interior solution. With $c(n)$ strictly increasing over the range of interior solutions, we have (by (1))$m^*(n)$ also strictly increasing. Thus those skills with all nonworkers, if any, are the lowest skills and those with no nonworkers, if any, are the highest skills.

Since $\partial m^*/\partial c$ and $G(m^*)/g(m^*)$ are positive the tax on earned income $n+b-c$ is positive if and only if λ is greater than U_1 (see (6)). For those skill levels n with g nonincreasing in m at $m^*(n)$ and positive tax, all terms in (6) are monotonic in the right way to imply positive marginal tax rates. Thus when $c(n)$ is continuous we can have the two possibilities shown in figs. 1 and 2.

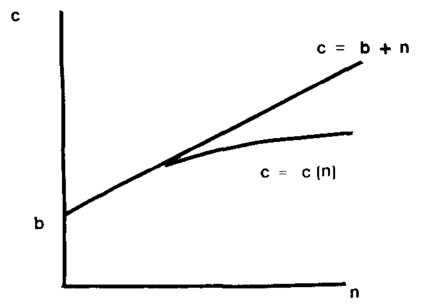

Fig. 1

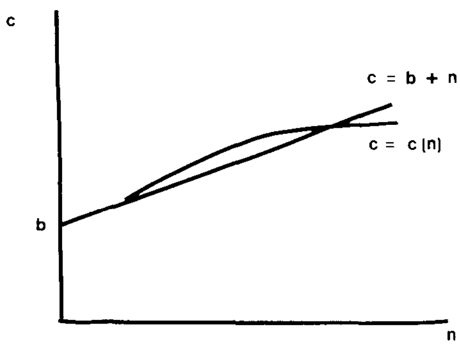

Fig. 2

While both possibilities have similar structures for high earners there are differences between them for low earners. In fig. 1, all workers pay positive tax on earnings $(c(n)-n-b<0)$. This is not the case in the other possibility. In fig. 2 the marginal tax rate is negative over a range. A major element in this structure of outcomes is that no work is the only substitute for work at one's skill level. Thus the benefit which can be paid nonworkers is limited by its effect on the supply of labor of many skills. Increasing the consumption of low skilled workers does not affect the behavior of the higher skilled. Thus the limitation on the pay of the low skilled in fig. 2 is the deadweight burden coming from the subsidization of labor of that skill. In the case of fig. 1, even the low skilled workers are enough better off than the nonworkers that they pay taxes on income rather than being subsidized to work.

As indicated above, the optimal consumption levels, $c(n)$, need not be continuous in skill. An example of a discontinuity at the bottom of the tax schedule is shown in fig. 3. For skills below the discontinuity there is no tax paid or subsidy received for work and no one works. For skills just above the discontinuity, labor is subsidized inducing work by some with this skill. This possibility is explored with a Cobb–Douglas example in the next section. Of course, other locations for discontinuities are possible.

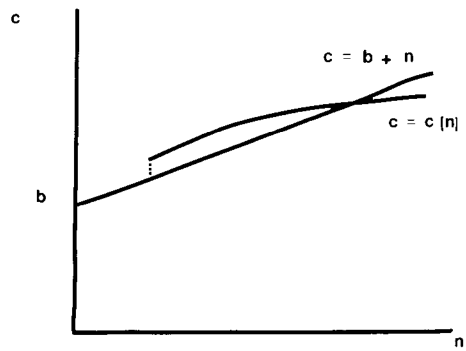

Fig. 3

Since consumption is nondecreasing in skill, there would be no change in the analysis if workers could fill jobs of lower skill. This assumes that workers do not prefer jobs of lower skill.

4. Cobb–Douglas example

To illustrate the possibilities shown in the three figures, let us consider the case of a Cobb–Douglas utility function, $U = \ln c + \ln(M - my)$, with $M > 1.5$. We assume that m is uniformly distributed between 0.5 and 1.5 for all n. When there is an interior solution for $m^*(n)$ (i.e. for this skill level some people work while others don't) consumption is given by

$$c(n) = 0.5\lambda^{-1} + 0.5[\lambda^{-2} - 4bM(\lambda^{-1} - n - b)(M - \tfrac{1}{2})^{-1}]^{1/2}, \qquad (10)$$

which is obtained by solving (6), making use of (1). $c(n)$ is increasing and concave. This implies an increasing marginal tax rate over the range of interior solutions. When the solution is interior, the cutoff level of disability satisfies (from (1))

$$m^*(n) = M(1 - b/c(n)). \qquad (11)$$

With this structure we have $c(n) = n + b$ at $n = \lambda^{-1} - b$. Since we are only considering solutions with $\lambda < U_1(b) = b^{-1}$, there is a positive value of n satisfying this condition. At $\lambda^{-1} - b$ the slope of $c(n)$ is $\lambda bM/(M-0.5)$. Except for the case of tangency between $c(n)$ and the line $b+n$ there is also a second value of n at which $c(n) = n + b$. We can use this fact to explore the different cases.

If, at the optimum, λ, b, and M are such that $c'(\lambda^{-1} - b)$ equals 1, then we have a situation as in fig. 1, with the consumption locus tangent to the 45° line. To find economies where this is the optimum we must find values of M, $h(n)$, and A to satisfy eq. (8) and the resource constraint in (2). These take the form

$$1 - \lambda b = \lambda \int_{\underline{n}}^{\bar{n}} (c(n) - b)(m^*(n) - 0.5)h(n)\mathrm{d}n, \tag{12}$$

$$\int_{\underline{n}}^{\bar{n}} [(m^*(n) - 0.5)(c(n) - n) + (1.5 - m^*(n))b]h(n)\mathrm{d}n = A. \tag{13}$$

Since $m^*(n)$ equals 0.5 for $n < \lambda^{-1} - b$, values of $\underline{n}$ and $\bar{n}$ satisfy (12) (with $\underline{n} < \lambda^{-1} - b < \bar{n}$) can always be found. This in turn permits a choice of A to satisfy (13). An example is shown in table 1. No taxes are paid for earned

Table 1
Optimal tax example ($b = 0.75$, $\lambda = 1$, $M = 2$).

Earned income	Income tax	Percentage working	Earned income	Income tax	Percentage working
0.25	0	0	2	0.84	72
0.50	0.04	26	3	1.52	83
0.75	0.13	40	4	2.25	90
1	0.25	50	5	3.01	95
			6.25	4.0	100

incomes below $\lambda^{-1} - b = 0.25$, which is n_1 (that is no one with skill lower than 0.25 chooses to work even though there is no tax on earned income). At the income level 0.25 the marginal tax is zero. The marginal tax rate rises steadily, reaching 100% at an earned income of 6.25, at which skill everyone is working.

For $c'(\lambda^{-1} - b)$ greater than one, the picture is essentially the same, with the difference that the marginal tax is positive at the income level where the average tax is zero.

In examples where $c'(\lambda^{-1} - b)$ is less than one, the average tax is negative for skill levels below $\lambda^{-1} - b$ and positive for higher skill levels. An example is shown in table 2 where the social marginal utility of consumption by nonworkers is 2.5 times the shadow price on resources. In the example there is no taxation of low incomes, and no work done. The marginal tax rate is negative at the point of the lowest earned incomes. This corresponds to the case shown in fig. 2.

Table 2

Optimal tax example ($b=0.4$, $\lambda=1$, $M=2$).

Earned income	Income tax	Percentage working	Earned income	Income tax	Percentage working
0.133	0	0	1	0.22	82
0.2	−0.09	34	1.5	0.55	91
0.4	−0.08	59	2	0.90	97
0.6	0	70	2.4	1.20	100

In table 3 is an example illustrating the case shown in fig. 3 – a discontinuity in the tax schedule. In this example, the first order conditions for consumption of workers with low skill, (10), imply subsidization of work. The other candidate for the optimum is no subsidization (and no work). To compare these two solutions we examine the social gain from inducing work of the individuals of each skill class. This is the gain in utility less the social cost of subsidizing their work:

$$\int_{0.5}^{m^*(n)} [\ln(c(n)/b)+\ln((M-m)/M)+\lambda(n-c(n)+b)]\mathrm{d}m$$
$$=(m^*(n)-0.5)(\lambda(n-c(n)+b)-1)+(M-0.5)\ln(c(M-0.5)/bM). \qquad (14)$$

For $m^*(n)>0.5$, this social return to work is increasing in skill. At an income level of approximately 0.0823 the social return to inducing work, (14), becomes positive. Thus there is a discontinuity in the tax structure at this point. No tax is paid or subsidy received on lower incomes. A sizeable subsidy (twice earnings) is paid on earnings just above the discontinuity.

5. Concluding remarks

Some workers make small adjustments to small tax changes while others make large adjustments. In particular both the poor and the aged may find full time employment and full withdrawal from the labor force to be the two

Table 3
Optimal tax example ($b=0.3$, $\lambda=1$, $M=2$).

Earned income	Income tax	Percentage working	Earned income	Income tax	Percentage working
<0.0823	0	0	0.7	0	90
0.0823	−0.17	42	0.9	0.13	94
0.1	−0.20	50	1.1	0.26	97
0.2	−0.22	67	1.3	0.40	100
0.3	−0.20	75			
0.5	−0.11	84			

most attractive alternatives.[4] In this case the subsidization of work may be optimal, while it is not optimal in the presence only of smooth work adjustment. To go further with this consideration, analysis of optimal tax structures needs to consider more complicated market settings reflecting the wide range of situations faced by different workers in modern economies.

[4]This can occur because of fixed costs of work [see Hausmann (1979)] or different wage rates for full and part time work.

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