# OPTIMAL TAX TREATMENT OF THE FAMILY: MARRIED COUPLES 

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Received August 1979, revised version received August 1982


#### Abstract

This paper examines the appropriate tax treatment of the family in a series of analytical models and numerical examples. For a population of taxpaying couples which differ in earning capacity, we derive the optimal tax rates for each potential earner. These rates depend crucially upon own and cross labor supply elasticities and the joint distribution of wage rates. Our results suggest that the current system of income splitting in the United States, under which husbands and wives face equal marginal tax rates, is nonoptimal. Using results from recent cconometric studies, and allowing for a sensitivity analysis, the optimal tax rates on secondary workers in the family are much lower than those on primary earners. Indeed, our best estimate is that the secondary earner would face tax rates only one-half as high as primary earners.


## 1. Introduction

The appropriate tax treatment of the family has been a basic issue in the design and implementation of federal taxation in every country employing direct taxation. Many countries use personal exemptions, deductions, and/or credits varying with family size. Several countries, the United States among them, allow some type of income splitting among family members. ${ }^{1}$ These provisions generally are defended by an appeal to differing economic circumstances across families of different size and economic characteristics.

Many countries rely on the individual rather than the family as the primary unit of account for personal taxation. Important examples include Canada, Australia, Japan and the Netherlands. Among OECD countries, only France requires families to file a joint return. Furthermore, several countries, such as Austria, Denmark, Italy and Sweden, recently have switched to the individual as the unit of account.

While the question of the appropriate unit of account for personal taxation has always been important, it is assuming increasing importance. Dramatic

[^0]changes have occurred in family size and composition in most countries. ${ }^{2}$ The combined effects in the United States of the post-World War II baby boom and the 1970's baby bust, the recent rapid increase in the labor force participation of married women, the postponement of marriage and childbearing, the increased life expectancy and the soaring of the divorce rate render the traditional one worker, one child raiser, continuously married several child family representative of a dwindling proportion of the population.

While the time thus seems ripe for a re-examination of the role of the unit of account in personal taxation, such studies not only are few and far between, ${ }^{3}$ but do not focus on what we consider the most important economic issues. For example, virtually no attention is paid to the efficient allocation of the time of family members between the market and the household. Nor is the assortive mating in society and the valuation of full income examined in an attempt to design an efficient and equitable tax system.

The purpose of the present paper is to begin to provide a theory of the optimal tax treatment of the family. Our procedure is to examine a series of models, each of which focuses on a particular aspect of the problem of the tax treatment of the family. Thus, section 2 presents a model of identical families and discusses the issue of the most efficient tax system to raise the government's required revenue. Various approaches to the taxation of the family are analyzed from this efficiency point of view.

Section 3 extends the analysis to the case of differences across households in the ability to produce market income. Introducing the joint density of the wage rates of husbands and wives and a representation of social welfare enables us to discuss the tradeoff between efficiency losses and redistribution inherent in designing a tax system.

Section 4 develops an instructive and illustrative special case of the analysis by confining attention to the Stone-Geary class of utility functions. Since the analyses of sections 2 and 3 reveal that certain empirical parameters - own and cross substitution effects on the labor supply of husbands and wives plus the parameters of the joint distribution of abilities - play a central role in the design of the optimal tax treatment of the family, we discuss econometric estimates of these parameters and employ them to estimate the optimal tax system.

Section 5 concludes with a discussion of issues in the taxation of families of different size and composition which we address in the sequel to this paper.

[^1]
## 2. Optimal tax treatment of identical families

We first examine the structure of the optimal tax treatment of the family in a world of identical families. This is the simplest case to begin the analysis and will prove particularly useful in generating insights in the subsequent analysis of the optimal tax treatment of nonidentical families.

Assuming identical preferences and endowments and income effects are taken care of elsewhere, we may treat social welfare as the welfare of a representative consumer. The simplest case which begins to yield some insight into the problem under consideration contains three goods: the labor (available time less leisure) of the husband, $L_{1}$, the labor of the wife, $L_{2}$, and other consumption, $C$. Without loss of generality, nonleisure consumption is taken as the untaxed numeraire.

Thus, the welfare of each family (and social welfare) may be represented by the usual well-behaved utility function:

$$
\begin{equation*}
U\left(C, L_{1}, L_{2}\right) \tag{1}
\end{equation*}
$$

Note that for analytical convenience we put labor supply rather than the more usual leisure (the remaining fraction of time) into the utility function; hence:

$$
\frac{\partial U}{\partial L_{i}}<0, \quad i=1,2
$$

the marginal utility of labor supply is assumed to be negative.
Within the context of such a model, we inquire under what conditions it is desirable to tax the earnings of husbands and wives (or primary and secondary earners) at the same rate, at different rates, or even to subsidize the earnings of one of the workers. In general, we derive the optimal tax rates for the two taxed commodities, $L_{1}$ and $L^{2}$.

In the United States, husbands and wives are allowed to pool their income and file a joint return which taxes them as if they had each earned one-half of the income. This income splitting provision (1) reduces the rate of progression for married couples, and (2) equates the marginal rate of income taxation on the earnings of husbands and wives. The provision thus has important consequences for the horizontal and vertical equity of the income tax. It is commonly argued that the concern over the effects of income splitting stems from the progressive rate structure imbedded in the current income tax. For example, Groves (1963) argues that 'The issue is associated with progressive taxation and would be of little or no importance under a proportional tax.' Under a flat rate tax, there would be no rate of progression to reduce; hence, the first effect noted above would be absent. The second effect of income splitting, equating the marginal tax rate for
husbands and wives, would not disappear under proportional taxation. We may then inquire as to whether it is desirable tax policy for husbands and wives (or primary and secondary workers in the family) to face the same marginal rate of tax.

The usual view on this matter is summed up by Pechman (1971):
The classic argument in favor if income splitting is that husbands and wives usually share their combined income equally.... Two conclusions follow from this view. First, married couples with the same combined income should pay the same tax irrespective of the legal division of income between them; second, the tax liabilities of married couples should be computed as if they were two single persons with their total income divided equally between them.

We shall demonstrate below that taxing the earnings of husbands and wives at the same rate is inefficient, in the sense that it produces a dead weight loss to society substantially in excess of that obtainable under a differentiated rate structure. The reason for this is straightforward: a tax on earnings distorts the work-leisure (or market vs. nonmarket work) choice; equal marginal tax rates [since the (income-compensated) supply of wives is much more elastic than that of husbands] induce a larger decline in the market work of wives relative to husbands than is socially optimal. ${ }^{4}$

Choosing scales of measurement so that initial net prices, which are the unit resource costs, equal unity for all goods, and that costs are constant in the relevant range, we seek the tax rates that minimize the dead weight loss from the tax system, subject to raising the required revenues per family, $R$. Under these assumptions, the government's problem is to ${ }^{5}$

$$
\begin{align*}
\min _{\beta_{1}, \beta_{2}} W=-\frac{1}{2} & {\left[\left(\frac{1-\beta_{1}}{\beta_{1}}\right)^{2} S_{11}+\left(\frac{1-\beta_{2}}{\beta_{2}}\right)^{2} S_{22}+2 S_{12}\left(\frac{1-\beta_{1}}{\beta_{1}}\right)\left(\frac{1-\beta_{2}}{\beta_{2}}\right)\right] } \\
& -\lambda\left[\left(\frac{1-\beta_{1}}{\beta_{1}}\right) L_{1}+\left(\frac{1-\beta_{2}}{\beta_{2}}\right) L_{2}-R\right] \tag{2}
\end{align*}
$$

[^2]$$
P_{i \mathrm{net}}\left(1+\frac{1-\beta_{i}}{\beta_{i}}\right)=P_{i \mathrm{gross}}=\frac{P_{i \mathrm{net}}}{\beta_{i}}
$$
where $S_{i j}$ is the $i j$ th Hicksian income-compensated cross-effect of a change in the net wage of $i$ with respect to the leisure (negative of labor supply) of $j$, and $1-\beta_{i}$ is the tax rate on the earnings of $i$.

Differentiating with respect to $\left(1-\beta_{1}\right) / \beta_{1}$ and $\left(1-\beta_{2}\right) / \beta_{2}$ and setting the derivatives equal to zero yields:

$$
\begin{align*}
& -\frac{\partial W}{\partial\left(\frac{1-\beta_{1}}{\beta_{1}}\right)}=S_{11}\left(\frac{1-\beta_{1}}{\beta_{1}}\right)+S_{12}\left(\frac{1-\beta_{2}}{\beta_{2}}\right)+\lambda L_{1}=0 \\
& -\frac{\partial W}{\partial\left(\frac{1-\beta_{2}}{\beta_{2}}\right)}=S_{12}\left(\frac{1-\beta_{1}}{\beta_{1}}\right)+S_{22}\left(\frac{1-\beta_{2}}{\beta_{2}}\right)+\lambda L_{2}=0  \tag{3}\\
& -\frac{\partial W}{\partial \lambda}=\left(\frac{1-\beta_{1}}{\beta_{1}}\right) L_{1}+\left(\frac{1-\beta_{2}}{\beta_{2}}\right) L_{2}-R=0
\end{align*}
$$

Solving by Cramer's rule we have:

$$
\begin{align*}
& \left(\frac{1-\beta_{1}}{\beta_{1}}\right)=\frac{\left|\begin{array}{ll}
\lambda L_{1} & S_{12} \\
\lambda L_{2} & S_{22}
\end{array}\right|,}{\left|\begin{array}{ll}
S_{11} & S_{12} \\
S_{12} & S_{22}
\end{array}\right|,} \\
& \left(\frac{1-\beta_{2}}{\beta_{2}}\right)=\frac{\left|\begin{array}{ll}
S_{11} & \lambda L_{1} \\
S_{12} & \lambda_{2} L_{2}
\end{array}\right|}{\left|\begin{array}{ll}
S_{11} & S_{12} \\
S_{12} & S_{22}
\end{array}\right| .} \tag{4}
\end{align*}
$$

Clearly, equal tax rates on the earnings of the husband and wife are optimal when the numerators in (4) are equal or when

$$
\begin{equation*}
\lambda L_{1} S_{22}-\lambda L_{2} S_{12}=\lambda L_{2} S_{11}-\lambda L_{1} S_{12} \tag{5}
\end{equation*}
$$

The homogeneity of demand curves may be used to reduce (5), via the row sum conditions on the Slutsky matrix, to

$$
L_{1} S_{23}=L_{2} S_{13} \quad \text { or } \quad \frac{S_{23}}{L_{2}}=\frac{S_{13}}{L_{1}}
$$

Given initial prices of unity, this tells us that the earnings of husbands and
wives should be taxed at the same rates as under the income-splitting provision, when the labor supplies of husbands and wives have identical compensated cross elasticities of demand with respect to the consumption of goods, or

$$
\eta_{23}=\eta_{13}
$$

Furthermore, the tax rate on the husband should exceed the tax rate on the wife if

$$
L_{1} S_{23}>L_{2} S_{13}
$$

(and conversely if $L_{1} S_{23}<L_{2} S_{13}$ ). This is equivalent to

$$
L_{1}\left(S_{22}+S_{12}\right)>L_{2}\left(S_{11}+S_{12}\right) .
$$

Thus, if the cross substitution effects are small relative to the own substitution effects, the higher tax rate should be levied according to whether

$$
\frac{S_{22}}{L_{2}} \gtrless_{i} \frac{S_{11}}{L_{1}},
$$

i.e. we should tax more heavily the earnings of the factor with the smaller own compensated wage elasticity.

The optimal tax rates need not both be positive. ${ }^{6}$ The denominators in (4) are necessarily non-negative, being equivalent to a second-order principal minor of the negative semi-definite Slutsky matrix. Examining, for example, the optimal tax rate on the wife's earnings, we note that it will be negative if

$$
L_{2} S_{11}<L_{1} S_{12}
$$

Since $S_{11}<0$ and $L_{2} \geqq 0$, complementarity of husbands' and wives' labor is sufficient for a wage subsidy for wives to be optimal. ${ }^{7}$

While we shall return to econometric estimates of the $S_{i j}$ in section 4 below, we note here that numerous econometric studies of labor supply have concluded that the own-substitution effect on labor supply for wives is much larger than that for husbands as well as the cross-substitution effect; ${ }^{8}$ since the typical hours of work of husbands exceeds that for wives, a higher tax rate for husbands than wives would appear desirable.

[^3]
## 3. Ability differences and the optimal tax treatment of the family

### 3.1. Household behavior

Households are assumed to have identical utility functions, $U\left(C, L_{1}, L_{2}\right)$, where again $C$ denotes total consumption, $L_{1}$ labor supply by the husband, and $L_{2}$ labor supplied by the wife, each measured as a fraction of total hours available. ${ }^{9}$ Households differ in their wage rate per hour, $w_{1}$ the husband's wage rate, and $w_{2}$ the wife's wage rate. Consumption is taken as numeraire with a unit price. There is no income apart from labor income, so $Y_{1}=w_{1} L_{1}$ and $Y_{2}=w_{2} L_{2}$. The household's before-tax total income, $Y$, is therefore $Y=Y_{1}$ $+Y_{2}=w_{1} L_{1}+w_{2} L_{2}$.

### 3.2. Tax instruments

The policy instrument examined in this paper is an income tax schedule which may be written in the general form $t\left(Y_{1}, Y_{2}\right)$, implying that the government can distinguish, for tax purposes, between a husband's and a wife's incomes. We examine, these possibilities by considering a general linear tax schedule of the form

$$
\begin{equation*}
t\left(Y_{1}, Y_{2}\right)=-\alpha+\left(1-\beta_{1}\right) Y_{1}+\left(1-\beta_{2}\right) Y_{2} \tag{6}
\end{equation*}
$$

where $\alpha$ is an income guarantee, and $1-\beta_{1}$ and $1-\beta_{2}$ are the marginal tax rates on $Y_{1}$ and $Y_{2}$, respectively. A requirement for joint returns without distinction between incomes would imply equal tax rates, i.e. $\beta_{1}=\beta_{2}$. A favorable tax treatment of, say, the wife's income implies $\beta_{2}>\beta_{1}$, etc.

If the joint distribution of $w_{1}$ and $w_{2}$ is given by $f\left(w_{1}, w_{2}\right)$, the government's budget constraint is given by

$$
\int_{0}^{\infty} \int_{0}^{\infty} t\left(w_{1} L_{1}, w_{2} L_{2}\right) f\left(w_{1}, w_{2}\right) \mathrm{d} w_{1} \mathrm{~d} w_{2}=R
$$

or, by (6),

$$
\begin{equation*}
\alpha=\int_{0}^{\infty} \int_{0}^{\infty}\left[\left(1-\beta_{1}\right) w_{1} L_{1}+\left(1-\beta_{2}\right) w_{2} L_{2}\right] f\left(w_{1}, w_{2}\right) \mathrm{d} w_{1} \mathrm{~d} w_{2}-R \tag{7}
\end{equation*}
$$

where $R$ is the net required revenue.

[^4]
### 3.3. Policy objectives

If it is assumed that the government's policy objectives are represented by a social welfare function, $W$, of the form

$$
\begin{equation*}
W=\int_{0}^{\infty} \int_{0}^{\infty} G\left[U\left(C, L_{1}, L_{2}\right)\right] f\left(w_{1}, w_{2}\right) \mathrm{d} w_{1} \mathrm{~d} w_{2}, \tag{8}
\end{equation*}
$$

where $G$ is a concave function. This formulation allows for a range of concern for equality from the strict utilitarian case $G(U)=U$ to the Rawlsian case $(G=\min U)$. The government maximizes (8) with regard to $\alpha, \beta_{1}$ and $\beta_{2}$ subject to (7).

Obviously, the government is also constrained by the fact that $C, L_{1}$ and $L_{2}$ are functions of $w_{1}, w_{2}$ and the tax parameters. Thus, the household's budget constraint is given by

$$
\begin{align*}
C & =Y_{1}+Y_{2}-l\left(Y_{1}, Y_{2}\right)  \tag{9}\\
& =\alpha+\beta_{1} w_{1} L_{1}+\beta_{2} w_{2} L_{2} .
\end{align*}
$$

Maximization of $U$ subject to (9) yields the first-order conditions:

$$
\begin{equation*}
U_{2}+\beta_{1} w_{1} U_{1}=0 \quad \text { and } \quad U_{3}+\beta_{2} w_{2} U_{1}=0 \tag{10}
\end{equation*}
$$

which, together with (9), determine the individual's equilibrium.
We may form the Lagrangian:

$$
\begin{equation*}
\mathscr{L}=W-\lambda\left[\alpha-\int_{0}^{\infty} \int_{0}^{\infty}\left[\left(1-\beta_{1}\right) w_{1} L_{1}+\left(1-\beta_{2}\right) w_{2} L_{2}\right] f\left(w_{1}, w_{2}\right) \mathrm{d} w_{1} \mathrm{~d} w_{2}+R\right] \tag{11}
\end{equation*}
$$

and obtain, using (9) and (10), the first-order condition for maximization of (8):

$$
\begin{align*}
\frac{\partial \mathscr{L}}{\partial \alpha}= & \int_{0}^{\infty} \int_{0}^{\infty}\left[G^{\prime} U_{1}-\lambda+\lambda\left(\left(1-\beta_{1}\right) w_{1} \frac{\partial L_{1}}{\partial \alpha}\right.\right. \\
& \left.\left.+\left(1-\beta_{2}\right) w_{2} \frac{\partial L_{2}}{\partial \alpha_{1}}\right)\right] f\left(w_{1}, w_{2}\right) \mathrm{d} w_{1} \mathrm{~d} w_{2}=0 \tag{12}
\end{align*}
$$

$$
\begin{align*}
& \frac{\partial \mathscr{L}}{\partial \beta_{1}}=\int_{0}^{\infty} \int_{0}^{\infty}\left[G^{\prime} U_{1} w_{1} L_{1}-\lambda w_{1} L_{1}+\lambda\left(\left(1-\beta_{1}\right) w_{1} \frac{\partial L_{1}}{\partial \beta_{1}}\right.\right. \\
&\left.\left.+\left(1-\beta_{2}\right) w_{2} \frac{\partial L_{2}}{\partial \beta_{1}}\right)\right] f\left(w_{1}, w_{2}\right) \mathrm{d} w_{1} \mathrm{~d} w_{2}=0  \tag{13}\\
& \frac{\partial \mathscr{L}}{\partial \beta_{2}}=\int_{0}^{\infty} \int_{0}^{\infty}\left[G^{\prime} U_{1} w_{2} L_{2}-\lambda w_{2} L_{2}+\lambda\left(\left(1-\beta_{1}\right) w_{1} \frac{\partial L_{1}}{\partial \beta_{2}}\right.\right. \\
&\left.\left.+\left(1-\beta_{2}\right) w_{2} \frac{\partial L_{2}}{\partial \beta_{2}}\right)\right] f\left(w_{1}, w_{2}\right) \mathrm{d} w_{1} \mathrm{~d} w_{2}=0 \tag{14}
\end{align*}
$$

Defining a function $h$ :

$$
\begin{equation*}
h\left(w_{1}, w_{2}\right) \equiv G^{\prime} \frac{U_{1}}{\lambda}+\left(1-\beta_{1}\right) w_{1} \frac{\partial L_{1}}{\partial \alpha}+\left(1-\beta_{2}\right) w_{2} \frac{\partial L_{2}}{\partial \alpha}-1, \tag{15}
\end{equation*}
$$

eqs. (12) and (13) may be rewritten:

$$
\begin{align*}
\frac{1}{\lambda} \frac{\partial \mathscr{L}}{\partial \alpha}= & \int_{0}^{\infty} \int_{0}^{\infty} h\left(w_{1}, w_{2}\right) f\left(w_{1}, w_{2}\right) \mathrm{d} w_{1} \mathrm{~d} w_{2}=0  \tag{16}\\
\frac{1}{\lambda} \frac{\partial}{\partial \beta_{1}}= & \int_{0}^{\infty} \int_{0}^{\infty} h\left(w_{1}, w_{2}\right)\left(-w_{1} L_{1}\right) f\left(w_{1}, w_{2}\right) \mathrm{d} w_{1} \mathrm{~d} w_{2} \\
& +\int_{0}^{\infty} \int_{0}^{\infty} w_{1}\left[\left(1-\beta_{1}\right) w_{1} S_{11}+\left(1-\beta_{2}\right) w_{2} S_{21}\right] f\left(w_{1}, w_{2}\right) \mathrm{d} w_{1} \mathrm{~d} w_{2}=0  \tag{17}\\
\frac{1}{\hat{\lambda}} \frac{\partial \mathscr{L}}{\partial \beta_{2}}= & \int_{0}^{\infty} \int_{0}^{\infty} h\left(w_{1}, w_{2}\right)\left(-w_{2} L_{2}\right) \mathrm{d} w_{1} \mathrm{~d} w_{2} \\
& +\int_{0}^{\infty} \int_{0}^{\infty} w_{2}\left[w_{1}\left(1-\beta_{1}\right) S_{12}+\left(1-\beta_{2}\right) w_{2} S_{22}\right] f\left(w_{1}, w_{2}\right) \mathrm{d} w_{1} \mathrm{~d} w_{2}=0 \tag{18}
\end{align*}
$$

where

$$
\begin{equation*}
S_{i j}=-\frac{\partial L_{i}}{\partial\left(\beta_{j} w_{j}\right)}+L_{j} \frac{\partial L_{i}}{\partial \alpha}, \quad i, j-1,2 \tag{19}
\end{equation*}
$$

are the Slutsky terms for husbands' and wives' leisure.
In order to simplify the interpretation of the optimum solution (16) (18), let us assume that $w_{2}$ is a nonrandom, strictly monotone function of $w_{1}$. This,
in effect, reduces the problem to one dimension, so we omit henceforth the subscript for $w$ in the density functions.

We may now provide an interpretation for $h\left(w_{1}, w_{2}\right)=H(w)$. The first term in (15) is the social marginal utility (expressed in money terms by division by i) of a lump-sum transfer to a household with wage $w$. This is clearly a decreasing function of $w$. The second term is the change in tax revenue due to such a transfer. Thus, if $T(w)=-\alpha+\left(1-\beta_{1}\right) w_{1} L_{1}+\left(1-\beta_{2}\right) w_{2} L_{2}$, then this term is equal to $\partial T(w) / \partial \alpha$. We assume that this expression is nonincreasing with $w$. This is a sufficient assumption to ensure that $H(w)$ is a decreasing function of $w$.

The interpretation of $H(w)$ is now straightforward. It is the net social marginal utility of an increase in $\alpha$ (the income guarantee). Condition (16) states that

$$
\begin{equation*}
\int_{0}^{\infty} H(w) F(w) \mathrm{d} w=0 \tag{20}
\end{equation*}
$$

where $F(w)$ is the marginal density of $w_{1}$, i.e. the optimum level of $\alpha$ is such that the social marginal utility of an increase in its value averages to zero over the population.

Eqs. (17) and (18) can now be rewritten:

$$
\begin{align*}
\int_{0}^{\infty} H(w)\left(-w_{1} L_{1}\right) F(w) \mathrm{d} w & +\int_{0}^{\infty} w_{1}\left[\left(1-\beta_{1}\right) w_{1} S_{11}\right. \\
& \left.+\left(1-\beta_{2}\right) w_{2} S_{21}\right] F(w) \mathrm{d} w=0  \tag{21}\\
\int_{0}^{\infty} H(w)\left(-w_{2} L_{2}\right) F(w) \mathrm{d} w & +\int_{0}^{\infty} w_{2}\left[\left(1-\beta_{1}\right) w_{1} S_{12}\right. \\
& \left.+\left(1-\beta_{2}\right) w_{2} S_{22}\right) F(w) \mathrm{d} w=0 . \tag{22}
\end{align*}
$$

Let us assume further that the wage income of the husband, $w_{1} L_{1}$, and of the wife, $w_{2} L_{2}$, are nondecreasing with the wage rate, $w^{10}$ Under these assumptions it can be shown that the first terms in (21) and (22) are nonnegative.

Proposition 1. If $H(w)$ is nonincreasing in $w$ and incomes are nondecreasing with the wage rate,
(a) when $S_{12}=0$ then $1-\beta_{1}>0$ and $1-\beta_{2}>0$;

[^5](b) when $S_{12}<0$ (two types of labor are complements), then either $1-\beta_{1}>0$ or $1-\beta_{2}>0$.

The proof is obvious.
Further simplification toward empirical application is obtained by assuming that the $S_{i j}$ are constant. In this case (21) and (22) simplify to:

$$
\begin{align*}
& \left(1-\beta_{1}\right) \delta_{11} S_{11}+\left(1-\beta_{2}\right) \delta_{12} S_{12}=\mathrm{A}  \tag{23}\\
& \left(1-\beta_{1}\right) \delta_{21} S_{21}+\left(1-\beta_{2}\right) \delta_{22} S_{22}=B \tag{24}
\end{align*}
$$

where

$$
A=\int_{0}^{\infty} H(w) w_{1} L_{1} F(w) \mathrm{d} w \leqq 0, \quad B=\int_{0}^{\infty} H(w) w_{2} L_{2} F(w) \mathrm{d} w \leqq 0
$$

and

$$
\delta_{i j}=\int_{0}^{\infty} w_{i} w_{j} F(w) \mathrm{d} w>0, \quad i, j=1,2 .
$$

The solution of (23) and (24) is given by

$$
\begin{align*}
& 1-\beta_{1}^{*}=\frac{A \delta_{22} S_{22}-B \delta_{12} S_{12}}{\delta_{11} \delta_{22} S_{11} S_{22}-\delta_{12}^{2} S_{12}^{2}}  \tag{25}\\
& 1-\beta_{2}^{*}=\frac{B \delta_{11} S_{11}-A \delta_{21} S_{21}}{\delta_{11} \delta_{22} S_{11} S_{22}-\delta_{12}^{2} S_{12}^{2}} \tag{26}
\end{align*}
$$

where an asterisk denotes the optimum.
Notice that the covariance between husband's and wife's wage rate, expressed by $\delta_{12}$, affects the optimum tax rates directly through $S_{12}$. (Indirectly, clearly, $A$ and $B$ also depend on $\delta_{12}$.)

Now, the sign of the denominator in (25) and (26) is positive by the second-order conditions of the individual $S_{11} S_{22}-S_{12}^{2}>0$, and by Schwartz's inequality $\delta_{11} \delta_{22}-\delta_{12}^{2}>0$.

Proposition 2. If $S_{i j}$ are constant, and the two types of labor are Hicksian substitutes, $S_{12}>0$, then $1-\beta_{1}^{*}>0$ and $1-\beta_{2}^{*}>0$.

It is seen that when $S_{12}<0$, i.e. when the husband's and the wife's labor are Hicksian complements, it is possible that one tax rate will be negative, that is, a wage subsidy is desirable. ${ }^{11}$

[^6]Note that $\beta_{1}^{*}=\beta_{2}^{*}$ when

$$
\begin{equation*}
A \delta_{22} S_{22}-B \delta_{12} S_{12}=B \delta_{11} S_{11}-A \delta_{21} S_{21} \tag{27}
\end{equation*}
$$

There is no a priori reason why this condition should be satisfied. Furthermore, when $A=B$ (equal social marginal utility for a lump-sum transfer to husband and wife) then (27) implies $\delta_{11} S_{11}=\delta_{22} S_{22}$, i.e. the product of the variance of earnings and the own-substitution effect on labor supply should be the same for husbands and wives. More generally, when $A$ $=B$, the ratio $\left(1-\beta_{1}^{*}\right) /\left(1-\beta_{2}^{*}\right)$ depends positively on the ratio $\delta_{22} S_{22} / \delta_{11} S_{11}$, i.e. the larger the elasticity of wives' relative to husbands' labor supply the larger should be the tax on husbands' relative to wives' income. Furthermore, given the elasticities of labor supply, the higher tax rate should be levied on the earnings with the smaller variance, ceteris paribus.

## 4. An example

Suppose that the utility function is of the Stone-Geary family, often used in empirical studies:

$$
U=b_{c} \log \left(C-C_{0}\right)+b_{1} \log \left(1-L_{1}\right)+b_{2} \log \left(1-L_{2}\right)
$$

where $b_{c}, b_{1}, b_{2}\left(b_{c}+b_{1}+b_{2}=1\right)$ and $C_{0}$ are positive constants. The regular demand functions derived from (9) and (10) for the case are given by:

$$
\begin{align*}
& C-C_{0}=b_{c} \cdot I \\
& \beta_{1} w_{1}\left(1-L_{1}\right)=b_{1} \cdot I  \tag{29}\\
& \beta_{2} w_{2}\left(1-L_{2}\right)=b_{2} \cdot I
\end{align*}
$$

where $I\left(w_{1}, w_{2}\right)=\alpha-C_{0}+\beta_{1} w_{1}+\beta_{2} w_{2}$ is full income.
The compensated labor supply functions are given by:

$$
\begin{align*}
& L_{1}=1-b_{1} A\left(\beta_{1} w_{1}\right)^{b_{1}-1}\left(\beta_{2} w_{2}\right)^{b_{2}} \mathrm{e}^{U}  \tag{30}\\
& L_{2}=1-b_{2} A\left(\beta_{1} w_{1}\right)^{b_{1}}\left(\beta_{2} w_{2}\right)^{b_{2}-1} \mathrm{e}^{U}
\end{align*}
$$

where $A=b_{c}^{b_{c}} b_{1}^{b_{1}} b_{2}^{b_{2}}$. Accordingly, the compensated leisure demand derivatives are:

$$
\begin{align*}
& S_{11}=\left(b_{1}-1\right)\left(\frac{1-L_{1}}{\beta_{1} w_{1}}\right), \quad S_{12}=b_{2}\left(\frac{1-L_{1}}{\beta_{2} w_{2}}\right)=b_{1}\left(\frac{1-L_{2}}{\beta_{1} w_{1}}\right),  \tag{31}\\
& S_{22}=\left(b_{2}-1\right)\left(\frac{1-L_{2}}{\beta_{2} w_{2}}\right)
\end{align*}
$$

Substituting (31) into eqs. (17) and (18), we obtain:

$$
\begin{align*}
& \int_{0}^{\infty} \int_{0}^{\infty} h\left(w_{1}, w_{2}\right)\left(-\beta_{1} w_{1} L_{1}\right) f\left(w_{1}, w_{2}\right) \mathrm{d} w_{1} \mathrm{~d} w_{2} \\
& +\int_{0}^{\infty} \int_{0}^{\infty} b_{1}\left[\left(\frac{1-\beta_{1}}{\beta_{1}}\right)\left(b_{1}-1\right)+\left(\frac{1-\beta_{2}}{\beta_{2}}\right) b_{2}\right] \\
& \times I\left(w_{1}, w_{2}\right) f\left(w_{1}, w_{2}\right) \mathrm{d} w_{1} \mathrm{~d} w_{2}=0, \tag{32}
\end{align*}
$$

and

$$
\begin{align*}
& \int_{0}^{\infty} \int_{0}^{\infty} h\left(w_{1}, w_{2}\right)\left(-\beta_{2} w_{2} L_{2}\right) f\left(w_{1}, w_{2}\right) \mathrm{d} w_{1} \mathrm{~d} w_{2} \\
& +\int_{0}^{\infty} \int_{0}^{\infty} b_{2}\left[\left(\frac{1-\beta_{1}}{\beta_{1}}\right) b_{1}+\left(\frac{1-\beta_{2}}{\beta_{2}}\right)\left(b_{2}-1\right)\right] \\
& \times I\left(w_{1}, w_{2}\right) f\left(w_{1}, w_{2}\right) \mathrm{d} w_{1} \mathrm{~d} w_{2}=0 . \tag{33}
\end{align*}
$$

Eqs. (32) and (33) can be viewed as two equations in the optimum $\left(\left(1-\beta_{1}^{*}\right) / \beta_{1}^{*}\right)$ and $\left(\left(1-\beta_{2}^{*}\right) / \beta_{2}^{*}\right)$.

A variety of studies have estimated family labor supply functions. As noted in section 2 , these studies invariably conclude that the labor supply of wives is much more elastic than that of husbands. ${ }^{12}$ For example, Pencavel and Johnson (1978) estimate a linear expenditure system with the $S_{i j}$ 's at mean values as follows: $S_{11}=73.9 ; S_{12}=-6.2 ; S_{22}=249.3$. Since $L_{1}$ exceeds $L_{2}$ by a substantial amount, the compensated wage elasticity of wives is five or six times as large as that for husbands. Furthermore, Pencavel and Johnson (1978 estimate a $b_{1}$ slightly larger than zero and a modest $b_{2}$.

Studies of the correlation in wage rates between husbands and wives, like the labor supply studies, must account for the fact that a large percentage of wives do not work in the market at any point in time. A recent careful study by Smith (1978) suggests a substantial positive correlation in wage rates of husbands and wives.

We approximate the net marginal utility of transfers, $h\left(w_{1}, w_{2}\right)$, by a linear function. Eqs. (32) and (33) involve only first and second moments of the joint earnings distribution function. We use plausible recent U.S. values in our numerical simulations. We present in tables 1 and 2 illustrative estimates

[^7]Table 1
Optimal tax rates on primary and secondary earner and optimal income guarantee ${ }^{\text {a }}\left\{\left(1-\beta_{1}\right)\right.$, $\left(1-\beta_{2}\right), \alpha($ in $\left.\$ 1000)\right\}$.

|  | Covariance in $\$ 1000)$ |  |  |  |
| :--- | :--- | :--- | :--- | :---: |
|  | 0 | 2 | 4 |  |
| $b_{2}=0.2$ | $(0.30,0.10,3.5)$ | $(0.34,0.12,4.0)$ | $0.37,0.14,4.4)$ |  |
| $b_{2}=0.3$ | $(0.30,0.08,3.3)$ | $0.34,0.10,3.8)$ | $(0.38,0.12,4.1)$ |  |

${ }^{\mathrm{a}}$ Assumes $\mu_{1}=\$ 12,000 ; \mu_{2}=\$ 8,000 ; \sigma_{1}^{2}=\$ 8,000 ; \sigma_{2}^{2}=\$ 4000 ; b_{1}=0.05$.
Table 2
Optimal tax rates on primary and secondary earners and optimal income guarantee ${ }^{a}\left\{\left(1-\beta_{1}\right)\right.$, $\left(1-\beta_{2}\right), x($ in $\left.\$ 1000)\right\}$.

|  | Variance $1\left(\sigma_{1}^{2}\right)($ in $\$ 1000)$ |  |  |  |  |  | 16 |
| :--- | :--- | :--- | :--- | :--- | :---: | :---: | :---: |
|  | 6 | 8 | 12 |  |  |  |  |
| Covariance $=2$ <br> (in $\$ 1000)$ |  |  |  |  |  |  |  |
| Variance $2\left(\sigma_{2}^{2}\right):$ <br> (in $\$ 1000)$ |  |  |  |  |  |  |  |
| 4 | $(0.31,0.12,3.8)$ | $(0.34,0.12,4.0)$ | $(0.38,0.13,4.4)$ | $(0.41,0.13,4.7)$ |  |  |  |
| 6 | $(0.33,0.14,4.0)$ | $(0.35,0.14,4.2)$ | $(0.38,0.15,4.5)$ | $(0.41,0.15,4.8)$ |  |  |  |
| 8 | $(0.34,0.16,4.1)$ | $(0.36,0.16,4.3)$ | $(0.40,0.17,4.7)$ | $(0.42,0.17,4.9)$ |  |  |  |


| Covariance $=4$ |
| :--- |
| (in $\$ 1000$ ) |

Variance $2\left(\sigma_{2}^{2}\right)$ :

(in $\$ 1000$ ) $\quad$|  |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
| 4 | $(0.35,0.14,4.2)$ | $(0.37,0.14,4.4)$ | $(0.40,0.14,4.7)$ | $(0.42,0.15,4.9)$ |
| 6 | $(0.37,0.16,4.4)$ | $(0.38,0.16,4.5)$ | $(0.41,0.16,4.8)$ | $(0.43,0.17,5.1)$ |
| 8 | $(0.37,0.17,4.5)$ | $(0.39,0.18,4.7)$ | $(0.42,0.18,5.0)$ | $(0.44,0.18,5.2)$ |

$$
{ }^{\mathrm{a}} \text { Assumes } \mu_{1}=\$ 12,000 ; \mu_{2}=\$ 8,000 ; b_{1}=0.05 ; b_{2}=0.20 \text {. }
$$

of the optimal tax rates on first and second earners in the family and of the optimal guarantee for interesting variations in the underlying parameters. ${ }^{13}$

Table 1 reports the sensitivity of the optimal tax rates and income guarantee for alternative values of the covariance in wages and the 'weight' of the second earners' leisure in the utility function (which varies inversely with the compensated labor supply elasticity). Reading across either row, we note that with the given means and variances, increases in the covariance of wages are associated with higher tax rates on both husbands and wives and larger income guarantees. This is as expected since the greater covariance

[^8]implies greater inequality in combined earnings. Also, note that the optimal tax rate on primary earners in these examples is on the order of three times as large as that on secondary workers.

Next note that as $b_{2}$ increases from 0.2 to 0.3 (with the corresponding decrease in $b_{c}$ ), the optimal tax rate on secondary earners declines slightly, as does the optimal income guarantee. The increase in $b_{2}$ implies a reduction in the compensated elasticity of labor supply (or leisure demand). Since $b_{1}$ remains at 0.05 in the example, and the sum of the $b$ 's is unity in the StoneGeary utility function, the increase in $b_{2}$ is offset by a decline in $b_{c}$ implying an increase in the compensated demand for consumption goods.

Table 2 focuses on the effects of inequality on the optimal tax rates and income guarantee. Given the assumed mean wages, covariance in wages and labor supply parameters, we note a series of interesting results. First, as in table 1 , the optimal tax rates on primary earners are between twice and three times as high as those on secondary earners. Second, reading across rows, note that for a given variance in the wages of the secondary earner, as the variance in the primary earners' wages increases, their tax rates and the income guarantee rise substantially. There is little effect on the (optimal) tax rate on secondary earners. All of the larger guarantee to offset the greater inequality comes at the expense of the primary earner.

Third, reading down columns, note that for a given variance in the primary earner's wages, increases in the variance of the secondary earner's wage results in an increase in their tax rate and the income guarantee, and a very slight increase in the tax on primary earners. This slight asymmetry results from the fact that much more is collected in taxes from primary earners.

Fourth, comparing corresponding sets of tax rates and guarantees in the upper and lower panels of table 2 reveals that for each given pair of variances of wages, the larger the covariance, and hence, the greater inequality in pooled family income, the higher the tax rates on each family member and a larger income guarantee.

Thus, our numerical examples reinforce the analytical results reported above and provide ample scope for scepticism concerning the desirability of income splitting and taxation based on pooled family income independent of source. The optimal taxation of two-earner families, when their labor supply elasticities and/or underlying distribution of wages varies, ${ }^{14}$ implies separate tax rates (or schedules of rates) on each separately. The optimal transfer payment system to reflect inequality aversion will also be affected by these same structural features of our economy and the tendency of earners to form families.

[^9]While alternative estimates would change these rates somewhat, our major qualitative feature - the higher rate of tax on husbands than wives results from all reasonable parameter values. We take this to be a strong indictment of the case for income splitting quoted in section 2 . In a world of married couples, the primary earner should face higher tax rates than the secondary worker.

## 5. Conclusion

We have examined the appropriate unit of account for personal income taxes in a world of married couples which differ in earning capacity. The typical second-best problem yielded both analytical results and empirical insights which suggest that equal marginal tax rates on husbands and wives, as under the U.S. income splitting provision, is nonoptimal. The primary earner in the family would face higher optimal tax rates than the secondary earner due to the relatively less elastic labor supply of primary workers. A numerical example based on recent parameter estimates derived from the Stone-Geary utility function suggests that the rate on husbands would be roughly twice that on wives.

Families differ in more than their ability to produce market income. They differ in the number of children, whether or not two adults are present, etc. The increase in divorce rates and other changes in family patterns suggest that the relative tax treatment of families of different size and composition is also becoming more important. Indeed, we hear more and more about incentives to marry or to divorce embedded in the tax codes of different countries. We shall deal with questions of family size and composition in a sequel to this paper. We hope that by focusing on the optimal tax treatment of the family in a world of married couples only, we have brought to attention the neglected, but crucial, issues of relative labor supply elasticities and wage rate variances and covariances of husbands and wives. ${ }^{15}$

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[^0]:    *We would like to thank Steve Penico and Dong-Ik Lee for valuable research assistance, and the U.S. Treasury Department and the Sloan Foundation for financial support.
    'Other countries allowing or requiring income splitting include Spain, Switzerland, France, Germany, Norway and the United Kingdom.

[^1]:    ${ }^{2}$ See the excellent summary by Michael, Fuchs and Scott (1980).
    ${ }^{3}$ Important recent exceptions include Munnell (1980), Brazer (1980) and Rosen (1977, 1978).

[^2]:    ${ }^{4}$ The allocative inefficiency induced by various tax devices has been a central issue in tax theory and policy for many decades. In modern form, it dates back to Ramsey (1927) and Pigou (1947), and includes important contributions by Rolph and Break (1949). Friedman (1959), Little (1951) and Corlett and Hague (1951). The most empirically relevant analyses are those of Harberger (1964a, 1964b, 1966). While earlier contributions tended to focus on how to achieve optimal taxation, Harberger extended the analysis to include the measurement of the dead weight loss associated with any set of non-neutral taxes. Renewed interest has been stimulated by the work by Diamond and Mirrlees (1971) and Atkinson and Stiglitz (1972).
    ${ }^{5}$ Here the tax rates are expressed as percentages of the net price of leisure; the more usual quotation in terms of gross earings may be derived once we note that a subsidy to leisure at the rate $\left(1-\beta_{i}\right) / \beta_{i}$ is equivalent to a subsidy at the rate $1-\beta_{i}$ on the gross price. Thus,

[^3]:    ${ }^{6}$ Obviously, one tax rate must be positive with a positive revenue requirement.
    ${ }^{7}$ For husbands, $L_{1} S_{22}<L_{2} S_{12}$.
    ${ }^{8}$ For example, see Boskin (19/3), Hall (19/3), Kosen (19/7), Heckman (19/4), Hurd (1976), Ashenfelter and Heckman (1973), and Pencavel and Johnson (1978).

[^4]:    ${ }^{9}$ Note we here normalize total time available to unity.

[^5]:    ${ }^{10}$ Thus, we assume the wage elasticity of labor supply exceeds minus one. The empirical studies discussed below render this a safe supposition.

[^6]:    ${ }^{11}$ Note the similar conclusion arrived at in section 2.

[^7]:    ${ }^{12}$ For example, see Boskin (1973), Bowen and Finegan (1969), Cain (1966), Hurd (1976), Mincer (1962), and Pencavel and Johnson (1978).

[^8]:    ${ }^{13}$ Since $a$ is wage-weighted, and $\int h(w) f(w) \mathrm{d} w$, presumably $a$ will be close to zero unless there is extreme social inequality aversion. The level of $a$ would affect the level of tax rates and transfers; since our focus is on the differential in these tax rates, we do not present results for other values of $a$. However, our calculations reveal that even with much larger values of $a$, the same pattern of much lower tax rates on wives occurs.

[^9]:    ${ }^{14}$ Obviously, the $S_{i j}$ and $\delta_{i j}$ depend upon the tax rates; the current estimated values will change if we change tax rates. However, the qualitative conclusions reported here are quite robust to modest changes in the $S_{i j}$ and $\delta_{i j}$.

[^10]:    ${ }^{15}$ Robert Hall points out to us that as formulated, our problem can be thought of as applying to any two-dimensional labor supply decision, e.g. optimal taxation of a worker early and late in life.

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