

## Optimal Taxation and Social Insurance with Endogenous Private Insurance<sup>†</sup>

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*We characterize welfare gains from government intervention when the private sector provides partial insurance. We analyze models in which adverse selection, pre-existing information, or imperfect optimization create a role for government intervention. We derive formulas that map existing empirical estimates into quantitative predictions for optimal policy. When private insurance generates moral hazard, standard formulas for optimal government insurance must be modified to account for fiscal externalities. In contrast, standard formulas are unaffected by “informal” private insurance that does not generate moral hazard. Applications to health and unemployment show that formal private market insurance can significantly reduce optimal government benefit rates. (JEL D82, G22, H21, H23, J65)*

What is the optimal amount of redistributive taxation and social insurance? A recent literature in public economics has come closer to providing quantitative answers to this central policy question. This literature has derived “sufficient statistic” formulas that map elasticities estimated in the modern program evaluation literature into predictions about optimal policies (e.g., Peter A. Diamond 1998, Saez 2001, Robert Shimer and Ivan Werning 2007, Chetty 2009). One important limitation of existing sufficient statistic formulas is that they do not allow for private market insurance, implicitly assuming that the government is the sole provider of insurance. In practice, private markets have historically been an important source of insurance and continue to remain active today despite the substantial size of social insurance programs.

Previous theoretical and empirical studies have recognized that the existence of private insurance can lower the optimal level of social insurance.<sup>1</sup> However, there is no method of mapping reduced-form empirical estimates such as those of Cutler and

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<sup>1</sup> For example, Mikhail Golosov and Aleh Tsyvinski (2007) make this point theoretically. David M. Cutler and Jonathan Gruber (1996a, 1996b) present evidence that crowd-out of private insurance is substantial in health care, and argue that this lowers the optimal level of public health insurance.

Gruber (1996a) into quantitative statements about the optimal level of government intervention in models such as those of Orazio Attanasio and Jose-Victor Rios-Rull (2000) or Golosov and Tsyvinski (2007). This paper takes a step toward filling this gap. We develop formulas for optimal taxation and social insurance in stylized models that allow for partial private insurance. The formulas are functions of reduced-form parameters that are frequently estimated in empirical studies, and can therefore be easily adapted to analyze policies ranging from optimal tax and transfer policy to unemployment and health insurance.

The starting point for our analysis is the specification of the limits of private market insurance and the potential role for government intervention. There are at least five reasons that government intervention could improve upon private insurance markets. First, private markets can only insure against shocks that occur *after* agents purchase private insurance. Only the government can provide redistribution across types revealed before private insurance contracts are signed. Second, informational asymmetries can lead to market unravelling through adverse selection (George A. Akerlof 1970). Third, even when private markets function perfectly, individuals may suffer from behavioral biases such as myopia or overconfidence that lead them to underinsure relative to the optimum (Louis Kaplow 1991; Stefano DellaVigna 2009; Johannes Spinnewijn 2008). Fourth, private firms generally cannot sign exclusive contracts, leading to inefficient outcomes because of multiple dealing (Mark V. Pauly 1974). Finally, some studies have argued that the administrative and marketing costs of private insurance exceed those of public insurance (Steffie Woolhandler, Terry Campbell, and David U. Himmelstein 2003; Uwe E. Reinhardt, Peter S. Hussey, and Gerard F. Anderson 2004) because of increasing returns and zero-sum strategic competition. As we discuss in Section I, these limitations of private insurance markets may explain the emergence and expansion of social insurance during the twentieth century.

In this paper, we characterize the welfare gains from government intervention under the first three private market limitations.<sup>2</sup> We analyze models in which the agent's earnings vary across states. This variation can be interpreted as uncertainty due to shocks, as in a social insurance problem, or as variation in earnings ability behind the veil of ignorance, as in an optimal taxation problem. The model permits suboptimal choice of private insurance—e.g., because of overconfidence among individuals—as well as market limitations due to pre-existing information or adverse selection. We derive formulas for the welfare gain from increasing the government tax rate (or social insurance benefit) that depends on five parameters: the variation in consumption across states and risk types, the curvature of the utility function, the elasticity of effort with respect to the tax or benefit rate, the size of the private insurance market, and the crowd-out of private insurance by public insurance. The first three parameters are standard elements of sufficient statistic formulas for optimal taxation and social insurance without private insurance. The last two are

<sup>2</sup> We do not consider multiple dealing as it is treated in detail by Pauly (1974). We do not consider administrative costs in the interest of space. The formulas we develop can be extended to incorporate such costs using estimates of loading factors for public and private insurance.

the new elements. In addition to offering a method of making quantitative predictions about welfare gains, our analysis yields two general qualitative lessons.

First, standard optimal tax and social insurance formulas overstate the optimal degree of redistribution in the presence of private insurance that generates moral hazard. A planner may observe substantial income inequality and conclude based on classic optimal tax results that redistributive taxation would improve welfare. However, if the observed earnings distribution already reflects implicit insurance provided by the private sector (e.g., through wage compression by firms) then making the redistribution through taxation could reduce welfare via its effects on the private insurer's budget. In the extreme case where private insurance markets function optimally, the planner would end up strictly reducing welfare by implementing such redistributive taxes (Kaplow 1991). Intuitively, the government exacerbates the moral hazard distortion created by private-sector insurance, and must therefore take into account the amount of private insurance and degree of crowd-out to calculate the optimal policy. Taking the observed earnings distribution as a reflection of marginal products, as is standard practice both in the theory of optimal taxation and in policy debates, may lead to misleading conclusions about optimal tax policy.

Second, it is critical to distinguish private insurance mechanisms that generate moral hazard from those that do not. While "formal" arms-length insurance contracts are likely to generate as much moral hazard as public insurance, "informal" risk sharing arrangements—such as borrowing from close relatives or relying on spousal labor supply to buffer shocks—may involve much less moral hazard. When private insurance does not generate moral hazard, the formula for optimal government benefits coincides exactly with existing formulas that ignore private insurance completely. This is because the effect of informal private insurance is already captured in the smaller consumption-smoothing effect of public insurance. This point is of practical importance because most existing empirical studies on private versus public insurance provision do not distinguish between formal and informal private insurance provision (e.g., Robert M. Townsend 1994; Julie Berry Cullen and Gruber 2000; Attanasio and Rios-Rull 2000; Robert F. Schoeni 2002).

To illustrate how our formula can be applied to obtain quantitative predictions about optimal policy, we present applications to unemployment insurance (UI) and health insurance. In the unemployment application, we focus on severance pay provided by private employers as a form of private insurance. Severance pay generates moral hazard because it can induce workers to shirk on the job, since they do not fully internalize the costs of being laid off. Using variation in UI benefit laws across states in the United States, we estimate that a 10 percent increase in UI benefit levels reduces private insurance against job loss (severance pay) by approximately 7 percent. Plugging this estimate into our formula along with other parameter estimates from the existing literature, we find that there is a wide range of parameters for which standard formulas that ignore private insurance and crowd-out imply that raising the benefit level would raise welfare when in fact it would lower welfare.

Our second application explores the welfare gains from expanding public health insurance (e.g., Medicare and Medicaid) using existing estimates of behavioral responses to health insurance. Calibrations of our formula suggest that the aggregate level of public health insurance is near the optimum given the amount of private

insurance and its response to public insurance. Accounting for private insurance is very important. The standard formula that ignores the private insurance provision overstates the welfare gain from an aggregate expansion of public health insurance by a factor of more than 100 with existing elasticity estimates. Note that these calibration results are based on a representative-agent model with elasticity estimates for the aggregate population. There are likely to be subgroups of the population that are underinsured, such as low-income individuals, and others that are overinsured. Our analysis should therefore be interpreted not as a policy recommendation but rather as a call for further work estimating the key elasticities by subgroup to identify how public health insurance should be reformed.

This paper builds on and relates to several strands of the literature on optimal insurance. One theoretical literature has considered optimal insurance and government redistribution problems jointly (e.g., Ake Blomqvist and Henrik Horn 1984, Jean-Charles Rochet 1991, and Helmuth Cremer and Pierre Pestieau 1996). These papers analyze models with heterogeneity in ability to earn (as in James A. Mirrlees 1971) coupled with ex post shocks to income (such as a health shock). In these models, the government is the sole provider of insurance and chooses both an optimal income tax schedule and a social insurance program.<sup>3</sup> In contrast, our paper considers a simpler model with a single source of earnings heterogeneity, which does not distinguish between risk and ability, but allows public and private insurance to coexist.

Models with private and public insurance have been considered in the literature on optimal health insurance (e.g., Timothy Besley 1989, Thomas M. Selden 1993, Blomqvist and Per-Olov Johansson 1997, Alessandro Petretto 1999, William Encinosa 2003, and Francesca Barigozzi 2006). We develop empirically implementable formulas for the welfare gains from public insurance in such models. Our formulas help to connect the theoretical work to the corresponding empirical literature on the interaction between private and public health insurance (e.g., Paul B. Ginsburg 1988; Amy K. Taylor, Pamela Farley Short, and Constance M. Horgan 1988; John R. Wolfe and John H. Goddeeris 1991; Cutler and Gruber 1996a, 1996b; Amy Finkelstein 2004). The formulas we derive are in the same spirit as recent sufficient statistic formulas in that they shed light on the essential features of the models that matter for welfare analysis. However, unlike typical sufficient statistic results, the formulas we derive here are based on more stylized models and therefore may not be fully robust to modifications of the primitive structure, an issue that we discuss in greater detail in the conclusion.

The most closely related paper to our study is that of Liran Einav, Finkelstein, and Mark R. Cullen (forthcoming), who develop a different method of characterizing welfare in a model with adverse selection in the private insurance market. Einav, Finkelstein, and Cullen (forthcoming) show that the slopes of the demand and cost curves for private insurance are together sufficient statistics for welfare. Our formula depends, instead, on ex post behavioral responses to change in government benefit levels. The two formulas are complements. Einav, Finkelstein, and Cullen's

<sup>3</sup> An exception is Robin Boadway et al. (2006), who allow for private insurance. However, they assume that private insurers observe ability while the government does not. In our model, private and public insurers have the same informational constraints.

(forthcoming) method is easier to implement when exogenous price variation in insurance markets and demand and cost data are available. Our formulas may be easier to implement when there is variation in government benefit levels that permits estimation of ex post behavioral responses.

The remainder of the paper is organized as follows. Section I provides a brief historical motivation for examining the interaction between social and private insurance. In Section II, we consider endogenous private insurance by studying a model in which the government and private sector have the same tools, but the level of private insurance is not necessarily set optimally. Section III considers a model with market limitations due to the inability of the private sector to insure pre-existing risks and adverse selection. The applications are presented in Section IV. Section V concludes.

### **I. Private and Social Insurance: Historical Background and Motivation**

To motivate our analysis, we briefly review the history of private insurance schemes that predated social insurance. We focus on the case of unemployment insurance because it is one of the oldest forms of both private and social insurance and illustrates the main intuitions underlying our model.

In the 1800s, private unemployment insurance was most commonly provided by trade unions. Some trade unions required all union members to pay a fee into a fund in exchange for receiving a payment from the fund if they were laid off (International Labor Organization 1955). Some insurance was also provided by social clubs or associations, which pooled risk among their members. Neither of these private insurance schemes was very successful, and they never covered more than a small minority of the workers in an economy. The trade union insurance only pooled risks within industry. As a result, industry-wide shocks, which were responsible for many layoffs, could not be handled. The voluntary social clubs suffered from attracting only those individuals with the highest risk of job loss. This adverse selection problem led to the demise of many of these clubs.

In view of these problems, municipalities began to introduce voluntary insurance schemes that were open to anyone in the city. One of the earliest such examples was the voluntary UI system of Berne, Switzerland, which began in 1893. This system, which was subsidized by the municipality, offered a benefit of 1 franc per day for individuals who had been out of work for at least 15 days if they had paid membership dues for 6 months. The system suffered from severe adverse selection. For example, factory employees, who had low layoff risk, did not participate at all despite the generous city subsidy (William Franklin Willoughby 1897). Authorities also had difficulty in collecting individuals' dues after they became members, creating substantial litigation costs.

These limitations of private and voluntary public insurance—problems in pooling risk across pre-existing types, adverse selection, and administrative costs—led to the introduction of mandatory social insurance programs in the early 1900s. As such, social insurance programs expanded, private insurance schemes became much less prevalent, presumably because of crowd-out (International Labor Organization 1955). Modern developed economies provide social insurance for unemployment, disability,

health, and old age, among other risks. In 1996, social insurance accounted for 22 percent of gross domestic product (GDP), on average, in Organization for Economic Co-Operation and Development (OECD) countries (International Labor Organization 2000).

Despite the dramatic expansion of social insurance over the past century, some private insurance persists. In the case of unemployment insurance, many firms voluntarily make severance payments when they lay off workers (Donald O. Parsons 2002). In the United States, 62 percent of individuals have private health insurance (Gruber 2007). Many countries have private annuity, disability, and life insurance markets.<sup>4</sup> In addition to these formal insurance contracts, “informal” private insurance schemes, such as loans from friends and relatives and spousal labor supply, are widespread (Cullen and Gruber 2000). This coexistence of private and public insurance motivates the question of how public redistribution and insurance programs should optimally be designed given that private insurance continues to be provided in equilibrium. The lessons from history discussed above—the limitations of private insurance markets and the crowd-out of private insurance by public intervention—are the central elements of the model we use to analyze this question.

## II. Optimal Public Insurance with Endogenous Private Insurance

### A. Setup

We analyze a model in which the government uses taxation or social insurance to redistribute income across individuals with different levels of earnings. Risk averse individuals would like to insure themselves against the risk of having low-income realizations. Following Mirrlees (1971), we study a “hidden skill” model, in which the uncertainty about output (skill) levels is resolved before individuals choose effort. To simplify the analysis, we restrict attention to optimal linear contracts throughout this paper. In the working paper version Chetty and Saez (2008), we show that the results proved below in Propositions 1–3 also hold in Hal R. Varian’s (1980) model of taxation as insurance, in which uncertainty about output is resolved after effort is chosen.

Consider an economy with a continuum of individuals of measure one who differ in skill levels  $n$ . Let the distribution of skills be given by a density  $f(n)$ . An individual of ability  $n$  must work  $z/n$  hours to generate output of  $z$ . The government imposes a linear tax rate  $m$  on income, which it uses to finance a lump sum grant  $R$ . The individual’s consumption is therefore  $c = (1 - m)z + R$ . The individual chooses a level of earnings  $z$  after he learns his skill level  $n$ , which is private information, to maximize utility

$$(1) \quad \max_z U(c(z), z; n).$$

<sup>4</sup> See OECD (2008) for statistics on private insurance market shares by country. These numbers must be interpreted cautiously because even though the insurance is provided by private companies, many countries have mandatory insurance requirements.

To simplify exposition, we assume the following form for utility in the theoretical section of the paper:

$$(2) \quad U(c, z, n) = u\left(c - h\left(\frac{z}{n}\right)\right),$$

where  $u(\cdot)$  is increasing and concave, and  $h(\cdot)$  is an increasing and convex function that captures disutility of work. This functional form is convenient because it permits risk aversion while eliminating income effects (Jeremy Greenwood, Zvi Hercowitz, and Gregory W. Huffman 1988; Diamond 1998). With this utility specification, the first-order condition for the agent's choice of  $z$  is  $n(1 - m) = h'(z/n)$ . The choice of  $z$  depends solely on  $1 - m$  and not on  $R$ . Let  $\bar{z} = \int z f(n) dn$  denote average earnings in the population and  $\varepsilon_{\bar{z}, 1-m}$  the elasticity of average earnings  $\bar{z}$  with respect to the net-of-tax rate. This behavioral response reflects the moral hazard distortion created by insurance policies.

We analyze optimal insurance contracts in this model in four steps. First, we characterize the optimal second-best contract, the optimal policy for a single insurer (government or private). This is the problem considered in previous studies that developed sufficient statistic formulas for taxation and social insurance (e.g., Saez 2001, Chetty 2006a). Second, we consider optimal government insurance in the presence of private insurance when private insurance is not necessarily optimized. Third, we consider the special case when private insurance is optimized. Finally, we consider a model where private insurance does not generate moral hazard.

### B. Second-Best Contract Benchmark

Suppose there is a single insurer who offers a linear insurance contract with tax rate  $m$  behind the veil of ignorance (i.e., before individuals learn about their ability  $n$ ). Tax revenue is rebated as a lump sum grant  $m\bar{z}$ . The insurer chooses  $m$  to maximize

$$(3) \quad W = \int u\left(z(1 - m) + m\bar{z} - h\left(\frac{z}{n}\right)\right) f(n) dn.$$

Because  $z$  maximizes individual utility, using the envelope theorem, the first-order condition with respect to  $m$  is:

$$(4) \quad 0 = \frac{dW}{dm} = \int u'(c) \left[ (\bar{z} - z) - m \frac{d\bar{z}}{d(1 - m)} \right] f(n) dn \\ = -\text{cov}(z, u') - \frac{m}{1 - m} \varepsilon_{\bar{z}, 1-m} \bar{u}' \bar{z},$$

where  $\bar{u}' = \int u' f(n) dn$  denotes the average marginal utility and  $\text{cov}(z, u')$  denotes the covariance between earnings  $z$  and marginal utility  $u'$ . Equation (4) yields the standard optimal tax formula:

$$(5) \quad \frac{m}{1 - m} = \frac{1}{\varepsilon_{\bar{z}, 1-m}} \frac{-\text{cov}(z, u')}{\bar{z} \bar{u}'}$$

The formula in (5) can be derived heuristically as follows. Suppose the insurer increases  $m$  by  $dm$ . The direct utility cost for individual with earnings  $z$  is  $-u'(c) z dm$ . The behavioral response to  $dm$  does not generate a first order effect on utility because of the envelope theorem. Therefore, in aggregate, the direct welfare cost is  $dW = -dm \int u' z f(n) dn$ . The mechanical increase in tax revenue (ignoring behavioral responses) due to  $dm$  is  $dM = \bar{z} dm$ . The behavioral response in earnings reduces tax revenue by  $dB = -m d\bar{z} = (-m/(1-m)) \varepsilon_{\bar{z}, 1-m} \bar{z} dm$ . Hence, the lump sum grant increases by  $dM + dB$ , increasing welfare by  $\bar{u}'(dM + dB) = \bar{u}' \bar{z} [1 - (m/(1-m)) \varepsilon_{\bar{z}, 1-m}] dm$ . At the optimum, these effects must all cancel so that  $dW + \bar{u}'(dM + dB) = 0$ , which yields (5).

### C. General Case: Private Insurance Not Necessarily Optimized

We now turn to the problem of optimal government insurance with endogenous private insurance. We begin by introducing notation to distinguish the private and government insurance contracts. Let  $\tau$  denote the tax rate chosen by the government and  $t$  the tax rate in the private insurance contract. Private insurance applies to raw output  $z$ . We denote by  $w = (1-t)z + t\bar{z}$  the net-of-private insurance income. Government taxation applies to the net incomes  $w$ , and we denote by  $c = (1-\tau)w + \tau\bar{w}$  final disposable income.

Concretely, the private insurer can be thought of as a firm that compresses its wage structure ( $w$ ) relative to true marginal products ( $z$ ) to provide insurance. The government can only observe earnings, not true underlying marginal products, and hence sets taxes as a function of  $w$ .<sup>5</sup>

Let  $m$  denote the total tax rate on output, defined such that  $1-m = (1-t)(1-\tau)$  and  $c = (1-m)z + m\bar{z}$ . If the private insurer and government cooperated to set  $m$  to maximize social welfare, the resulting contract would be identical to that described in the single insurer setting above. However, in practice, private insurers take the government contract  $\tau$  as given when they choose  $t$ . Let  $t(\tau)$  denote the private insurer's choice of  $t$  as a function of the government tax. In this subsection, we take the function  $\tau \rightarrow t(\tau)$  as given, and do not assume that  $t(\tau)$  is chosen optimally to maximize the agent's expected utility. The level of private insurance level might not be optimized because of individual failures. Left to their own devices, individuals may purchase too little insurance. For example, individuals could be too optimistic about the probability they will obtain a high-skill level ( $n$ ). As a result,  $t(\tau)$  differs from the optimal level that would maximize the agent's expected utility in the presence of overconfidence. Let  $r = -d \log(1-t)/d \log(1-\tau)$  denote the empirically observed rate at which public insurance crowds out private insurance. If  $r = 0$ , there is no crowd-out. If  $r = 1$ , there is perfect crowd-out.

The government chooses  $\tau$  to maximize the agent's expected utility (3), taking into account the private insurer's response. Let  $\varepsilon_{\bar{w}, 1-\tau} = d \log \bar{w} / d \log(1-\tau)$  denote the elasticity of average (post insurance) earnings  $\bar{w}$  with respect to  $1-\tau$ , taking

<sup>5</sup> Because the individuals' effort decision depends solely on the return net of all taxes  $(1-t)(1-\tau)$ , the analysis below goes through with no changes if government taxes are levied on the true marginal products  $z$  and private insurance is based on net-of government tax incomes.

into account the endogenous response of private insurance  $t$  to a change in  $\tau$ . The following proposition characterizes the marginal welfare gain from increasing the tax rate ( $dW/d\tau$ ) in terms of empirically estimable elasticities.

PROPOSITION 1: *The welfare gain from raising the government tax is*

$$(6) \quad \frac{dW}{d\tau} = -(1-r)\bar{w} \left[ \left( \frac{\tau}{1-\tau} + t \right) \frac{\varepsilon_{\bar{w},1-\tau}}{1-r} + \frac{\text{cov}(w, u')}{\bar{w} \bar{u}'} \right],$$

and the optimal tax rate satisfies

$$(7) \quad \frac{\tau}{1-\tau} = -t + \frac{1-r}{\varepsilon_{\bar{w},1-\tau}} \frac{-\text{cov}(w, u')}{\bar{w} \bar{u}'}$$

PROOF:

This problem is identical to (3). Hence, (4) and (5) remain valid. The government sets  $\tau$  so that the total tax rate  $m$  satisfies the standard formula. Because the government does not observe  $z$  directly, it is useful to rewrite (4) and (5) as a function of  $w$  instead of  $z$ . To do this, first note that  $\bar{w} = \bar{z}$  and hence  $\varepsilon_{\bar{w},1-m} = \varepsilon_{\bar{z},1-m}$ . Second,  $\text{cov}(w, u') = \text{cov}(z(1-t) + t\bar{z}, u') = (1-t)\text{cov}(z, u')$ . Third,  $\varepsilon_{\bar{w},1-\tau} = \varepsilon_{\bar{w},1-m} (1-r)$  as a 1 percent increase in  $1-\tau$  translates into a  $1-r$  percent increase in  $1-m$  because of crowd-out effects. Similarly,  $(dm/d\tau) = (1-r)(1-t)$  and hence,  $(dW/d\tau) = (1-r)(1-t) (dW/dm)$ . Finally  $m/(1-m) = [\tau/(1-\tau) + t]/(1-t)$ . Using these expressions, we can rewrite (4) as (6) and (5) as (7).

Proposition 1 shows that private insurance affects the formula for the optimal tax rate in two ways. First, the added  $-t$  term on the right side of (7) reflects the mechanical reduction in the optimal level of government taxation given the presence of private insurance. Formula (7) shows that the *sum* of private and public insurance should be set according to the standard formula, and hence the optimal  $\tau$  is reduced in proportion to  $t$ . The second effect is that the inverse elasticity term is multiplied by  $1-r$ . Since  $r > 0$ , this effect also makes the optimal government tax rate smaller. Intuitively, the elasticity relevant for optimal taxation is the fundamental elasticity of output with respect to total taxes  $\varepsilon_{\bar{z},1-m} = \varepsilon_{\bar{w},1-m}$ , which measures the total moral hazard cost (to both the private and public insurers) of redistributing \$1 of tax revenue. To recover the fundamental output elasticity in the presence of crowd-out, one must rescale the observed elasticity  $\varepsilon_{\bar{w},1-\tau}$  by  $1/(1-r)$ .<sup>6</sup>

An important implication of (7) is that if the wage structure already reflects implicit insurance provided by the private sector, then standard optimal tax formulas that are functions of the observed wage distributions and wage earnings elasticities are invalid. Intuitively, the private sector already bears a moral hazard cost for the

<sup>6</sup> Therefore, the crowd-out rate  $r$  only matters for rescaling the observed elasticity, and plays no fundamental economic role in the formula. If one could measure the fundamental elasticity  $\varepsilon_{\bar{w},1-m}$  directly, an estimate of crowd-out would be unnecessary.

insurance it provides. The government exacerbates this pre-existing distortion by introducing additional insurance. Therefore, one must take into account the amount of private insurance to back out the optimal policy.<sup>7</sup>

In the simple model considered here, the planner can replicate the second-best optimal allocation by choosing the level of  $\tau$  that generates the optimal  $m$  given the private insurer's response. The second best can be achieved because private and public insurance are identical tools in this model; the role for government emerges if private insurance is not set optimally. When the government and private sector have different tools, as in the adverse selection model analyzed later in this paper, the second best level of welfare can no longer be achieved.

#### D. Special Case: Optimized Private Insurance

To gain further insight into the effects of government intervention and connect our results to the existing literature, we now consider the special case where private insurance is chosen optimally to maximize expected utility. The following proposition characterizes the effects of government intervention on social welfare when private insurance is optimized:

PROPOSITION 2: *If private insurance is set optimally,*

- *The optimal government tax rate is  $\tau = 0$ , and the welfare cost of introducing a small tax  $\tau$  is second-order:  $(dW/d\tau) (\tau = 0) = 0$ .*
- *Government intervention strictly reduces welfare when  $\tau$  is positive:  $(dW/d\tau) (\tau > 0) < 0$ .*
- *The effect of a tax increase on welfare is given by*

$$(8) \quad \frac{dW}{d\tau} = -\varepsilon_{\bar{z}, 1-\tau} \frac{\tau}{1-\tau} \bar{u}' \bar{z},$$

where  $\varepsilon_{\bar{z}, 1-\tau}$  is the elasticity of average earnings with respect to  $1 - \tau$ , taking into account the endogenous response of  $t$  to  $\tau$ .

PROOF:

*Optimal Private Contract.*—In a competitive market in which insurers are price takers, the private insurer takes the government tax rate  $\tau$  and the lump sum grant  $R = \tau \bar{w}$  as given when setting  $t$ . Therefore,  $t$  is chosen to maximize:

$$(9) \quad W = \int u \left( z(1-t)(1-\tau) + t(1-\tau)\bar{z} + R - h\left(\frac{z}{n}\right) \right) f(n) \, dn.$$

<sup>7</sup> In redistributive taxation applications, it is challenging to measure the private insurance rate  $t$  because the difference between  $w$  and  $z$  is not directly observed. Conceptually, one must estimate the difference between observed wages and marginal products. In social insurance applications, private insurance typically involves explicit transfers, and can therefore be measured directly as we shall see in the applications section.

Using the envelope condition for individuals'  $z$ , the first-order condition with respect to  $t$  is:

$$(10) \quad 0 = \frac{dW}{dt} \Big|_{R,\tau} = -(1-\tau) \int z u' f(n) dn + (1-\tau) \bar{z} \bar{u}' \\ + t(1-\tau) \bar{u}' \frac{d\bar{z}}{dt} \Big|_{R,\tau}.$$

Because, there are no income effects, we have  $\varepsilon_{\bar{z},1-t|R,\tau} = \varepsilon_{\bar{z},1-m}$ , and hence

$$(11) \quad \frac{t}{1-t} = \frac{1}{\varepsilon_{\bar{z},1-m}} \frac{-\text{cov}(z, u')}{\bar{z} \bar{u}'}$$

This expression shows that the private insurer follows a rule analogous to the second-best single insurer choice in (5) in setting  $t$ .

*Effect of Government Intervention with Optimized Private Insurance.*—Noting that the government grant is  $R = \tau \bar{z}(1-m)$ , it follows from (9) that

$$W = \int u \left( z(1-m) + m\bar{z} - h\left(\frac{z}{\bar{n}}\right) \right) f(n) dn.$$

Using the envelope condition for  $z$  and the equation  $dm/d\tau = (1-t)(1-r)$ , we obtain

$$\frac{dW}{d\tau} = (1-t)(1-r) \bar{u}' \bar{z} \left[ \frac{-\text{cov}(z, u')}{\bar{z} \bar{u}'} - \frac{m}{1-m} \varepsilon_{\bar{z},1-m} \right].$$

The first term in the square bracket can be rewritten using the first-order-condition in (11) to obtain

$$\frac{dW}{d\tau} = (1-t)(1-r) \bar{u}' \bar{z} \varepsilon_{\bar{z},1-m} \left[ \frac{t}{1-t} - \frac{m}{1-m} \right].$$

Finally, observing that  $t/(1-t) - m/(1-m) = -\tau/((1-\tau)(1-t))$  and that  $\varepsilon_{\bar{z},1-\tau} = (1-r)\varepsilon_{\bar{z},1-m}$ , we obtain (8) which proves the third point in Proposition 2. The first two points in Proposition 2 follow immediately from the third.

This proposition shows that the standard lessons of the theory of excess burden apply in our model. The welfare cost of taxation is proportional to the size of the behavioral response to taxation, and the marginal cost of taxation increases linearly with the tax rate. However, there is one important difference relative to the traditional analysis. In standard models of taxation and social insurance without endogenous private insurance, the deadweight burden of taxation is an efficiency cost that the government would be willing to trade-off against the benefits of more insurance. In the present model, the level of redistribution through market insurance is already optimal

given incentive constraints, and thus the net welfare gain *equals* the deadweight burden of taxation. Hence, a benevolent government should do precisely nothing.

When private insurance markets function optimally, government insurance via taxation or social insurance *strictly* reduces welfare because crowd-out of private insurance is imperfect. To understand the intuition, it is helpful to consider an example. Suppose the government naïvely thinks it can achieve the second-best optimum by setting  $\tau$  equal to  $m$  given by the standard formula (5). This tax system would achieve the optimum described in Section IIA if the private insurance market chose not to provide any insurance. However, zero private insurance is not the contract that will emerge in an economy with a private insurance market. Since  $\tau < 1$ , redistribution is incomplete and hence  $-\text{cov}(z, u') > 0$ , and hence (11) implies that  $t > 0$ , i.e., the private insurer does offer additional insurance. This additional insurance would increase the expected utility of the agent and the insurance companies would break even. But the individual and private insurer do not internalize the effect of their choices on the government's budget. The added private insurance reduces effort below the second-best optimum and hence leads to a lower average output than the government was expecting. As a result, the government goes into deficit and its lump sum grant  $R$  needs to be reduced.<sup>8</sup>

We are not the first to make the point that government intervention is undesirable when private insurance markets function efficiently. When private insurance is set optimally, the market equilibrium with no government intervention is information-constrained Pareto efficient. Abstractly, it is well known that when the private market equilibrium is constrained efficient, government price distortions lead to inefficiency (Edward C. Prescott and Townsend 1984a, 1984b). Kaplow (1991) makes a similar point about the welfare losses from government intervention in a simple social insurance model, and Blomqvist and Johansson (1997) and Barigozzi (2006) obtain similar results in a model with public and private health insurance. The main contribution of the present paper is to develop formulas for the welfare gains from government intervention when the private market does not reach the second-best optimum.

Three additional remarks on Proposition 2 deserve mention. First, we expect that the relationship between the degree of crowd-out  $r$  and risk aversion will generally have an inverted U-shape. When individuals are risk neutral ( $u(c) = c$ ), there is no private insurance ( $t = 0$ ), so there is by definition no crowd-out ( $r = 0$ ). At the other extreme, with infinite risk aversion, and assuming that the support of  $z$  includes zero,  $\text{cov}(z, u') = -\bar{z} \bar{u}'$  and (11) implies that  $t/(1-t) = 1/\varepsilon_{z,1-m}$ . If the elasticity does not vary with the net-of-tax rate  $1-m$ , the private insurance rate  $t$  is independent of  $\tau$ . Therefore, in the case of infinite risk aversion, there is also no crowd-out. For any interior case with nonzero, finite risk aversion, crowd-out will be positive and incomplete. Hence, government intervention is likely to induce the greatest welfare losses when risk aversion is very low or very high, because these are the cases in which crowd-out will be small and individuals will end up being most over-insured.

<sup>8</sup> These results contrast with the analysis of Golosov and Tsyvinski (2007), who argue that government insurance has no effect on social welfare in static models because of 100 percent crowd-out. The reason for the difference is that Golosov and Tsyvinski (2007) study government intervention, where the government controls directly the final consumption outcomes, whereas we consider the effects of distortionary taxation, which is typically the nature of policies used in practice.

Second, a corollary of Proposition 2 is that in the absence of private market failures, public goods should be financed via a uniform lump-sum tax that generates the desired amount of revenue, even if agents have different marginal utilities of income in each state. The private insurance market will then set redistribution to the optimal level. A distortionary tax to finance the public good would lead to lower expected utility than a lump-sum tax that generated an equivalent amount of revenue. This point underscores the substantial effect that endogenous private insurance arrangements can have on standard intuitions in the literatures on optimal taxation and social insurance.

Third, if there are fixed administrative costs of setting up an insurance policy, small private insurers will typically have higher administrative costs per capita than a single social insurance scheme. In such an environment, the optimum would be for the government to be the sole insurance provider. However, because of the effects discussed in Proposition 2, private insurers will generally provide positive amounts of insurance in equilibrium even though they have higher administrative costs than the government. Reaching the second-best optimum therefore requires *forbidding* private insurance with administrative costs.

#### E. Private Insurance Without Moral Hazard

Thus far, we have considered private insurance contracts that generate the same amount of moral hazard as public insurance. However, “informal” private risk-sharing mechanisms may involve less moral hazard. Examples include self insurance through spousal labor supply or insurance through relatives and neighbors who can monitor effort. Firms themselves may provide insurance against shocks that involves less moral hazard by conditioning wage payments on noisy signals of effort rather than purely on output. In this section, we explore how the degree of moral hazard in private insurance affects our formulas.

When private insurance does not generate moral hazard, reaching the first-best of full insurance would be feasible in principle, completely eliminating the role for government intervention. In practice, there are costs of informal insurance, such as limits to liquidity, costs of borrowing from relatives, or relying on spousal labor supply, that prevent full insurance. We model such costs by a loading factor on informal insurance transfers. Informal private insurers can implement lump-sum taxes and transfers  $T_n$  to individuals with ability  $n$  by paying a loading cost  $s(T_n)$ . We assume that  $s(\cdot)$  is convex with  $s(0) = 0$  and  $s(T) > 0$  for  $T \neq 0$  to ensure that the private insurance optimization problem is concave. The government applies a tax at rate  $\tau$  on net-of-insurance earnings  $w = z - (T + s(T))$  and rebates taxes collected with a lump sum  $R = \tau \bar{w}$ .

**PROPOSITION 3:** *Irrelevance of informal private insurance for optimal social insurance formulas. If informal private insurance is optimized, the optimal government tax rate  $\tau$  is given by*

$$(12) \quad \frac{\tau}{1 - \tau} = \frac{1}{\varepsilon_{\bar{w}, 1 - \tau}} \frac{-\text{cov}(w, u')}{\bar{w} \bar{u}'}$$

## PROOF:

The private insurer and individuals take  $\tau$  and  $R = \tau \bar{w}$  as given, and simultaneously choose  $z_n$  and  $T_n$  to maximize

$$\int u \left[ [z_n - (T_n + s(T_n))](1 - \tau) + R - h\left(\frac{z_n}{n}\right) \right] f(n) \, dn$$

subject to the budget constraint  $\int T_n f(n) \, dn \geq 0$ . We denote by  $\lambda$  the multiplier of this private insurance constraint. The first-order condition for  $z_n$  implies that  $h'(z_n/n) = n(1 - \tau)$ , i.e., the government tax reduces incentives to work, but the amount of informal insurance does not. The first-order condition for  $T_n$  implies that  $u'(c_n) (1 + s'(T_n)) (1 - \tau) = \lambda$ : marginal utility is equalized up to the loading factor.

The government chooses  $\tau$  to maximize

$$W(\tau) = \int u \left[ [z_n - (T_n + s(T_n))](1 - \tau) + \tau \bar{w} - h\left(\frac{z_n}{n}\right) \right] f(n) \, dn.$$

Using the envelope conditions for  $z_n$ , we have

$$\begin{aligned} \frac{dW}{d\tau} &= \int u' \left[ -w + \bar{w} - \tau \frac{d\bar{w}}{d(1 - \tau)} - (1 + s'(T_n))(1 - \tau) \frac{dT_n}{d\tau} \right] f(n) \, dn \\ &= \left[ -\bar{w} \bar{u}' \frac{\tau}{1 - \tau} \varepsilon_{\bar{w}, 1 - \tau} + \bar{w} \bar{u}' - \int w u' f(n) \, dn \right] - \lambda \int \frac{dT_n}{d\tau} f(n) \, dn, \end{aligned}$$

where the last term is obtained using  $u'(c_n) (1 + s'(T_n)) (1 - \tau) = \lambda$ . The last term cancels out as  $\int T_n f(n) \, dn = 0$  is constant with  $\tau$ . Hence, we obtain, again, the standard formula (12).

Why does only private insurance that generates moral hazard change the formulas for optimal taxation? When private insurance generates moral hazard, the changes in effort induced by government intervention have a first-order externality on the private insurer's budget and must therefore be taken into account directly in the formula. If private insurance does not generate moral hazard, the fiscal externality term disappears because effort is chosen jointly with  $T$  to optimize  $W$ . Thus, the informal insurance mechanisms emphasized in the structural models of Attanasio and Rios-Rull (2000) and Attanasio, Hamish Low, and Virginia Sánchez-Marcos (2005) are already taken into account in existing optimal tax and social insurance formulas.

The irrelevance of informal insurance challenges the existing literature, which views crowd-out of all types of private insurance as reducing the welfare gains from government intervention equally. Informal insurance reduces the welfare gains from social insurance simply by reducing the empirically observed correlation between  $u'$  and  $w$  in (12).<sup>9</sup> In contrast, formal insurance reduces this correlation and creates a

<sup>9</sup> This result is consistent with Chetty (2006a), who shows that Martin Neil Baily's (1978) formula is robust to allowing for arbitrary choices in the private sector as long as they are constrained efficient.

fiscal externality, further reducing the welfare gain from government intervention. It is therefore important to distinguish crowd-out of the two types of private insurance for policy analysis.

### III. Failures in Private Insurance Markets

In this section, we extend our formulas to allow for two market failures: pre-existing information and adverse selection. Pre-existing information refers to *public* information about individuals' ability before insurance contracts are signed. Adverse selection refers to *private* information about individuals' ability before insurance contracts are signed.

#### A. Pre-Existing Information

We model pre-existing public information as follows. The population is partitioned into  $K$  exogenous groups  $k = 1, \dots, K$ . For instance, the groups could represent health status or region of birth. Group  $k$  contains a fraction  $p_k$  of the population and has (conditional) density of abilities  $f_k(n)$  so that  $f(n) = \sum_k p_k f_k(n)$ . Group identity is known by both private insurers and individuals before signing insurance contracts, but the realization of  $n$  within a group is unknown to both insurers and individuals when signing the contract. As a result, private insurance is offered *within* each group  $k$ . The private insurer offers an insurance rate  $t_k$  conditional on group  $k$ . When there is a single group, this corresponds to the model in Section II. At the other extreme, when the number of groups is as large as the number of individuals, all information is revealed before contracts can be signed, and there is no scope for private insurance contracts. In contrast, the government can impose redistributive taxation on the entire population.

We assume that the government is restricted to using a uniform tax rate  $\tau$  on earnings. This is a strong assumption in the present model because the government could potentially improve welfare by conditioning tax rates on groups, but it is useful to gain insight into the key features of the problem.<sup>10</sup>

We now introduce notation for the multiple-group case. Let  $m_k$  denote the total tax rate in group  $k$  such that  $1 - m_k = (1 - \tau)(1 - t_k)$ . Individual  $n$  in group  $k$  optimally chooses earnings such that  $h'(z/n) = (1 - m_k)n$ . We denote by  $\bar{z}_k = \int z f_k(n) dn$  average earnings in group  $k$  and by  $\varepsilon_k = ((1 - m_k)/\bar{z}_k) (d\bar{z}_k/d(1 - m_k))$  the elasticity of  $\bar{z}_k$  with respect to  $1 - m_k$ . As in Section II, let  $w = (1 - t_k)z + t_k \bar{z}_k$  denote earnings post private insurance (but before government taxation). Note that  $\bar{w}_k = \bar{z}_k$  and  $\bar{w} = \bar{z}$  where  $\bar{w}$  and  $\bar{z}$  are the full population means as in Section II. Let  $c = (1 - \tau)w + \tau \bar{w} = (1 - \tau)(1 - t_k)z + t_k(1 - \tau)\bar{z}_k + \tau \bar{z}$  denote disposable income after government taxation.

The crowd-out rate  $r_k$  of private insurance in group  $k$  is  $1 - r_k = ((1 - \tau)/(1 - m_k)) \times (d(1 - m_k)/d(1 - \tau))$ . Note that this implies that  $d(1 - m_k)/d(1 - \tau) = (1 - t_k)(1 - r_k)$  so that we have  $dm_k/d\tau = (1 - t_k)(1 - r_k)$ . It also implies that

<sup>10</sup> In practice, horizontal equity considerations appear to prevent the government from using pre-existing public information such as height, age, family background, or education for tax or redistribution purposes.

$((1 - \tau)/(1 - t_k)) (d(1 - t_k)/d(1 - \tau)) = d \log(1 - t_k)/d \log(1 - \tau) = -1 + (d \log(1 - m_k)/d \log(1 - \tau)) = -1 + 1 - r_k = -r_k$  so that we have  $dt_k/d\tau = -r_k (1 - t_k)/(1 - \tau)$ . As above, let  $\varepsilon_{\bar{w}, 1-\tau} = ((1 - \tau)/\bar{z}) (d\bar{z}/d(1 - \tau)) = ((1 - \tau)/\bar{w}) (d\bar{w}/d(1 - \tau))$  denote the elasticity of aggregate earnings with respect to the net-of-government tax rate  $1 - \tau$ , taking into account crowd-out. Note also that, as  $1 - r_k = ((1 - \tau)/(1 - m_k)) (d(1 - m_k)/d(1 - \tau))$ , the elasticity of  $\bar{z}_k$  with respect to  $1 - \tau$  is  $((1 - \tau)/\bar{z}_k) (d\bar{z}_k/d(1 - \tau)) = (1 - r_k)\varepsilon_k$ .

We denote by  $\text{cov}(\bar{u}'_k, \bar{w}_k) = \sum_k p_k \bar{u}'_k [\bar{w}_k - \bar{w}]$  the covariance between average marginal utilities ( $\bar{u}'_k$ ) and wages ( $\bar{w}_k$ ) across the  $K$  groups. Let  $\text{cov}_k(u', w) = \int u'(w - \bar{w}_k) f_k(n) dn$  denote the covariance between  $u'$  and  $w$  within group  $k$ . Finally, in the case where private insurance does not generate moral hazard, it has the same lump-sum and loading factor design as in Section IIE within each group  $k$ .

The government chooses  $\tau$  to maximize social welfare under a utilitarian criterion:

$$W = \sum_k p_k \int u \left( z(1 - m_k) + t_k(1 - \tau)\bar{z}_k + \tau\bar{z} - h\left(\frac{z}{n}\right) \right) f_k(n) dn.$$

The following proposition characterizes the solution to this problem.

PROPOSITION 4:

- In the general case where private insurance is not necessarily optimized, the optimal government tax rate  $\tau$  satisfies

$$(13) \quad \frac{\tau}{1 - \tau} = - \frac{1}{\varepsilon_{\bar{w}, 1-\tau}} \left[ \sum_k p_k (1 - r_k) \frac{\text{cov}_k(u', w) + \varepsilon_k t_k \bar{u}'_k \bar{w}_k}{\bar{u}' \bar{w}} + \frac{\text{cov}(\bar{u}'_k, \bar{w}_k)}{\bar{u}' \bar{w}} \right].$$

- When private insurance is optimized within each group,  $\text{cov}_k(u', w) + \varepsilon_k t_k \bar{u}'_k \bar{w}_k = 0$  for each  $k$ , and hence the optimal government tax rate  $\tau$  satisfies

$$(14) \quad \frac{\tau}{1 - \tau} = - \frac{1}{\varepsilon_{\bar{w}, 1-\tau}} \frac{\text{cov}(\bar{u}'_k, \bar{w}_k)}{\bar{u}' \bar{w}}.$$

- When private insurance does not generate moral hazard, the optimal government tax rate  $\tau$  follows the standard formula:

$$(15) \quad \frac{\tau}{1 - \tau} = - \frac{1}{\varepsilon_{\bar{w}, 1-\tau}} \frac{\text{cov}(u', w)}{\bar{u}' \bar{w}}.$$

PROOF:

We start with the general case. In the first-order condition with respect to  $\tau$  for the government, using the envelope condition for individual  $z$ , we can ignore the effects of  $\tau$  on individual  $z$ , so that we have

$$0 = \frac{dW}{d\tau} = \sum_k p_k \int u'(c) \left[ -z \frac{dm_k}{d\tau} - t_k \bar{z}_k + (1 - \tau) \frac{dt_k}{d\tau} \bar{z}_k + t_k(1 - \tau) \frac{d\bar{z}_k}{d\tau} + \bar{z} + \tau \frac{d\bar{z}}{d\tau} \right] f_k(n) dn.$$

Using  $dm_k/d\tau = (1 - t_k)(1 - r_k)$  and  $dt_k/d\tau = -r_k(1 - t_k)/(1 - \tau)$  obtained above, we can rewrite the first-order condition as:

$$0 = \sum_k p_k \int u'(c) \left[ -z(1 - t_k)(1 - r_k) - t_k \bar{z}_k - r_k(1 - t_k) \bar{z}_k \right. \\ \left. + t_k(1 - \tau) \frac{d\bar{z}_k}{d\tau} + \bar{z} + \tau \frac{d\bar{z}}{d\tau} \right] f_k(n) \, dn.$$

Rewriting each term (in the same order), and noting that  $-t_k - r_k(1 - t_k) = (1 - t_k) \times (1 - r_k) - 1$ , we obtain

$$0 = \sum_k p_k \left[ -(1 - t_k)(1 - r_k) \int u' z f_k \right. \\ \left. + \left\{ [(1 - t_k)(1 - r_k) - 1] \bar{z}_k - t_k(1 - \tau) \frac{d\bar{z}_k}{d(1 - \tau)} \right\} \int u' f_k \right] \\ + \left[ \bar{z} - \tau \frac{d\bar{z}}{d(1 - \tau)} \right] \int u' f(n) \, dn.$$

Using the elasticities discussed above,  $\varepsilon_{\bar{w}, 1-\tau} = ((1 - \tau)/\bar{z}) (d\bar{z}/d(1 - \tau))$  and  $\varepsilon_k = ((1 - m_k)/\bar{z}_k) (d\bar{z}_k/d(1 - m_k))$ , and noting that  $((1 - \tau)/\bar{z}_k) (d\bar{z}_k/d(1 - \tau)) = (1 - r_k) \varepsilon_k$ , we can rewrite the first-order condition as:

$$0 = \sum_k p_k \left[ (1 - t_k)(1 - r_k) \left\{ - \int u' z f_k + \bar{z}_k \int u' f_k \right\} \right. \\ \left. + \left\{ -\bar{z}_k - t_k(1 - r_k) \varepsilon_k \bar{z}_k \right\} \int u' f_k \right] \\ + \left[ 1 - \frac{\tau}{1 - \tau} \varepsilon_{\bar{w}, 1-\tau} \right] \bar{z} \int u' f(n) \, dn.$$

Using the average marginal utilities  $\bar{u}' = \int u' f$  and  $\bar{u}'_k = \int u' f_k$ , and the covariance  $\text{cov}_k(z, u') = \int u' z f_k - \bar{z}_k \int u' f_k$ , we obtain

$$0 = - \sum_k p_k (1 - t_k)(1 - r_k) \text{cov}_k(z, u') - \sum_k p_k \bar{u}'_k \bar{z}_k \\ - \sum_k p_k \bar{u}'_k \bar{z}_k t_k (1 - r_k) \varepsilon_k + \bar{u}' \bar{z} - \frac{\tau}{1 - \tau} \varepsilon_{\bar{w}, 1-\tau} \bar{u}' \bar{z}.$$

Reordering terms, we obtain

$$\begin{aligned} \frac{\tau}{1-\tau} \varepsilon_{\bar{w}, 1-\tau} \bar{u}' \bar{z} &= \bar{u}' \bar{z} - \sum_k p_k \bar{u}'_k \bar{z}_k \\ &\quad - \sum_k p_k (1-t_k)(1-r_k) \text{cov}_k(z, u') \\ &\quad - \sum_k p_k \bar{u}'_k \bar{z}_k t_k (1-r_k) \varepsilon_k. \end{aligned}$$

Note that  $\bar{z} = \bar{w}$ ,  $\bar{z}_k = \bar{w}_k$ , and  $\text{cov}_k(w, u') = (1-t_k) \text{cov}_k(z, u')$ . Hence, using  $\text{cov}(\bar{u}'_k, \bar{w}_k) = \sum_k p_k \bar{u}'_k \bar{w}_k - \bar{u}' \bar{w}$ , we finally obtain

$$\frac{\tau}{1-\tau} \varepsilon_{\bar{w}, 1-\tau} \bar{u}' \bar{w} = -\text{cov}(\bar{u}'_k, \bar{w}_k) - \sum_k p_k (1-r_k) [\text{cov}_k(w, u') + \varepsilon_k t_k \bar{u}'_k \bar{w}_k],$$

which demonstrates (13).

When the private insurer sets  $t_k$  optimally, it takes  $\tau$  and  $R = \tau \bar{z}$  as given and chooses  $t_k$  to maximize

$$\int u \left( z (1-t_k)(1-\tau) + t_k(1-\tau) \bar{z}_k + R - h\left(\frac{z}{n}\right) \right) f_k(n) \, dn.$$

This is the same problem as in Section IID, and the first-order condition leads to the same formula:

$$\frac{t_k}{1-t_k} = \frac{-\text{cov}_k(z/\bar{z}_k, u'/\bar{u}'_k)}{\varepsilon_k},$$

which can be rewritten as  $t_k \varepsilon_k = -\text{cov}_k(w/\bar{w}_k, u'/\bar{u}'_k)$ . Therefore, the first term in the square bracket expression in (13) vanishes, yielding the simpler formula (14).

If there is no moral hazard in the private insurer choice, then, as in Section IID, lump-sum transfers  $T_n^k$  and  $z_n$  are chosen simultaneously to maximize

$$\int u \left[ \left[ z_n - (T_n^k + s_k(T_n^k)) \right] (1-\tau) + R - h\left(\frac{z_n}{n}\right) \right] f_k(n) \, dn,$$

subject to the budget constraint  $\int T_n^k f_k(n) \, dn \geq 0$  (with multiplier  $\lambda_k$ ). The first-order condition for  $T_n^k$  implies that  $u'(c_n^k) (1 + s'_k(T_n^k)) (1-\tau) = \lambda_k$ . The government chooses  $\tau$  to maximize

$$W(\tau) = \sum_k p_k \int u \left[ \left[ z_n - (T_n^k + s_k(T_n^k)) \right] (1-\tau) + \tau \bar{w} - h\left(\frac{z_n}{n}\right) \right] f_k(n) \, dn.$$

Using the envelope conditions for  $z_n$ , and noting that  $w_n = z_n - (T_n^k + s_k(T_n^k))$ , we have

$$\begin{aligned} \frac{dW}{d\tau} &= \sum_k p_k \int u' \left[ -w + \bar{w} - \tau \frac{d\bar{w}}{d(1-\tau)} - (1 + s'_k(T_n^k))(1-\tau) \frac{dT_n^k}{d\tau} \right] f_k(n) \, dn \\ &= -\bar{w} \bar{u}' \left[ \frac{\tau}{1-\tau} \varepsilon_{\bar{w}, 1-\tau} + \text{cov}(w/\bar{w}, u'/\bar{u}') \right] - \sum_k p_k \lambda_k \int \frac{dT_n^k}{d\tau} f_k(n) \, dn. \end{aligned}$$

The last term cancels out as  $\int T_n^k f_k(n) \, dn = 0$  for each  $k$  and any value of  $\tau$ , yielding (15).

Note that the three points in Proposition 4 nest exactly the results obtained in Propositions 1, 2, 3 in the special case where  $K = 1$ , i.e., when there is no pre-existing information. Equation (13) shows that the welfare gain from government intervention in this more general model consists of two elements: (1) increased insurance within groups, which is captured by parameters analogous to those analyzed in the baseline case, and (2) increased insurance across groups, which reflects the gains from pooling risk across types. In the present model, (1) only is operative if private insurance is not optimized; if it was, this term would be zero and only the second term is operative.

When private insurance is optimized, the optimal tax rate follows a modified formula in which individuals in group  $k$  are treated as a *single* individual with marginal utility and earnings equal to the average in group  $k$ . Intuitively, private insurance takes care of within group insurance as well as the government could. Therefore, the government should focus solely on redistribution *across* groups, which cannot be insured by the private sector because of pre-existing information. Finally, (15) shows that the government can ignore “informal” private insurance that does not generate moral hazard and apply the standard optimal tax formula.

### B. Adverse Selection

Now suppose that each individual knows which group he belongs to before signing insurance contracts, but insurers do not have this information. This model is an extension of the classic adverse selection models proposed by Michael Rothschild and Joseph E. Stiglitz (1976) and Charles A. Wilson (1977). If private insurers were to offer a menu of competitive insurance contracts with insurance rates  $(t_1, \dots, t_M)$ , individuals in group  $k$  would self-select the insurance rate that yields the highest expected utility and private insurers would make negative profits. Hence, private insurers are forced to adjust contracts to respect incentive compatibility constraints. A strong equilibrium is defined as a set of contracts such that no firm can make positive profits by offering a new insurance contract. When such strong equilibria exist, they are always separating in the sense that each group  $k$  selects a single and specific insurance rate  $t_k$  (Rothschild and Stiglitz 1976). When such strong equilibria do not exist, it is necessary to weaken the concept of equilibrium to obtain existence of an equilibrium (Wilson 1977).

Here, we restrict attention to the region of the parameter space where an equilibrium exists. In this equilibrium, each group  $k$  is offered private insurance at tax rate  $t_k$ . The  $t_k$  can be common across some groups if there is partial pooling. Furthermore, the  $t_k$  depend on the government tax rate  $\tau$ . Assume that the  $t_k$  are smooth functions of  $\tau$  so that crowd-out rates  $r_k$  are well defined and welfare gains are smooth.<sup>11</sup> The revelation principle implies that in equilibrium each individual will truthfully reveal his type  $k$ . Hence, the equilibrium in this model can be viewed as a special case of the pre-existing information model analyzed above, where private insurance contracts were set arbitrarily. It follows that the general formula (13) in Proposition 4 holds in this environment.

Adverse selection does, however, affect the formula with optimized private insurance (14) because it distorts the provision of private insurance within groups. This leads to a violation of the within-group private insurance optimality condition  $t_k^* = -\text{cov}_k(w/\bar{w}_k, u'/\bar{u}'_k)/\varepsilon_k$ . To obtain further intuition, rewrite (13) as:

$$(16) \quad \frac{\tau}{1-\tau} = -\frac{1}{\varepsilon_{\bar{w},1-\tau}} \frac{\text{cov}(\bar{u}'_k, \bar{w}_k)}{\bar{u}'\bar{w}} + \frac{1}{\varepsilon_{\bar{w},1-\tau}} \sum_k p_k (1-r_k) \varepsilon_k (t_k^* - t_k) \frac{\bar{u}'_k \bar{w}_k}{\bar{u}'\bar{w}},$$

where  $t_k^*$  is the optimal private insurance rate in the absence of adverse selection constraints. There are two components in (16). The first term reflects the value of redistribution across the groups, as in (14). The second term reflects the value of fixing the distortions created by adverse selection *within* each group. Typically, adverse selection leads to under-provision of private insurance:  $t_k < t_k^*$  (Rothschild and Stiglitz 1976). Hence, there is value to increasing within-group redistribution even if private contracts are optimized in the presence of adverse selection.

More generally, (16) implies that the government tax rate  $\tau$  should be *higher* than in the case with no adverse selection (14) because it fixes two market failures rather than one.

#### IV. Empirical Applications

We now apply the formula in Proposition 1 to characterize the welfare gains from expanding unemployment and health insurance programs in the United States. These calibrations are intended primarily to illustrate the potential impacts of endogenous private insurance on calculations of welfare gains from government intervention rather than for policy analysis. These simple calculations do not account for all margins of behavioral responses and for heterogeneity across individuals.

Because social insurance problems typically involve two states, e.g., employment and unemployment, we first adapt the analysis above to an environment in which agents choose between two levels of income. We then show that the formulas derived above can be expressed in terms of the parameters estimated in existing empirical studies of social insurance in this extensive-margin model.

<sup>11</sup> In cases with multiple equilibria, this assumption requires that the allocation never jumps across equilibria following a small change in  $\tau$ .

### A. Welfare Gain from Social Insurance

We consider the general model in (1), but from this point forward assume that there are only two feasible earnings levels:  $z \in \{z_L, z_H\}$ . For consistency with the prior literature on unemployment and health insurance, we use a separable utility specification in the empirical applications:

$$U(c, z; n) = u(c) - h(z/n).$$

In the binary earnings model, insurance contracts can be characterized by a premium (or tax) that agents pay insurers when earnings are high in exchange for a benefit (or transfer) that insurers pay agents when earnings are low. Before agents realize their value of  $n$ , they sign contracts with a private insurer who charges a tax  $\tau_p$  if the agent has high earnings ( $z_H$ ) and pays a transfer  $b_p$  if he has low earnings. All agents hold the same private insurance contract  $(\tau_p, b_p)$  because there are no ex ante differences across individuals. The government charges a tax  $\tau$  to finance a public insurance system and pays a transfer  $b$  if the agent has low earnings. With this notation, an individual works to obtain high earnings  $z_H$  if and only if  $n > n^*$ , where  $n^*$  satisfies

$$u(z_H - \tau - \tau_p) - u(z_L + b + b_p) = h(z_H/n^*) - h(z_L/n^*).$$

The social welfare function aggregates utility over the continuum of agents in the economy:

$$\begin{aligned} W &= \int_0^{n^*} [u(z_L + b + b_p) - h(z_L/n)] dF(n) \\ &\quad + \int_{n^*}^{\infty} [u(z_H - \tau - \tau_p) - h(z_H/n)] dF(n). \end{aligned}$$

Let the fraction of agents who have high earnings  $z_H$  be denoted by

$$e = 1 - F(n^*) = \int_{n^*}^{\infty} dF(n).$$

Let  $F^{-1}$  denote the inverse of  $F$ . Then  $n^* = F^{-1}(1 - e)$ , and we can write social welfare as

$$W(e) = eu(z_H - \tau - \tau_p) + (1 - e)u(z_L + b + b_p) - \psi(e),$$

where

$$\psi(e) = \int_0^{\infty} h(z_L/n) dF(n) + \int_{F^{-1}(1-e)}^{\infty} [h(z_H/n) - h(z_L/n)] dF(n),$$

is the aggregate disutility from working at the higher earnings level. In this model, the fraction of agents who work ( $e$ ) is effectively chosen to maximize  $W$ , taking the government and private insurance contracts as given. The aggregate economy is isomorphic to standard social insurance models (e.g., Baily 1978), where the individual controls his probability of being in the low earnings state by choosing effort  $e$ .

Let  $\varepsilon_{1-e,B}$  denote the elasticity of the probability of the low state ( $1 - e$ ) with respect to the total benefit level,  $B = b + b_p$ . The private insurance benefit level  $b_p$  depends on the government benefit  $b$  according to a function  $b_p(b)$ , which may not necessarily be set optimally. For example, suppose individuals perceive their ex ante probability of having a skill level  $n > n^*$  as  $e + \pi$ , where  $\pi$  measures the individual's degree of overconfidence. Spinnewijn (2008) shows that when  $\pi > 0$ , the private market equilibrium yields  $b_p$  below the second-best level of  $B$ .<sup>12</sup> The formula below nests the case with  $\pi > 0$ .

Because the benefits are additive rather than multiplicative, it is convenient to define the crowd-out parameter as  $r = -db_p/db$  for the social insurance applications. With endogenous private insurance, the government chooses  $b$  to maximize

$$(17) \quad W = eu \left( z_H - \frac{1-e}{e} (b_p(b) + b) \right) + (1-e)u(z_L + b_p(b) + b) - \psi(e).$$

The following proposition is the analog of Proposition 1 for this binary earnings model.

**PROPOSITION 5:** *The welfare gain from raising the government social insurance benefit is*

$$(18) \quad \frac{dW}{db} = (1-e)(1-r) u'(c_H) \left[ \frac{u'(c_L) - u'(c_H)}{u'(c_H)} - 1 - \frac{\varepsilon_{1-e,b}}{e} \frac{1 + b_p/b}{1-r} \right].$$

**PROOF:**

It is straightforward to establish that  $\psi(e)$  is an increasing and convex function of  $e$ .<sup>13</sup> Therefore,  $e$  is effectively set at an interior optimum by agents. As noted in Section IIC, choosing  $b$  is equivalent to choosing the total benefit  $B$ . Differentiating (17) yields  $dW/dB = (1-e)u'(c_H)[((u'(c_L) - u'(c_H))/u'(c_H)) - \varepsilon_{1-e,B}/e]$ . Observe that  $dW/db = (dW/dB)(1-r)$ . Likewise,  $de/db = (de/dB)(1-r)$ , and thus  $\varepsilon_{1-e,B} = -(b + b_p)(de/dB)/(1-e) = (1 + b_p/b) \varepsilon_{1-e,b}/(1-r)$ . Plugging these expressions into the expression for  $dW/dB$  yields (18).

The first term in (18) measures the gap in marginal utilities across the two states, which captures the marginal value of insurance. The second term captures the cost of insurance through the behavioral response. Analogous to the tax scenario, private

<sup>12</sup> Spinnewijn (2008) analyzes private insurance and social insurance separately in a model with biased beliefs. He does not consider the optimal level of social insurance with endogenous private insurance as we do here. In Spinnewijn's (2008) terminology, our analysis permits "baseline" optimism, but not "control" optimism because the latter would distort effort choices.

<sup>13</sup> Note that  $\psi'(e) = -(dF^{-1}(1-e)/de)[h(z_H/n^*) - h(z_L/n^*)]f(n^*) = h(z_H/n^*) - h(z_L/n^*) > 0$  as  $z_H > z_L$ . We have  $dn^*/de = -1/f(n^*)$ , hence  $\psi''(e) = [(z_H/n^*)h'(z_H/n^*) - (z_L/n^*)h'(z_L/n^*)]/[n^*f(n^*)] > 0$  as  $h(\cdot)$  is increasing.

insurance amplifies the second term and makes  $dW/db$  smaller through two channels: the crowd-out effect  $1 - r$  in the denominator and the mechanical effect  $b_p/b$  in the numerator. The crowd-out term reflects a rescaling to recover the fundamental elasticity  $\varepsilon_{1-e,B}$ , and the  $b_p/b$  term reflects the reduction in  $b$  required to achieve the optimal level of  $B$ .

The expression for  $dW/db$  measures the marginal welfare gain of changing social insurance benefits in utils. To convert this expression into an interpretable money metric, we normalize the welfare gain from a \$1 (balanced budget) increase in the size of the insurance program by the welfare gain from raising the wage bill in the high state by \$1:

$$(19) \quad G(b) = \frac{dW}{db} \frac{1}{1-e} \Big/ \frac{dW}{dz_H} \frac{1}{e}$$

$$(20) \quad = (1-r) \left[ \frac{u'(c_L)}{u'(c_H)} - 1 - \frac{\varepsilon_{1-e,b}}{e} \frac{1 + b_p/b}{1-r} \right].$$

### B. Application 1: Unemployment Insurance

The existing literature on optimal unemployment insurance essentially ignores private insurance. Most private insurance against unemployment is provided through informal risk sharing that is unlikely to generate much moral hazard, and hence can be ignored in the calculation of optimal benefits according to the results in Section IIE. However, many US private firms provide unemployment insurance in the form of severance payments—lump-sum cash grants made by firms to workers who are laid off. Unlike government provided unemployment benefits, severance pay does not distort job search behavior after job loss because it does not affect marginal incentives to search. Severance pay can, however, distort effort choices while working by changing the relative price of being unemployed relative to having a job.

In this section, we calibrate the welfare gain from raising the UI benefit level when the response of severance pay to UI benefits is taken into account. To map the optimal UI problem into the static framework in Section IVA, we ignore the job search decision, treating search effort after job loss as invariant to the UI benefit level. Instead, we focus on the distortion in the probability of job loss (e.g., due to shirking) caused by UI benefits and severance pay. In our static model, both UI benefits and severance pay act as transfers to the unemployed state, and are financed by taxes in the employed state.<sup>14</sup> Employed agents earn  $z_H > 0$  and unemployed agents earn  $z_L = 0$ . An agent with ability  $n$  must pay an effort cost  $h(z_H/n)$  to keep his job. The formula in Proposition 1 can be directly applied to this environment with the following empirical analogs for its arguments: the fraction of agents who

<sup>14</sup> A more precise calibration would take into account the fact that UI benefits are conditioned on duration, and thus are larger when a worse “state” is realized. This calibration would require separate estimates of the effect of UI benefits and severance pay on the probability of job loss.

are unemployed is  $1 - e$ ; the elasticity of the probability of job loss with respect to the UI benefit level is  $\varepsilon_{1-e,b}$ ; and  $c_H$  and  $c_L$  are consumption when employed and unemployed, respectively; and  $r$  measures the crowd-out of severance pay ( $b_p$ ) by government UI benefits ( $b$ ).

*Estimation of Crowd-out Elasticity.*—As an illustration of the data and empirical strategy needed to implement our formula with endogenous private insurance, we begin by estimating the two key parameters: the size of the private insurance market ( $b_p/b$ ) and the crowd-out effect ( $r$ ). To do so, we use data on severance pay from a survey conducted by Mathematica on behalf of the Department of Labor. The dataset (publicly available from the Upjohn Institute) is a sample of unemployment durations in 25 states in 1998 that oversamples UI exhaustees. We reweight the data using the sampling weights to obtain estimates for a representative sample of job losers. The dataset contains information on unemployment durations, demographic characteristics, and an indicator for receipt of severance pay. There are 3,395 individuals in the sample, 508 of whom report receiving a severance payment. See Chetty (2008) for further details on the dataset and sample construction.

To calculate  $b_p/b$ , first note that 15 percent of job losers report receiving severance pay in our data. According to calculations reported in Chetty (2008), the mean severance payment conditional on receipt of severance pay is equal to 10.7 weeks of wages, the mean UI benefit level is 50 percent of the wage, and the mean unemployment duration is 15.8 weeks. Hence, in the aggregate population, the ratio of total private insurance to total public insurance is

$$\frac{b_p}{b} = \frac{0.15 \times 10.7}{0.5 \times 15.8} = 0.20.$$

To estimate  $r$ —the effect of an increase in the UI benefit level on severance pay—we exploit variation in UI benefit levels across states. An OLS regression of severance pay receipt on individual UI benefit levels is unlikely to yield a consistent estimate of  $r$  because individuals with higher wages have a higher probability of receiving severance pay and higher UI benefits (Lee Hecht Harrison 2001; Chetty 2008). We account for the confound created by wage heterogeneity across individuals in two ways. First, we isolate policy-driven UI benefits by instrumenting for individual benefits with the maximum benefit level in their state. Most states pay a fixed wage replacement rate up to a maximum, which varies considerably across states and thereby creates variation in UI benefit levels. Second, we control for individual wages flexibly throughout our analysis using a 10 piece spline for the individual log wage. The specifications we estimate are analogous to those of Bruce D. Meyer (1990), with severance pay receipt rather than unemployment durations as the dependent variable.

We begin with a simple graphical analysis to illustrate the estimation of the crowd-out effect. Figure 1 plots the relationship between average severance pay receipt and the maximum UI benefit level, conditioning on wages. To construct this

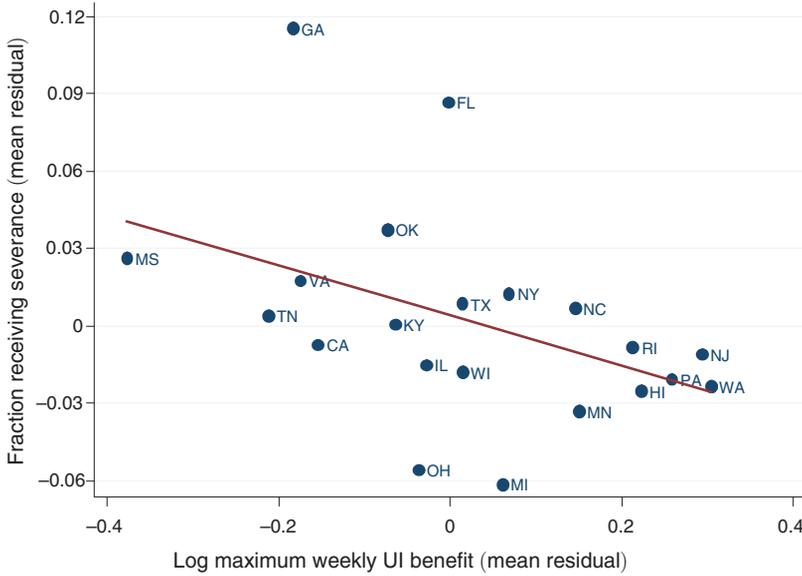


FIGURE 1. EFFECT OF UI BENEFITS ON SEVERANCE PAY

*Notes:* Figure plots relationship between fraction of individuals receiving severance pay in each state versus maximum state UI benefit level, conditioning on wages. Figure shows a scatter plot of the mean residuals by state from a regression of severance pay receipt and log maximum weekly benefit level on a log wage spline (see text for details).

*Source:* Mathematica survey of UI exhaustees in 25 states in 1998. States with fewer than 50 individual observations are excluded from this figure.

figure, we first regress the severance pay dummy on the individual wage spline and the maximum UI benefit level on the wage spline and compute residuals. We then compute mean residuals of both variables by state. The figure is a scatter plot of the mean residuals. We exclude states that have fewer than 50 individuals from this figure to reduce the influence of outliers on the graph. All observations are included in the regression analysis below. The figure shows that states with higher UI benefit levels have fewer severance payments, indicating that private insurance is crowded out to some extent by public insurance.

To quantify the amount of crowd-out, we estimate regression models of the following form:

$$(21) \quad sev_i = \alpha + \beta \log b_i + f(w_i) + \gamma X_i + \varepsilon_i,$$

where  $sev_i$  is an indicator for whether individual  $i$  received a severance payment,  $b_i$  is a measure of the UI benefit level for individual  $i$ ,  $f(w_i)$  denotes the wage spline, and  $X_i$  denotes a vector of additional controls.

Specification 1 of Table 1 reports estimates of (21) without any additional controls  $X$ , with  $b_i$  equal to the maximum benefit level in the state where individual  $i$  lives. Standard errors in this and all subsequent specifications are clustered by state to adjust for arbitrary within-state correlation in errors. The estimated coefficient

TABLE 1—EFFECT OF UI BENEFITS ON SEVERANCE PAY: REGRESSION ESTIMATES

Dependent variable:	Reduced-form OLS		TSLS
	No controls	With controls	With controls
Severance pay	(1)	(2)	(3)
log max UI benefit	−0.074 (0.030)	−0.065 (0.030)	
log individual UI benefit			−0.105 (0.054)
Sample size	2,996	2,733	2,733

*Notes:* Specifications 1 and 2 report estimates from an OLS regression; specification 3 reports estimates from a two-stage least squares regression using log state max benefit as an instrument for actual individual benefit reported in data. Specifications 2 and 3 include the following controls: job tenure, age, gender, household size, education, dropout, industry, occupation, and race dummies.

of  $\beta = -0.074$  implies that doubling the UI benefit maximum would reduce the fraction of individuals receiving severance pay by 7 percent. Specification 2 replicates 1 with the following individual-level covariates: job tenure, age, gender, household size, education, dropout, industry, occupation, and race dummies. The point estimate on the UI benefit level is not affected significantly by the inclusion of these controls.

Specifications 1 and 2 are reduced form regressions which show the effect of the instrument (maximum benefit levels) on severance pay. To obtain an estimate of the effect of a \$1 increase in the benefit level on the probability of severance pay receipt, we estimate a two-stage least squares regression, instrumenting the log individual benefit level with the log state maximum. The estimated coefficient on the log individual benefit, reported in column 3 of Table 1, is  $\beta = -0.105$ . Doubling the UI benefit level would reduce severance receipt by 10.5 percentage points, relative to a mean value of 15 percent, implying  $\varepsilon_{b_p,b} = -0.7$ . We conclude that  $r = -db_p/db = -\varepsilon_{b_p,b} b_p/b = 0.7 \times 0.2 = 0.14$ .

The identification assumption underlying these regressions is that the cross-state variation in UI benefit maximums is orthogonal to other determinants of severance pay receipt conditional on wage levels. Most plausible endogeneity stories would work toward attenuating our estimate of the crowd-out effect. For example, suppose states with higher UI benefit maximums are populated by individuals who are more risk averse and therefore place higher value on insurance. Such states would also have higher private insurance, biasing downward our crowding out estimate. Given these concerns about policy endogeneity, our simple empirical analysis should be viewed as illustrative. Future work should exploit within-state variation in UI benefits to obtain a more credible and precise estimate of the crowd-out effect.

*Calibration.*—We calibrate (20) to calculate the marginal welfare gain of public UI using the following inputs drawn from the empirical literature on unemployment insurance and the estimates above:

$e = 0.95$  from CPS statistics (5 percent unemployment rate)

$r = -\frac{db_p}{db} = 0.14$  from calculations above

$\frac{b_p}{b} = 0.2$  from calculations above

$\frac{c_e}{c_u} = \frac{1}{0.9}$  from Gruber (1997)

$\gamma = 2$  from Chetty (2006b),

under the approximation that utility exhibits constant relative risk aversion between  $c_u$  and  $c_e$ ,  $u'(c_u)/u'(c_e) = (1/0.9)^\gamma$ , where  $\gamma$  denotes the coefficient of relative risk aversion. The remaining parameter, for which we have no existing estimate, is  $\varepsilon_{1-e,b}$ —the elasticity of the probability of job loss with respect to the UI benefit level  $b$ . Leaving this parameter unspecified and plugging in the remaining values into the formula for  $dW/db$ , we obtain

$$G(b) = (1 - 0.14)(0.23 - 1.47\varepsilon_{1-e,b}).$$

It follows that if the job loss elasticity  $\varepsilon_{1-e,b} > 0.15$ ,  $dW/db < 0$  at present UI benefit levels when crowd-out is taken into account. In contrast, if we were to apply a formula that does not take crowd-out of private insurance into account, we would obtain

$$G(b) = (0.23 - 1.05\varepsilon_{1-e,b}).$$

Hence, an analyst who ignores crowd-out would conclude that the welfare gain from raising the UI benefit level is negative only if  $\varepsilon_{1-e,b} > 0.25$  (ignoring distortions in unemployment durations). We conclude that in this application, there is a significant but modest range of parameters for which adjusting the formula for endogenous private insurance leads to different policy implications.

### C. Application 2: Health Insurance

*A Model of Health Shocks.*—To adapt the model of social insurance in Section IVA to health insurance, consider an economy with a continuum of ex ante identical agents, each of whom has utility  $u(c)$  when healthy and  $u(c) - v$  when sick. By purchasing health care, which costs  $\$C$ , an agent who is sick can return to the healthy state and erase the utility cost  $v$ . Both sick and healthy individuals earn a fixed income  $z$ . In this setting,  $v$  measures the agent's gross valuation of health care. The valuation of health care  $v$  is distributed according to a smooth cumulative distribution function  $F(v)$ .

The structure of insurance contracts is the same as above. Insurers collect premia from "healthy" individuals who choose not to purchase health care and pay (net of premium) benefits to individuals who do purchase health care. An agent buys health

care if and only if  $u(z - C + b + b_p) > u(z - \tau - \tau_p) - v$ , i.e., if and only if  $v > v^*$ , where

$$v^* = u(z - \tau - \tau_p) - u(z - C + b + b_p).$$

Letting the fraction of agents who do not buy health care be denoted by  $e = F(v^*)$ , we can write social welfare as

$$(22) \quad W(e) = eu\left(z - \frac{1-e}{e}(b_p(b) + b)\right) + (1-e)u(z - C + b_p(b) + b) - \psi(e),$$

where  $\psi(e) = \int_0^{F^{-1}(e)} v dF$  is the aggregate utility gain from consumption of health care. The government chooses  $b$  to maximize (22), a problem that is identical to (17) with  $z_H = z$  and  $z_L = z - C$ . Therefore, the formula in Proposition 1 can be applied to calculate  $dW/db$  with the following empirical analogs for its arguments: the fraction of agents who purchase health care is  $1 - e$ ; the elasticity of health care utilization with respect to the government insurance benefit is  $\varepsilon_{1-e,b}$ ;  $c_H$  and  $c_L$  are consumption when healthy and sick, respectively; and  $r$  measures the crowd-out of private health insurance ( $b_p$ ) by public health insurance ( $b$ ).

*Calibration.*—We calibrate (20) to calculate the marginal welfare gain of public health insurance using the following inputs drawn from the empirical literature on health insurance:

$$\varepsilon_{1-e,C} = -0.2 \text{ from Willard G. Manning et al. (1987)}$$

$$1 - e = 0.1 \text{ from Manning et al. (1987) for inpatient usage rate}$$

$$r = -\frac{db_p}{db} = 0.5 \text{ from Cutler and Gruber (1996a)}$$

$$\frac{b_p}{b} = 0.89, \frac{b}{C} = 0.45 \text{ from National Health Care Statistics Table 6 (2006)}$$

$$\frac{c_e}{c_u} = \frac{1}{0.85} \text{ from John H. Cochrane (1991)}$$

$$\gamma = 2 \text{ from Chetty (2006b).}$$

Under CRRA utility, these parameters imply that  $u'(c_u)/u'(c_e) = (c_e/c_u)^\gamma = (1/0.85)^2 = 1.384$ . Also note that  $\varepsilon_{1-e,b} = -\varepsilon_{1-e,C}(b/C) = 0.2 \times 0.45 = 0.09$ . Hence,

$$G(b) = (1 - 0.5) \times \left(0.384 - \frac{0.2 \times 0.453}{0.9} \frac{1 + 0.89}{0.5}\right) = 0.0017.$$

If we had ignored crowd-out, we would have obtained

$$G(b) = \left( 0.384 - \frac{0.2 \times 0.453}{0.9} \right) = 0.28.$$

Taking crowd-out into account lowers the estimate of  $G(b)$  by a factor of more than 100. An analyst who ignored crowd-out and applied existing formulas (e.g., Chetty 2006a) would infer that a \$1 million expansion in public health insurance programs would generate \$280,000 in surplus net of the required tax increase needed to finance the expansion. This analyst would mistakenly conclude that an expansion in the overall level of public health insurance would yield substantial welfare gains. Taking crowd-out into account implies that we are near the optimum in terms of *aggregate* public health insurance levels, as a \$1 million across-the-board expansion would generate only \$1,700 in net social surplus.

There are several important caveats to this calibration that should be kept in mind when evaluating the policy implications of this simple calibration. First, this calculation does not fully account for pre-existing information and adverse selection, as it neglects the benefits of redistributing across pre-existing risk types via government insurance reflected in the first term of (16). Second, the calibration ignores the correlation between health shocks and income inequality, which could potentially increase the welfare gains from health insurance (Cremer and Pestieau 1996). Third, private health insurance benefits are already tax subsidized in the United States. Finally, and perhaps most importantly, our aggregate welfare gain calculation ignores substantial heterogeneity across types of people, conditions, Medicare versus Medicaid, etc. For some subgroups, such as the uninsured, there could clearly be substantial welfare gains from increasing public insurance benefits, whereas for others there could be substantial welfare gains from cutting benefits.

## V. Conclusion

This paper has characterized the welfare gain from public insurance in the presence of endogenous private insurance. The formulas for optimal tax and social insurance policies derived here highlight two general parameters as the determinants of how private insurance impacts the welfare gains from social insurance: the size of the formal private insurance market, and the crowd-out of formal private insurance by public insurance. The crowd-out and size of informal insurance—that is, insurance that does not generate moral hazard—does not enter the formulas. It is therefore crucial to distinguish between formal and informal insurance empirically.

Like recent “sufficient statistic” formulas for welfare analysis, the formulas we have derived can be implemented using reduced-form empirical evidence without full identification of the model’s primitives. However, unlike existing formulas in models without private insurance, we are unable to obtain a single formula that is robust across a range of models. In models with endogenous private insurance, the source of the deviation from constrained efficiency—e.g., asymmetric information, imperfect optimization, or formal versus informal private insurance—affects the

formula. We believe that the parameters we have identified are likely to matter in more general models, but other factors may also be relevant.

Our theoretical analysis can be generalized in three broad directions. First, one should characterize the effects of government intervention on the equilibria in the adverse selection model, as in Rothschild and Stiglitz (1976) and Wilson (1977). We have implicitly assumed that contracts and behavior will respond smoothly to changes in government policies, but in a setting with multiple equilibria, there could be jumps between equilibria that would invalidate our formula. Second, it would be useful to extend the analysis to allow for different loading factors for private and public insurance, micro-founded via increasing returns or wasteful marketing costs. Conversely, one could allow for different levels of efficiency, reflecting the possibility that private insurers could be more efficient because of competitive pressure. Finally, in the simple model we analyzed here, the best policy is simply to rule out formal private insurance and have the government provide all insurance. However, there are some areas in which private insurers have an informational advantage relative to the government. For instance, employers have more information on effort on the job, making the moral hazard problem smaller for the employer. Characterizing the optimal mix of government and private insurance is an important next step.

If government and private insurers optimize along the lines described by our analysis, our model makes testable predictions about the pattern of insurance contracts we should observe. For instance, private insurance should be more prevalent in economies with low job mobility (such as Japan), where firms have the ability to insure shocks through a compressed wage structure without facing as much adverse selection. Another prediction is that government insurance should be more prevalent for shocks that occur prior to the point at which insurance contracts can be purchased, such as disability at birth, or for shocks where optimization of insurance purchases is unlikely. It would be interesting to test empirically whether observed contracts match these theoretical prescriptions.

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