Econ 240A: Problem Set 4
Solutions to Selected Problems from Chapter 3

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45.

a.

Given a sample of observations $X_1, \ldots, X_n$, empirical expectations $E_n[f(X)]$ (i.e., sample averages) will under certain regularity conditions described in detail in Chapter 4 converge almost surely to the corresponding population expectation $E[f(X)]$. Therefore if we assume that the sample size of 7806 in this problem is "large enough" in the appropriate sense, the closeness of the sample skewness[1] and kurtosis[2] to zero should serve as a rough indication of whether the observations were generated by a member of the normal family of distributions.

For RNYSE we have $E_n\left(\frac{X-\mu}{\sigma}\right)^3 = -1.2706$ and $E_n\left(\frac{X-\mu}{\sigma}\right)^4 - 3 = 30.9208$; while for RTB90 we have a sample skewness and kurtosis of 1.3088 and 1.9719, respectively. Therefore one would not assume that the return to investments on stocks and bonds are generated by normal distributions.

b.

This question is motivated by the result shown in section that the empirical cdf is a good approximation of the population cdf when the sample size is large:

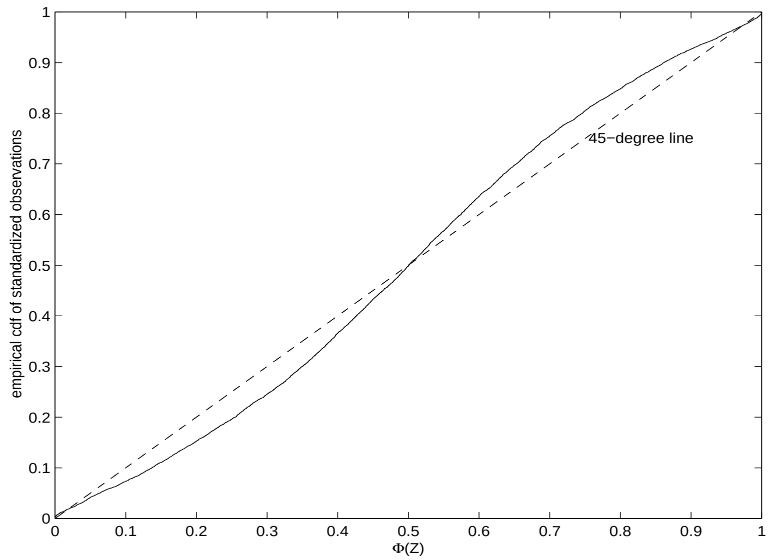
Note that the empirical cdf is the cdf associated with a probability distribution that assigns probability $\frac{1}{n}$ to each observation. In other words, the empirical cdf is given by the function

$$\begin{aligned}\hat{F}_n(a) &= \frac{\text{number of observations} \leq a}{n} \\ &= \frac{1}{n}\sum_{i=1}^{n} 1(X_i \leq a).\end{aligned}$$

[1] A measure of symmetry, where a value of zero indicates perfect symmetry.

[2] A measure of tail thickness where large values indicate the presence of a significant number of outliers.

Figure 1: Plot of Y against $\Phi(Z)$



Since $E[1(X_i \leq a)] = F(a)$ for $i = 1, \ldots, n$, where $F(\cdot)$ is the cdf of X_i, it follows from Kolmogorov's law of large numbers that $\hat{F}_n(a) \stackrel{a.s.}{\to} F(a)$. Therefore in this question we would expect Y, the empirical cdf of $Z = \frac{\text{RNYSE}-\mu}{\sigma}$ to cluster tightly around the 45-degree line in a graph of Y against $\Phi(Z)$ if Z really was generated by a normal distribution (i.e., if $F(\cdot) = \Phi(\cdot)$). This is not the case, since the graph actually looks like it does in Figure 1.

Y has a very definite tendency to be greater than $\Phi(Z)$ when $\Phi(Z)$ is less than about 0.5 and less than $\Phi(Z)$ when $\Phi(Z) > .5$. This is clear evidence against the normality of the observed Z's.