Electoral Incentives and the Allocation of Public Funds

Frederico Finan
UC-Berkeley

Maurizio Mazzocco
UCLA

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Abstract
Politicians allocate public resources in ways that maximize political gains, and potentially at the cost of lower welfare. In this paper, we quantify these welfare costs in the context of Brazil's federal legislature, which grants its members a budget to fund public projects within their states. Using data from the state of Roraima, we estimate a model of politicians' allocation decisions and find that 26.8% of the public funds allocated by legislators are distorted relative to a social planner's allocation. We then use the model to simulate three potential policy reforms to the electoral system: the adoption of approval voting, imposing a one-term limit, and redistricting. We find that a one-term limit and redistricting are both effective at reducing distortions. The one-term limit policy, however, increases corruption, which makes it a welfare-reducing policy. (JEL: D72, H00, C72, C82)

1. Introduction

A central function of governments is the provision of public goods and services. In 2020, governments throughout the world spent on average more than 32.4% of their countries' GDP on these goods and services.[1] When allocated efficiently, these expenditures can be important drivers of economic development and key determinants of quality of life. But, public expenditures are often allocated across regions by politicians who compete for reelection, and this competition can create political incentives that distort how public funds are spent relative to the social optimum. This raises the following questions: How large are these distortions? And, what types of reforms might help to reduce them?

The main contribution of this paper is to address these two questions. To do so, we develop a model that characterizes the allocation decisions of federal legislators in Brazil, who receive an annual budget to fund local public goods

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E-mail: ffinan@berkeley.edu (Finan); mmazzocc@econ.ucla.edu (Mazzocco)

1. This statistic comes from the 2020 Index of Economic Freedom.

across regions within their state. Our model starts with the basic premise that citizens value public goods and vote for the politicians who they believe will provide their region with more resources in the future to fund these goods. With this feature, the model can explain the strong correlation we observe in our data between the votes politicians receive and the amount of funds they allocated to a particular region during their previous term.[2] In the model, voters' decisions are also affected by a politician's electoral appeal. Politicians with higher electoral appeal fare better at the polls, all else equal.

In most models of distributive politics, politicians care exclusively about getting elected (e.g. Myerson (1993); Lizzeri and Persico (2001)). In our model, politicians care differentially about not only their election probabilities, but also the welfare of the people in their state. This heterogeneity in the degree of a politician's altruism enables us to account for the fact that, while in our data some politicians mostly target places with many voters, as has been documented in other settings (e.g. Atlas et al. (1995); Rodden (2002)), others politicians predominantly target regions that are less populated and less developed.

At the end of a political term, we model an incumbent's decision to run for reelection. By modeling this choice, we can account for the observation that incumbents who run, and thus have stronger electoral incentives, allocate their public funds differently than those who do not. This last pattern is also consistent with a large literature documenting the importance of reelection incentives on policy choices (e.g. Besley and Case (1995); List and Sturm (2006); Ashworth (2012); Lim (2013)).

Our model also allows for the funds that politicians allocate not to translate fully into pure local public goods. This can happen for two reasons. First, politicians can divert some of the funds, as the media frequently reports. Second, the public goods may have some degree of rivalry, in that the welfare value of the goods may decrease as more people use them.

Lastly, we consider a setting in which multiple politicians are simultaneously elected to represent their state. Thus, when incumbents decide where to allocate their funds and whether to run for reelection, they must consider the decisions of the other incumbents within the state. We account for this institutional feature in the model by having politicians make their decisions simultaneously and with incomplete information about the degree of altruism and electoral appeal of their opponents.

We estimate the model by Simulated Method of Moments (SMM) using data on the universe of public funds allocated from 1996 to 2013 by federal legislators representing the state of Roraima in Brazil. Brazil's federal legislature and the state of Roraima in particular, provide an ideal setting to estimate a model in which politicians allocate funds across regions. Each year the

2. Evidence that voters reward politicians for transfers has been documented in other contexts as well (e.g. Levitt and Snyder (1997); Manacorda et al. (2011)).

Brazilian Constitution grants each federal legislator a budget of BRL$1.5 million (US$750,000) to fund public projects in the state from which the legislator is elected. This constitutional provision allows us to investigate the effect of political competition on the politicians' allocation decisions without worrying about the endogeneity of who has access to these funds, which is an important issue in other contexts, such as the U.S. Congress. Moreover, local governments use these funds for large-scale development projects that have important welfare consequences.

We chose the state of Roraima because it is a poorer and less populated state, and a place where the welfare consequences of the allocation decisions of politicians are likely to be more important. Additionally, the computational burden of estimating our model increases exponentially with the number of incumbent politicians who compete for votes. In Brazil, each state is a single electoral district that elects a fixed number of legislators to the federal government. Roraima elects only eight legislators, which makes the estimation of our model challenging but feasible. We also use the estimated model to derive insights about the allocation of public funds that apply to more general settings.

We find that deputies misallocate 26.8% of their public funds relative to a social planner's allocation. We decompose these political distortions into three channels. Electoral incentives explain almost 30% of these distortions and corruption accounts for 13.5%. The remaining source of distortions stems from incomplete information. In our model, deputies allocate their funds in response to what they believe other deputies will do, but without knowing each other's types. Thus, even if we eliminate electoral incentives and corruption, deputies will still misallocate public funds because of the uncertainty surrounding the actions of the other politicians.

Our estimates suggest that we can explain much of the variation in how deputies allocate their funds with four types of politicians: altruistic high-valence types, altruistic low-valence types, egoistic high-valence types, and egoistic low-valence types. The egoistic politicians target regions with more votes at the cost of poorer and more productive places and are thus responsible for a larger fraction of the distortions. Had the pool of politicians only consisted of altruistic individuals, the distortions would have declined by 24%. These findings highlight the importance of unobserved heterogeneity in the allocation of public funds and the need to account for it when trying to understand allocation decisions.

Given the size of our estimated distortions, we investigate what types of reforms might help to reduce them. We simulate three possible policy reforms. We first consider the effects of adopting approval voting, which is an electoral system that allows people to vote for multiple candidates. We find that even though approval voting does reduce distortions, the effects are minimal. For example, if Brazil adopted a system in which voters could vote for five candidates instead of one, distortions would decrease by less than a percent.

The second policy we consider is to limit politicians to a single term. Brazil currently allows legislators to be elected indefinitely. But several countries have argued for, and in some cases implemented, term limits as a way to improve representation and reduce politicians' pandering. In our model, the advantage of a one-term-limit policy is that electoral incentives would no longer influence how funds are allocated. The disadvantage is that, according to our model, incumbents who forgo reelection divert 39% more funds than those who still face reelection incentives. The ability to determine which of these two effects dominates is an important contribution of our model. When we compare our results to a counterfactual situation in which deputies cannot run for reelection, we find that political distortions decrease by 34%. However, because of the increase in corruption, welfare as a whole goes down by 4.5%, suggesting that a one-term limit policy would be welfare reducing.

The last policy we consider is a redistricting policy that was considered by the legislature in 2009. The objective of the proposal was to transform states from a single multi-member district into several sub districts. Importantly, this policy would reduce the number of representatives in a given district and thus the uncertainty surrounding the actions of other incumbents. This allows the incumbents who care about welfare considerations to achieve an allocation that is closer to the social planner's. Under this policy, we find that the level of distortions decreases significantly and monotonically as we reduce the number of representatives in a given district. For example, when we go from eight to four deputies, distortions decrease by 24%.

Although these findings are specific to the state of Roraima, our model captures economic and political forces that apply broadly. We use the model to document their effects and provide three general insights that extend beyond the state of Roraima. Our first insight is that smaller disparities in electoral gains significantly reduce the size of the distortions among regions with similar welfare gains. We also document that larger disparities in welfare gains increase the size of the distortions among regions with similar electoral gains. Finally, we find that distortions increase with the amount stolen by deputies who do not run.

Overall, our findings contribute to two broad strands of the literature. First, our study relates to an extensive literature in both economics and political science that investigates the causes and consequences of distributive politics. As Golden and Min (2013) report in a comprehensive review of this vast literature, numerous studies have documented the importance of electoral incentives in the allocation of public goods and services. Yet our study is, to our knowledge, the first to quantify the welfare consequences of electoral incentives and to show how electoral institutions can help reduce potential deviations from a social planner's allocation.

Our focus on electoral rules naturally relates to a more specific literature within distributive politics that examines the importance of the electoral system for public goods provision.[3] The empirical studies have been almost entirely reduced-form (e.g. Milesi-Ferretti et al. (2002); Besley and Case (2003); Persson and Tabellini (2005); Beath et al. (2014)). This paper instead uses a structural approach to understand the effects of electoral institutions on the allocation of public funds. In this regard, our paper relates to a growing literature on the structural estimation of political economy models. For instance, Stromberg (2008) structurally estimates how U.S. presidential candidates allocate their campaign resources across states to maximize their election chances. We complement this study by examining the allocation of public resources, which besides providing electoral returns introduces important welfare considerations. Diermeier et al. (2005), Lim (2013), Aruoba et al. (2015) and Sieg and Yoon (2017) also estimate structural models of political choices to understand how electoral institutions, such as term limits, affect politicians' behavior. While these studies model many of the dynamic aspects of politicians' decisions, in our paper we account for them only through the incumbents' choice to run for reelection. But, we complement these studies in two important ways. First, differently from these studies, politicians in our model allocate public resources across regions that differ in the number of voters and demand for public funds. Second, we consider the interactions among the politicians' decisions by estimating a strategic game with incomplete information, whereas in those studies politicians make independent decisions.

The paper proceeds as follows. Section 2 provides background on Brazil's federal legislature and presents the reduced-form findings that motivate the model. Section 3 describes the model. Section 4 discusses our estimation approach and the identification of the model's parameters. Section 5 presents estimation results and policy simulations. Section 6 concludes the paper.

2. Background and Data

In this section, we discuss Brazil's political system, focusing on the institutional features that are relevant to our analysis. We then present our data and the patterns in the data that motivate our modeling choices.

2.1. Brazil's Federal Legislature

Brazil's federal legislature, also referred to as the Chamber of Deputies (we will use the terms "Deputy" and "Legislator" interchangeably), consists of 513

3. See Persson and Tabellini (2000) for a general review of the literature. The implications of different vote-counting schemes for candidate behavior is also reviewed in a 1995 JEP issue, see for example Levin and Nalebuff (1995).

seats allocated across 26 states according to population size. Each state is a multi-member district. During the elections, incumbents face competition from both new challengers as well as from the other incumbents.

National elections for the legislature take place on a four-year cycle and incumbents can be reelected indefinitely. Brazil uses the D'hondt open-list proportional representation method to first allocate seats across parties. Conditional on a party's allotment of seats, candidates within the party are elected based on their individual vote totals. Voting in Brazil is mandatory, and although citizens can vote for a political party, they usually vote directly for a single candidate. It is also common for several elected officials to change parties during their electoral terms. In the 49^{th} parliamentary session, for example, 55% of deputies switched parties during their term. With such a low degree of party loyalty, both from the standpoint of the politician and the electorate, party objectives do not play an important role in the allocation of public funds, as we will later show.

The primary responsibility of federal deputies is to allocate public funds. Brazil's legislature is comparatively weak and seldom legislates on issues of national concern (Ames 1995). As a Federal Deputy from Ceará stated in the Brazilian newspaper *Folha de São Paulo* on February 21, 1988: "A political career in Brazil is closely connected to success in bringing home material benefits ... Especially in the poorest regions, communities judge their deputies on what they bring home". A similar opinion was expressed by Federal Deputy Joaquim Haickel: "The primary function of a deputy is getting resources; legislating comes second." (Mainwaring and for Scholars. Latin American Program 1997).

In Brazil, access to these public funds is exogenous. Starting from 1996, the first year in our sample period, Brazil's constitution has granted a fixed yearly budget of BRL$1.5 million to each deputy.[4] In total, these budgetary amendments amount to 0.2% of total discretionary spending in a given year. Although this is a small fraction of Brazil's GDP, these projects represent an important injection of public goods for small and medium-sized states. For example, during our sample period the average municipality in Roraima had a yearly budget of BRL$6.18 million and received BRL$1.51 million in budgetary amendments.

2.2. Data

Our study combines administrative data from three sources. The budgetary amendment data come from the Chamber of Deputies. These data describe each budgetary amendment issued from 1996 to 2013, including the author's name,

4. The total amount has increased over time to adjust for inflation. It is also the only source of public funding deputies receive to allocate.

the amount, type, and location of the public investment. We use the author's name to merge these data with election results from the Tribunal Superior Eleitoral (TSE). The election data include vote totals for each candidate by municipality, along with various individual characteristics, including gender, education, occupation, and party affiliation. We use the election results to construct our primary measure of political support – municipal vote share – as well as various other measures of electoral performance and competition, such as the candidate's rank and vote total. Our final data source comes from the Brazilian Institute of Geography and Statistics (Instituto Brasileiro de Geografia e Estatística). The 2000 and 2010 population censuses provide several socioeconomic and demographic characteristics such as poverty rates, income inequality, and population size.

In Table 1, we present summary statistics for three different samples. The sample used in column 1 is for Brazil's 50^{th} legislature, which issued budgetary amendments during 1996-1999 and faced reelection in 1998. We use this sample for the reduced-form analysis, presented below. The restriction to one term is for convenience.[5] In column 3, we restrict the sample to Roraima, as a point of comparison. The sample in column 4 is also only for Roraima, but covers the period 1996-2013. We use this sample to estimate our model.

From 1996-1999, federal deputies across Brazil issued, on average, approximately 16 budgetary amendments per year, totaling $1.3 million reais.[6] There is considerable geographic variation in the distribution of these public works (see Appendix Figure A.1). More than 10% of municipalities did not receive a single public work during the 1996-1999 term, with the median municipality only receiving BRL$280,000 per year in budgetary amendments. In contrast, the top one percent of municipalities receive BRL$10,000,000 per year.

Elections in Brazil are highly contested. For example, during the 1998 elections, over six candidates per seat competed and 68% of incumbents ran for reelection. Conditional on running, reelection rates are 72%.

Brazil has over 26 political parties, which resulted in about 1.4 political parties per seat during the 1998 elections. With a proportional representation (PR) system, a small fraction of candidates will get elected because of their party's vote totals. This occurs when a party has earned a sufficient number

5. The match between the budgetary data and the election data has to be done by hand, which is why we limit the reduced-form analysis to a single term. The study of Brazil's budgetary amendments has a long tradition in the comparative politics literature. Since Ames (1995) classic study, there have been numerous empirical papers investigating the allocation of these budget appropriations and their electoral returns (e.g. Samuels (2003); Pereira and Renno (2003); Firpo et al. (2012).

6. As mentioned previously, federal deputies were allowed outlays totaling up to 1.5 million per year. Deputies generally reach the limit. However, in our analysis, we only consider outlays targeted to a municipality and exclude the ones designed to benefit either the state or the country as a whole.

of seats to elect members from its list whose vote totals would not have been otherwise sufficient to elect them. In 1998, this occurred for 13% of the seats. Although this fraction is relatively small, it does create some uncertainty when candidates assess their ex-ante probabilities of winning. We will account for this type of electoral uncertainty in the model.

For Roraima, the average municipality has a GDP per capita of around R$4,752 and population of 21,418 inhabitants. These numbers are comparable to the rest of Brazil.

2.3. Descriptive Evidence

We observe several features in the data that guide our modeling choices. In the model, we assume that voters care about the public funds they receive and reward the politicians who supply them. Two patterns in the data support these assumptions. First, we observe a strong association between where deputies receive their votes and where they allocated their funds. In panel A of Figure 1, we see a clear positive relationship between the share of votes deputies received in a municipality during the 1998 elections and the amount of public funds they had allocated during the previous term. The simple correlation implies that an increase of BRL$100,000 in funds is associated with a 2.8 percentage point increase in a deputy's vote share. In addition, this correlation is robust to controlling for both municipality and deputy fixed effects (see columns 1 and 2 of panel A in Table A.1).

The second pattern we observe is that voters are more likely to vote for the incumbent who allocated the largest amount independent of its size. To see this, we rank deputies based on how much they allocated within each municipality. We then regress their vote share on a set of indicators for their rankings while also controlling for municipality fixed effects. In panel B, we plot these coefficients. Deputies ranked first receive vote shares that are 27 percentage points higher than those ranked above 22 (the excluded category). The electoral returns to coming in second fall considerably, as second-place finishers only have a 7 percentage-point advantage. The vote shares of deputies ranked third and above are not statistically distinguishable from the excluded category. These results indicate that when casting their ballots, voters prefer the candidate who provides them with the most public funds. Given the relationships depicted in panels A and B, politicians have incentives to target municipalities with more voters, which is precisely what we see in panel C.

Another key assumption in our model is that politicians who run for reelection allocate resources differently from those who do not. We document evidence of this in panel (a) of Figure 2, where we plot the distribution of public funds by poverty levels for the two groups of incumbents. Panel (b) plots a similar figure using the municipality's human development index, which is a commonly used composite index of a regions's life expectancy, educational attainment, and income level. As both graphs indicate, incumbents who do not run for reelection are more likely to target poorer and less developed municipalities compared to incumbents with electoral concerns.

Finally, our model assumes that politicians allocate their funds independent of party affiliation. Deputies compete with their own party members in the same way they do against non-party members. We believe this assumption is reasonable for at least three reasons. First, in an open-list PR system politicians compete for votes both across parties and within parties. Second, as others and we have documented, there is little party allegiance in Brazil. Third, we do not find convincing reduced-form evidence that the allocation decisions of members of a deputy's party affect either his or her own allocation decisions or vote shares.

We can see this in columns 4 and 5 of Table A.1. In column 4, we regress a deputy's vote share in a municipality on how much she or he allocated, how much the other party members allocated, and the number of votes received by other party members. We include this last variable to proxy for any other forms of party spillovers, such as campaigning, etc. The inclusion of these additional variables has no effect on the relationship between a deputy's vote share and how much she or he allocated. For example, in panel A, we see that a one standard deviation increase in funds is associated with a 2.8 percentage point increase in vote share. In contrast, the coefficient on the other party members' funds is more than an order of magnitude smaller and negative, which is consistent with the fact that open-list PR systems foster intra-party competition. In column 5, we test whether a deputy's allocation decision depends on how much other party members have allocated. We would expect this if, for instance, party members were coordinating on their allocation choices. Instead, we find that the coefficient on the amount allocated by other party members is small in magnitude and statistically insignificant. We do find a positive association between how much deputies allocate and how much other parties allocate, which is consistent with the fact that parties will often compete over votes in larger cities. Comparing panel A to panel B, we see that these patterns for all of Brazil also hold when we only consider the state of Roraima.

In sum, the descriptive evidence suggests that deputies may have both electoral and non-electoral motives when allocating their public funds and voters reward them at the polls for the resources they provide. These patterns are present for Brazil as a whole, but also for Roraima in particular, as documented in panel B of Table A.1 and in the moments we use to estimate the model.

3. Model

Consider an economy in which, in term t, J deputies decide how to allocate a fixed amount of resources $\bar{Q}$ among M municipalities and whether to run for reelection at the end of the term. We model these decisions as a repeated static game that deputies play anew at the start of each term. Thus, we ignore the dependence on the time subscript t in the rest of the paper. Let $q^{j,m}$ denote the amount of resources deputy j allocates to municipality m, $q^j = \left\{q^{j,1}, \ldots, q^{j,M}\right\}$ the collection of allocations chosen by deputy j, $q = \left\{q^1, \ldots, q^J\right\}$ the allocations of all deputies, and $q^{-j} = \left\{q^1, \ldots, q^{j-1}, q^{j+1}, \ldots, q^J\right\}$ the allocations of all deputies except j.

3.1. Preferences

Voters' Preferences. Voters receive resources from deputies to fund local public goods. Voters have municipality-specific preferences over these goods or, equivalently, over the amount of allocated resources Q^m. We represent these preferences with the welfare function $w^m\left(Q^m, K^m\right)$, where w^m is increasing and concave in Q^m. The variable K^m accounts for all other factors in the municipality that affect a voter's welfare. We can calculate the total welfare of municipality m by multiplying w^m by the number of people living in the municipality, N^m:

$$W^m = N^m w^m\left(Q^m, K^m\right).$$

The welfare function is allowed to vary across municipalities to account for possible productivity differences in their use of public funds. This feature of the model enables us to distinguish between productivity versus electoral motives when politicians target places with a lot of voters.

Public funds may not translate entirely into welfare gains. Deputies may steal some of the funds or the resulting public goods may not be fully non-rival, in that the utility an individual derives from the public good may depend on the number of people who use it. For example, an individual may enjoy a park more if fewer people visit it. Therefore, we redefine Q^m to be the actual amount of resources used to fund local public goods after accounting for stealing and rivalry concerns. Formally, we define Q^m as follows:

$$Q^m = \varphi^{'}\left[\sum_{j=1}^{J_R} \varphi^{'}_{R} q^{j,m} + \sum_{j=1}^{J_{NR}} \varphi^{'}_{NR} q^{j,m}\right], \tag{1}$$

where the parameter $\varphi^{'}_{R} \in [0,1]$ measures the fraction of total resources not stolen by the J_R deputies who run for reelection, the parameter $\varphi^{'}_{NR} \in [0,1]$ represents the analogous fraction for the J_{NR} deputies who do not run, and the parameter $\varphi^{'} \in [0,1]$ measures the degree of rivalry.

Because only two of the parameters in Equation (1) can be identified, we rewrite Q^m as follows:

$$Q^m = \varphi^{'}\varphi^{'}_{R}\left[\sum_{j=1}^{J_R} q^{j,m} + \sum_{j=1}^{J_{NR}} \frac{\varphi^{'}_{NR}}{\varphi^{'}_{R}} q^{j,m}\right] = \varphi\left[\sum_{j=1}^{J_R} q^{j,m} + \sum_{j=1}^{J_{NR}} \varphi_{NR} q^{j,m}\right], \tag{2}$$

where φ measures the degree of rivalry times the fraction actually transferred by incumbents who run and φ_{NR} measures the fraction of funds not diverted by incumbents who do not run relative to those who do.

To estimate the model, we will assume the following functional form for the individual welfare function:

$$w^m = \rho_m \log\left(y^m + Q^m\right),$$

where y^m is per-capita income of municipality m. Despite its parsimony, this welfare function captures several important features. The logarithmic specification allows for decreasing returns. The municipality per-capita income y^m captures the existence of other goods and services that may affect the individual's welfare. Lastly, with the coefficient ρ_m, we can account for two types of heterogeneity across municipalities: (i) productivity differences and (ii) differences in the preferences deputies might have for different regions. These two sources of heterogeneity cannot be separately identified without additional data on the productivity of projects located in different municipalities, which currently do not exist. This identification issue is a general result that is independent of our functional form assumption.

Deputies' Preferences. We first describe the preferences of deputies who run for reelection. If deputy j chooses to run, which we denote by $d^j = 1$, j's utility function is composed of four parts: (i) the expected utility from running for reelection; (ii) the welfare of the people living in j's state; (iii) the utility cost of running for reelection; and (iv) preference shocks.

Specifically, let v_p^j be deputy j's utility from winning the election at the end of the term, v_{np}^j the utility if deputy j loses reelection, $\bar{\bar{C}}_R$ the utility cost of running for reelection, and ε_R^j a preference shock. The values v_p^j, v_{np}^j, and $\bar{\bar{C}}_R$, and the distribution of ε_R^j do not vary across terms. Lastly, let p^j denote the probability that deputy j wins the election at the end of the term, which we will derive in the next subsection. For a particular allocation of resources $q = \{q^1, \ldots, q^J\}$ and decisions to run $d = \{d^1, \ldots, d^J\}$ by all incumbents, we can write deputy j's utility as follows:

$$\bar{U}_R^j(q,d) = p^j(q,d)\, v_p^j + \left(1 - p^j(q,d)\right) v_{np}^j + \alpha_j \sum_{m=1}^{M} W^m(q) - \bar{\bar{C}}_R + \varepsilon_R^j,$$

where α_j is the weight that deputy j assigns to the welfare of the state's residents.

The first part of the utility function, $p^j(q,d)\, v_p^j + \left(1 - p^j(q,d)\right) v_{np}^j$, measures the expected utility of running for reelection and represents the egoistic motive of politician j. Provided that $v_p^j \geq v_{np}^j$ – the only case in which the deputy will choose to compete for reelection – this part captures the fact that politicians tend to allocate more resources to municipalities with higher electoral returns. The second part of the utility function, $\alpha_j \sum_m W^m$, describes the altruistic motive of a politician and it enables us to explain why politicians who run for reelection transfer a large fraction of their budget to poor municipalities with few voters.

For the model to be sufficiently flexibility, we assume that the preference shock ε_R^j comprises two parts. The first part is a shock that is allocation specific, i.e. each possible allocation draws a different shock from the same distribution. We denote this component by $\bar{\varepsilon}_{q,R}^j$. The second part, which we denote by $\bar{\nu}_R^j$, is common across allocation choices, but specific to the decision to run for reelection. With these two shocks, we can account for any variation we observe across allocation choices and running decisions that our model cannot explain. We can then rewrite deputy j's utility in the following form:

$$\bar{U}_R^j(q,d) = p^j(q,d)\left(v_p^j - v_{np}^j\right) + \alpha_j \sum_{m=1}^{M} W^m(q) - \bar{C}_R^j + \bar{\varepsilon}_{q,R}^j + \bar{\nu}_R^j,$$

where $\bar{C}_R^j = \bar{\bar{C}}_R - v_{np}^j$. Because v_p^j and v_{np}^j do not vary with the allocation chosen by deputy j, we can divide the politician's utility by $\alpha_j + v_p^j - v_{np}^j$ and obtain

$$U_R^j(q,d) = (1-\beta_j)\, p^j(q,d) + \beta_j \sum_{m=1}^{M} W^m(q) - C_R^j + \varepsilon_{q,R}^j + \nu_R^j,$$

where $\beta_j = \frac{\alpha_j}{\alpha_j + v_p^j - v_{np}^j}$. We assume that $\varepsilon_{q,R}^j$ and ν_R^j are independently distributed, with $\varepsilon_q^j \sim N(0, \sigma_{\varepsilon,R}^2)$ and $\nu_R^j \sim N(0, \sigma_{\nu_R}^2)$.

This alternative formulation of the politician's utility highlights the trade-off deputies face when choosing how to allocate their budget across municipalities. They can allocate their resources to increase municipal welfare or to increase their probability of being reelected. How much a deputy is willing to trade off between these two considerations depends on the parameter β_j. We interpret β_j as the degree of altruism of deputy j. In the estimation, we assume that the degree of altruism can take on two values: β_L for egoistic politicians and β_H for altruistic politicians. The probability that a politician is altruistic is given by π_β.[7]

A deputy who decides not to run has the same form of utility as a politician who chooses to run, except that the probability of winning the elections and the cost of running are now equal to zero. Specifically,

$$\bar{U}_{NR}^j(q) = v_{np}^j + \alpha_j \sum_{m=1}^{M} W^m(q) + \bar{\varepsilon}_{q,NR}^j + \bar{\nu}_{NR}^j,$$

where $\bar{\varepsilon}_{q,NR}^j$ and $\bar{\nu}_{NR}^j$ denote the corresponding preference shocks for deputies who do not run. It will later be clear that the deputy's allocation decisions

7. We have experimented with more than two degrees of altruism, but have found that two degrees was sufficient to explain the allocation patterns we observe in the data.

and choice to run are only affected by the difference $\bar{\nu}_R^j - \bar{\nu}_{NR}^j$. Thus, we set $\bar{\nu}_{NR}^j = 0$ and reinterpret $\bar{\nu}_R^j$ as the difference between the two shocks.

If we divide the utility by the same value used for a deputy who participates in the election, $\alpha_j + v_p^j - v_{np}^j$, we have a utility function that depends on the degree of altruism β_j:

$$U_{NR}^j(q) = \bar{v}_{np}^j + \beta_j \sum_{m=1}^{M} W^m(q) + \varepsilon_{q,NR}^j,$$

where $\bar{v}_{np}^j = \frac{v_{np}^j}{\alpha_j + v_p^j - v_{np}^j}$ and $\varepsilon_q^j \sim N(0, \sigma_{\varepsilon,NR}^2)$. Notice that electoral incentives do not play a role and only welfare considerations affect the allocation decisions of deputies who do not run. This feature of the model allows us to generate the observed pattern that deputies who do not run are more likely to allocate resources to poorer municipalities with fewer votes. Even though these incumbents care only about welfare, they will not necessarily maximize aggregate municipal welfare because they can steal part of their funds.

Note that diverted funds only enter a deputy's utility through the welfare function. This simplification is without loss of generality. Because the fraction diverted does not vary across regions, the amount stolen is the constant $\left(1 - \varphi_R^{'}\right)\bar{Q}$ for deputies who run and $\left(1 - \varphi_{NR}^{'}\right)\bar{Q}$ for deputies who do not. If we were to add these constants to the deputy's utility function, it would not affect the allocation decisions.

3.2. Voting Decisions and Strategic Interactions

Voting Decisions. Resident i of municipality m vote on the basis of three factors: the resources municipality m expects to receive from a candidate in the next term, the candidate's ability to appeal to voters during the elections δ_j, and a voting preference shock, $\xi^{i,j,m}$.

Voters form expectations over future levels of public funds differently depending on whether a politician is an incumbent or a challenger. For incumbents, we assume that voters use observed allocation choices, which is all the information available at the time of the election, to predict the type of the incumbents running for reelection and, hence, their allocation choices in the next term. To make the estimation of the model manageable, we assume that conditional on the allocation decisions to municipality m observed in the current term, past allocations and allocations to other municipalities provide no additional information on the politician's type.[8] We therefore model the

8. The data support this assumption. Conditional on municipality fixed-effects, a region will receive 0.41 cents (robust standard error = 0.09) for every dollar they received in the previous term, but less than 1 cent (coefficient = 0.004, robust standard error=.010) for every dollar they received 2 terms ago. Variables constructed using allocations to other

amount a voter in municipality m expects to receive from deputy j in the next term as follows:

$$E_j\left(q^{j,m'}\left|q^{j,m},q^{-j,m}\right.\right)=f^j\left(q^{j,m},q^{-j,m}\right).$$

In the estimation of the model, the expected allocation function f^j satisfies the following conditions. First, f^j is independent of $q^{-j,m}$ and linear in $q^{j,m}$.[9] Second, the constant term in f^j varies between incumbents (R) and challengers (C), to account for possible incumbency effects in the estimation of the model. Third, the coefficient on the current allocation $q^{j,m}$ varies across municipalities. Consequently, conditional on $q^{j,m}$, voters are allowed to have different expectations about future allocations depending on where they reside. These conditions imply the following form for $f^j\left(q^{j,m},q^{-j,m}\right)$:

$$f^j\left(q^{j,m},q^{-j,m}\right)=\gamma_{0,j}+\gamma_{1,m}q^{j,m},$$

where $\gamma_{0,j}=\gamma_{0,R}$ if j is an incumbent and $\gamma_{0,j}=\gamma_{0,C}$ otherwise. We normalize $\gamma_{0,C}=0$.

Since the J_C challengers are not in power during the current term, voters cannot condition on their previous decisions. Voters form their expectations for challengers by using the probability with which incumbents choose each feasible allocation in the current term. This assumption guarantees consistency of the deputies' choices across terms.

The voters' decisions also depend on the candidate's ability to appeal to voters during the elections, δ_j. We assume δ_j is revealed at the time of the elections. It is therefore known to voters when they cast their ballot, but not to deputies when they make their allocation decisions.[10] In the estimation, we assume that δ_j can take on two values: δ_H with probability π_δ for candidates with high electoral appeal and δ_L with probability $1-\pi_\delta$ for candidates with low electoral appeal. We normalize $\delta_L=0$. We therefore have four types

municipalities are not statistically significant. If one looks at the positive and statistically significant correlation with the allocation in the previous term through the lenses of our model, it is explained by the type of the politician, which is the omitted variable in our regression.

9. We also experimented with an increasing and concave transformation of $q^{j,m}$ by taking the log of $q^{j,m}$, which resulted in similar findings. In addition, we found from various explorations that once we controlled for $q^{j,m}$, variables constructed using $q^{-j,m}$, such as the maximum amount of resources allocated by deputy j's opponents, did not predict subsequent allocations.

10. Computationally, it makes little difference to assume that deputies observe the δ_j's of their rivals. We have opted for the alternative assumption because we believe that, for most politicians, the ability to connect with voters varies depending on the economic and social conditions. Therefore, it becomes public knowledge only during the electoral campaign.

of candidates: (i) high-altruism and high-appeal; (ii) high-altruism and low-appeal; (iii) low-altruism and high-appeal; and (iv) low-altruism and low-appeal. Finally, voters' choices are also affected by a preference shock $\xi^{i,j,m}$, which we assume is drawn from a type I extreme-value distribution.

We can now formalize the voters' decisions. Let $J_E = J_R + J_C$ denote the number of politicians running for office. Individual i in municipality m votes for politician j, if

$$\begin{aligned} j = \operatorname{argmax}_{j \in J} \{ & f^1 \left(q^{1,m}, q^{-1,m} \right) + \delta_1 + \xi^{i,1,m}, \ldots \\ & \ldots, f^J \left(q^{J_E,m}, q^{-J_E,m} \right) + \delta_{J_E} + \xi^{i,J_E,m} \} . \end{aligned} \tag{3}$$

This voting decision is consistent with the residents' preferences. Because voters' welfare is increasing in public funds, it is optimal for residents to vote for the politician who is expected to transfer the largest amount of resources to their municipality, all else equal. With this voting rule, we can rationalize the relationship highlighted in panel (b) of Figure 1 between vote shares and the candidate's ranking in terms of the amount allocated to a municipality.

Using Equation (3), we can calculate the total number of votes each candidate receives. Let $\zeta^{i,j}$ equal 1 if resident i plans to vote for candidate j and 0 otherwise. Also, let $\eta^j \sim U[0, \sigma_\eta]$ be a state-level voting shock that determines the share of politician j's supporters who abstain from voting, with $\sigma_\eta \leq 1$. This shock can be interpreted as the arrival of news about the candidate that leads his or her supporters not to vote. The total number of votes for politician j can then be computed as

$$nv(j) = \left(1 - \eta^j\right) \sum_{i=1}^{N} \zeta^{i,j},$$

where N is the total number of citizens in the state.

The random variable $nv(j)$ can be used to rank all of the candidates in the election and determine who is elected. Specifically, in the absence of party effects, given the incumbents' choices to run for reelection, their allocations, and the realization of the shocks, a politicians is elected if she or he is ranked in the top S positions, where S is the total number of available seats. Thus, we can write the probability that a deputy is elected as:

$$p^j(q, d) = P\left(\eta, \xi : nv(j) > nv(k) \, \text{for all } k \text{ except at most } S - 1 \,|q, d\right). \tag{4}$$

But, as we discussed in Section 2, Brazil's PR system does not necessarily elect all of the top vote getters. This can happen when a candidate receives a lot of votes, but her or his party did not receive enough votes to earn an extra seat. In Roraima, which is represented by 8 deputies, one elected candidate per term was not ranked among the top 8 in terms of vote totals during our sample period. To account for these party effects, we modify the probability in Equation (4) as follows. We first compute in the data the probabilities that a candidate loses a seat if ranked 1 through 8 based on the vote total $nv\left(j\right)$. Since these probabilities are similar for adjacent positions we compute three probabilities: the probability of losing a seat if ranked first or second; the corresponding probability if ranked third through sixth; and the analogous probability if positioned seventh or eight. These probabilities are: $p_{out} = [0.0, 0.0, 0.05, 0.05, 0.05, 0.05, 0.40, 0.40]$. We then compute the probability a candidate gains a seat if positioned 9 through J_E to obtain $p_{in} = [0.40, 0.40, 0.05, 0.05, 0.05, 0.05, 0.0, \ldots, 0.0]$. We then proceed in three steps: (i) we rank-order all the candidates based on their vote totals $nv\left(j\right)$; (ii) with the probabilities given by p_{out} we move one person out of the top 8 places, and replace this candidate with one person not in the top 8 according to the probabilities given by p_{in}; (iii) we use the new ranking to determine the probability that a candidate wins the election. Although this approach is reduced-form, it is consistent with the data and obviates the complications of having to add parties to the model, which would make the estimation infeasible.

Strategic Interactions. In our model, the choices of deputy j depend on the decisions of the other deputies. To deal with these strategic interactions, we make two assumptions. First, deputies decide simultaneously. Second, deputies do not observe the degree of altruism and electoral appeal of the other legislators. They only know the probabilities π_β and π_δ with which the types are independently drawn. To simplify the exposition, we will include in the deputy's type θ_h also the preference shocks, i.e. $\theta_h = \left(\beta_h, \delta_h, \varepsilon_q^h, \nu_R^h\right) \in \Theta$. Let $\sigma\left(\theta_h\right)$ denote the probability that deputy h is of type θ_h. Since β_h, δ_h, ε_q^h, and ν_R^h are drawn from distributions that are independent across deputies, the probability that j's rivals are of type θ_{-j} can be written as follows:

$$\sigma\left(\theta_{-j}\right) = \Pi_{h \neq j} \sigma\left(\theta_h\right).$$

3.3. Deputies' Optimal Decisions

Given the preferences, the voting rule, and the nature of the strategic interactions, we can describe the optimal decisions of the politicians. We proceed in two steps. We first determine their optimal allocation decisions conditional on whether they run for reelection. Given these choices, we then describe whether it is optimal to run.

Let $s\left(\cdot\right)$ be a strategy profile for the game, i.e. a function mapping any value of the vector θ to the pair of vectors $\left(q\left(\theta\right), d\left(\theta\right)\right)$. Then, conditional on running and a strategy profile for the other incumbents $s^{-j}\left(\cdot\right)$, politician j chooses the allocation $q^j = \left\{q^{j,1}, \ldots, q^{j,M}\right\}$ that solves the following problem:

$$V_R\left(\theta_j, s^{-j}(\cdot)\right) = \max_{q^j} \int \left[(1-\beta_j)\, p^j\left(q^j, s^{-j}(\theta_{-j})\right) + \right.$$
$$\left. + \beta_j \sum_{m=1}^{M} W^m\left(q^j, q^{-j}(\theta_{-j})\right)\right] d\sigma(\theta_{-j}) - C_R^j + \varepsilon_{q,R}^j + \nu_R^j$$
$$s.t. \sum_{m=1}^{M} q^{j,m} \leq \bar{Q}, \tag{5}$$

where $V_R\left(\theta_j, s^{-j}(\cdot)\right)$ denotes the value of running and the subscript q in $\varepsilon_{q,R}^j$ is a reminder that each allocation choice draws a different preference shock from the same distribution.

Conditional on not running and a strategy profile for the other incumbents $s^{-j}(\cdot)$, politician j chooses the allocation $q^j = \left\{q^{j,1}, \ldots, q^{j,M}\right\}$ that solves the following problem:

$$V_{NR}\left(\theta_j, s^{-j}(\cdot)\right) = \max_{q^j} \int \left[\bar{v}_{np}^j + \beta_j \sum_{m=1}^{M} W^m\left(q^j, q^{-j}(\theta_{-j})\right)\right] d\sigma(\theta_{-j}) + \varepsilon_{q,NR}^j$$
$$s.t. \sum_{m=1}^{M} q^{j,m} \leq \bar{Q}, \tag{6}$$

where $V_{NR}\left(\theta_j, s^{-j}(\cdot)\right)$ denotes the value of not running.

Given $s^{-j}(\cdot)$, deputy j chooses to compete in the election, $d^j = 1$, if

$$V_R\left(\theta_j, s^{-j}(\cdot)\right) \geq V_{NR}\left(\theta_j, s^{-j}(\cdot)\right).$$

Timing and Equilibrium. Deputies play a game within a term. Its timing is as follows: (i) nature privately reveals β_j and δ_j to the politicians; (ii) the deputies' preference shocks are realized and incumbents simultaneously decide their budget allocations and whether to run for reelection; (iii) the voting preference shocks, the electoral appeal of candidates, and the state-level shock are revealed to the voters, who then cast their vote. The game is repeated in each term.

Then, the Bayesian-Nash equilibrium that characterizes our model can be defined as follows.

Definition 1. The strategy profile $s^*(\cdot)$ is a Bayesian-Nash equilibrium if, for all j and θ_j, the choice $s^{j*}(\theta_j)$ maximizes deputy j's expected utility if of type θ_j, given the strategies $s^{-j*}(\cdot)$ of the other deputies.

3.4. The Social Planner's Problem

Before we discuss how we estimate the model, it is useful to define the benchmark we will use to measure the size of the distortions. Denote by $Q^{TOT} =$

$J\bar{Q}$ the total amount of funds available to the social planner (number of deputies times funds available to each deputy). We then define our benchmark as the allocation $Q^{sp} = \left\{Q^{sp,1}, \ldots, Q^{sp,M}\right\}$ of aggregate funds Q^{TOT} to municipality that maximizes aggregate welfare, i.e.

$$Q^{sp} = \underset{Q^1,\ldots,Q^M}{\arg\max} \sum_{m=1}^{M} W_m\left(Q^m\right) \qquad (7)$$
$$s.t. \sum_{m}^{M} Q^m \leq \varphi\, Q^{TOT},$$

where φ accounts for the degree of rivalry of the local public good. Deviations from this benchmark in the allocation of public funds will be our measure of distortions.[11]

4. Model Estimation and Identification Discussion

4.1. Model Estimation.

To make the estimation of the model computationally tractable, we impose the following additional assumptions. First, as it is common in the estimation of strategic games, we will assume that only one equilibrium is observed in the data (Draganska et al. 2008; Bajari et al. 2010).[12] Second, we discretize the provision of public goods into four choices. Specifically, a deputy can choose to give 0%, 33.33%, 66.66%, or 100% of their budget to a given municipality, subject to the constraint that the allocations must add up to BRL$1.5 million. Allowing deputies to choose among four possibilities is a restrictive but necessary assumption to make the estimation of the model feasible. Third, we aggregate Roraima's 15 municipalities into 4 macro-regions.[13] Region 1, which contains the capital city, is the wealthiest and most populated of the four regions. It has a population of 80,293 inhabitants and its GDP per capita is BRL$5,833. Region 2 is the least populated with 9,658 inhabitants, followed by region 4 (10,495) and region 3 (10,820). These other regions are also similar in terms of their GDP per capita. Besides easing the computational burden,

11. Given our normalization $\varphi_R = 1$, in the social planner's problem, the parameter φ measures the degree of rivalry only if the fraction of resources actually transferred by deputies who run, φ_R, is equal to one. If in reality $\varphi_R < 1$, we underestimate the distortions because in this case the social planner can allocate more funds than the deputies.

12. Bajari et al. (2010) argue that in static games with incomplete information, the number of equilibria tend to decrease as the number of actions and players increase. In their numerical example of a static entry game, they find that with only 5 players a unique equilibrium occurred in 93% of the models they considered.

13. See Figure A.3 for a map of Roriama and its macro-regions.

the aggregation is important for mitigating the spillover effects that may arise from a municipality's public project benefiting its surrounding municipalities.

The final simplification relates to the electoral appeal of politicians. To choose their optimal allocations, deputies have to compute the probability of winning the elections for any possible combination of electoral appeal of all the other candidates, both incumbents and non-incumbents. With two types of electoral appeal, eight incumbents, and twenty challengers, allowing for this form of heterogeneity increases the number of iterations required to compute the optimal choices of each incumbent by a factor of $2^{(7+20)} = 134,217,728$. Thus, it is simply not feasible to estimate this model without additional assumptions.[14] To simplify the calculations, we assume that for a given set of parameter values, incumbents compute the probability of winning the elections using as the electoral appeal of their rivals the average $\pi_\delta \delta_H + (1 - \pi_\delta)\,\delta_L$. With this assumption we have to perform only one iteration for each type of incumbent.[15]

The following Proposition establishes that, given our assumptions, a Bayesian-Nash equilibrium exists.

Proposition 1. *The model with a discretized provision of public goods has a Bayesian-Nash equilibrium in mixed strategies.*

Proof. In the Appendix. □

Despite these simplifying assumptions, our model is still computationally demanding to estimate due to the strategic interactions among 8 players. With 4 regions and 4 possible choices per region, each of the 8 deputies can select among 20 feasible allocations. To compute the expected utility of a single deputy, we have to consider for each one of the deputy's possible choices, all possible combinations of allocations by the seven rivals. In total, this involves computing $20^8 = 2.56e^{10}$ possible combinations. In addition, for each set of parameters, we have to iterate over beliefs to ensure that they are consistent with the deputies' choices. Because of this, even with the use of Fortran, MPI, and between 96 and 148 processors, a single iteration of the model takes on average more than 4 minutes.

We estimate the model's parameters by simulated method of moments (SMM) using data on allocation choices, the decision to run, and electoral outcomes. To compute the simulated moments, we proceed as follows. For a

14. Without additional assumptions, using Fortran and Message Passing Interface with 128 processors, it takes approximately 12 hours to solve the model for one set of parameter values.

15. We can test how sensitive our results are to this assumption by computing the full problem at the final set of estimated parameters. We find that the differences are negligible and do not affect any of the main results of the paper.

given set of parameters, we simulate the deputies' decisions for an initial set of beliefs. Given these optimal decisions, we then compute the set of beliefs consistent with those decisions. If the distance between the two sets of beliefs is large, we re-simulate the model using as initial beliefs the newly generated beliefs. We repeat this procedure until the distance between the initial and simulated beliefs is sufficiently small. Once this fixed point has been achieved, we compute the simulated moments used in the estimation and compare them with the corresponding data moments. We compute the standard errors using the asymptotic distribution of the estimated parameters.

4.2. Moments and Identification

We estimate a vector of 19 parameters. In this section, we discuss the identification of these parameters. Given the model's complexity, a formal proof of identification is not possible. Instead, we provide a heuristic argument for the variation we use to identify each one of the parameters.

Productivity Parameters: ρ_1, ρ_2, ρ_3, ρ_4. The welfare function consists of four parameters: ρ_1, ρ_2, ρ_3, and ρ_4. Because allocations must sum up to BRL$1.5 million per deputy, we can only identify three out of the four parameters. We therefore normalize their sum to 1 and estimate only ρ_1, ρ_2, and ρ_3. In our model, deputies who do not run for reelection allocate resources based only on welfare considerations. Thus, we exploit their allocation decisions to identify the productivity parameters. Accordingly, we use as moments the average share of resources allocated to regions 1, 2, and 3 by incumbents who do not run.

In principle, incumbents who do not run may have other electoral motives that affect their allocation decisions. In practice, however, these motives are limited for the deputies of Roraima. Among the deputies who do not run for reelection, 65% have remained out of politics. The electoral motives for these deputies are likely to be nonexistent, or at best minimal. Of the remaining 35%, 85% sought an elected office in the capital city, such as vice mayor or vice governor, and 15% ran for the state legislature or for a federal seat in the Senate. These politicians do have electoral motives and, given their career choices, had an incentive to target region 1 where a majority of the voters reside and the capital city is located. In this case, we should interpret our estimate of the size of the political distortions as a lower bound.

We also performed a reduced-form check of this assumption by estimating whether deputies gave more to municipalities where they run for a new office either in the past or in the future. We do not find that they do.[16]

16. Specifically, we regress the amount of funds a deputy allocated in a particular municipality during the 1996-1999 term on indicator for whether they ran or will run for office in that municipality. We estimated a statistically insignificant coefficient of 1.070

Another possibility is that deputies who choose not to run allocate more resources to their hometown for personal reasons, such as to fund projects that will benefit themselves and their neighbors. All the deputies from Roraima come from region 1. Thus, if these hometown motives are present, we again estimate a lower bound for political distortions.

Altruism Parameters: β_L, β_H, π_β. To identify the altruism parameters, we rely on differences in allocations between incumbents who run versus those who do not. To see how this variation identifies these parameters, suppose first that all deputies have the same degree of altruism. In this case, the difference in allocations between those who run and those who do not identifies the parameter β. If there is no difference, the degree of altruism will be one. If deputies who run transfer a larger fraction of resources to municipalities with higher electoral gains, β will be less than one and will approach zero as this difference increases.

With two degrees of altruism, the model can account for two groups of deputies who run for reelection and that favor regions with higher electoral gains differently. We identify the fraction of deputies with a higher degree of altruism π_β by using the difference in average funds between the region with the highest electoral gain and the remaining regions, conditional on running. If a larger difference is observed, the model requires a higher fraction of highly egoistic deputies to rationalize the data. Thus, to identify the altruism parameters, we add to the three moments used in the identification of the welfare parameters, the average share of resources allocated to regions 1, 2, and 3 by incumbents who decide to run for reelection.

Voting Function Parameters: γ_0, $\gamma_{1,1}$, $\gamma_{1,2}$, $\gamma_{1,3}$, $\gamma_{1,4}$, σ_θ, δ, π_δ. To identify the incumbency effect γ_0, we use the difference between incumbents and challengers in the average probability of getting elected. A larger difference implies a higher value for γ_0, all else equal.

To identify the effect of public funds on votes $\gamma_{1,m}$, $m = 1, \ldots, 4$, we use two sets of moments: (i) the probability that an incumbent wins conditional on transferring at least 2/3 of their funds to region 1 and the corresponding probability conditional on allocating at least 2/3 of their funds to region m and zero to region 1, for $m = 2, 4$;[17] (ii) the difference in the average share of resources allocated to region m, for $m = 1, 2, 3$, between reelected and non-reelected politicians.[18] Once we condition on the number of voters in region m and a deputy's electoral appeal, large values in both sets of moments indicate

(standard error = 1.106), which implies that deputies only gave about 1.3% more of their budget to those places.

17. For the latter moments, we condition on transferring zero funds to region 1 in order to isolate the choices in which deputies allocated all of their resources to the poor regions with minimal electoral gains.

18. The corresponding moment for region 4 is a linear combination of the moments for the other 3 regions.

that funds allocated to that region translate into a high number of votes. We should therefore identify a larger $\gamma_{1,m}$.

We also use the first set of moments to identify the support of the district-specific shock σ_θ. If σ_θ is large, then the electoral shock will play an important role in determining a deputy's probability of winning. In that case, we should expect to see small differences in the probability of winning between deputies who allocated most of their resources to region 1 and deputies who allocated most of their budget to the other regions. If instead, σ_θ is small, then transfers will have a larger impact on vote totals, and the difference in the probabilities will be larger.

We identify the electoral appeal parameter δ using the probability of winning conditional on allocating zero funds to region 1. If deputies who ignore the high electoral-return region have a high probability of reelection, then the parameter measuring high electoral appeal should be large. Similarly, we can identify the proportion of deputies with high electoral appeal in the population π_δ using the fraction of incumbents who ran for reelection but did not allocate to region 1. If the fraction is large, then the proportion of incumbents with high appeal in the population should also be large.

Rivalry and Diversion of Resource Parameters: φ, φ_{NR}. To identify φ and φ_{NR}, we first compute the difference in per-capita GDP between the region with the highest per-capita GDP and every other region and the corresponding difference in the amount of funds received, conditional on running and not running. We then use the covariance between these two variables for deputies who run and those who do not to identify the two parameters. Specifically, after having computed the two variables, we calculate their ratio conditional on running and not running for each term and $m = 2, 3, 4$. We then derive the two moments used in the estimation by averaging the ratios over terms and $m = 2, 3, 4$, for deputies who run and do not run for reelection.

We use these moments because they exploit the concavity of the welfare function. Deputies without electoral incentives will transfer public funds to the regions with lower per-capita GDP until its residents' marginal welfare equals the marginal welfare of residents of the region with the highest per-capita GDP. The same argument holds for deputies with electoral incentives and some degree of altruism, except that these deputies will only narrow the distance between the marginal welfare of the regions, where the distance depends on the relevance of the electoral incentives and the degree of altruism. Now consider the case in which φ is low, and hence there is a high degree of rivalry in Q_m. In this environment, conditional on the productivity parameters, the transfers to the regions with lower per-capita GDP have to be significantly larger to reduce the difference in marginal welfare with the region that has the highest GDP. If instead, φ is high, a smaller difference in transfers is sufficient to generate the required outcome. We can therefore pin down the parameter φ using the described variation in per-capita GDP and allocated funds.

The identification of the diversion parameter φ_{NR} requires a similar type of variation for incumbents who do not run. If these deputies divert more funds than incumbents who run, a small difference in per-capita GDP requires a larger difference between the funds transferred to region 1 and to region m, for $m = 2, \ldots, 4$, to equate marginal welfare.

Cost of Running Parameter and Variance Parameters: C_R, σ_ν, $\sigma_{\varepsilon,R}$, $\sigma_{\varepsilon,NR}$. We identify the cost of running C_R using the fraction of deputies who run for reelection: the higher the cost of running, the lower the fraction of deputies who choose to run.

To identify the variance of the shocks to the decision to run σ_ν, we use the probability of running conditional on allocating at least 2/3 of one's budget to region 1 and the probability of running conditional on allocating at least 2/3 of resources to region m, for $m = 2, 3$, and 0 to region 1. In our model, the decision to run for reelection depends mainly on two variables: (i) the allocation of funds, which determines the probability of winning a seat and, therefore, the expected benefits of running; and (ii) the size of the shock to the decision to run, which affects directly the utility value of running. If σ_ν is low, the running shocks are generally small and the decision to run is mostly affected by the allocation of funds. In this case, deputies who allocate most of their funds to region 1 have probabilities of running that are much larger than deputies who allocate to other regions. For higher values of σ_ν, the running shocks are generally large and the allocation decisions have smaller effects on the decision to run. The difference between the probability of running for deputies who allocate most of their resources to region 1 and the corresponding probability for deputies who allocate all of their resources to other regions is therefore smaller.

Finally, to identify the variances of the preference shocks $\sigma_{\varepsilon,R}$ and $\sigma_{\varepsilon,NR}$, we use the variances in allocations conditional on running and conditional on not running.

5. Results

5.1. Parameter Estimates

Productivity Parameters: ρ_1, ρ_2, ρ_3, ρ_4. We use the allocation decisions of deputies who do not run for reelection to identify our productivity parameters. We plot these allocations in Panel (a) of Figure 3. Region 1 receives only 10% of the funds, regions 2 and 3 receive more than twice that amount, and region 4 receives the largest proportion of funds at 40%. Our estimates follow a similar pattern: region 1 has the lowest productivity parameter at 0.016, regions 2 and 3 have significantly larger estimated productivity parameters at 0.243 and 0.160, and region 4 has the highest at 0.581 (see Table 2). With these estimates, our model matches the observed allocations quite well, as displayed in Panel (a) of Figure 3 and reported in the first three rows of Table 3. The largest difference between the simulated data (lighter bars) and the actual data (darker bars) is only 0.8 percentage points. To interpret the magnitude of these parameters, we compute the change in welfare generated by reallocating one dollar from the lowest productivity region (region 1) to one of the other three regions. The welfare effect from such a reallocation is 3.03 times larger for region 4 than for region 2 and 6.20 times larger for region 4 than for region 3.

Altruism Parameters: β_L, β_H, π_β. Conditional on the allocation decisions of incumbents who do not run, we can identify the altruism parameters using the allocation decisions of those who do run. We plot these allocation decisions in Panel (b) of Figure 3. Regions 1 and 4 receive the largest fractions of resources, 30% and 29% respectively. Region 2 receives 24% of the funds, and region 3 receives the lowest amount with only 17% of the funds. To match this U-shaped pattern, the model needs two types of deputies with different degrees of altruism: an egoistic type, who cares mostly about reelection incentives ($\beta_H = 0.070$), to explain the large fraction allocated to region 1; and an altruistic type, who cares about both altruism and electoral incentives ($\beta_L = 0.133$), to explain the large fraction allocated to region 4, which has the highest productivity, but relatively few voters. These estimates imply that an altruistic deputy is willing to substitute one vote for 84 dollars of welfare, whereas an egoistic deputy requires a welfare improvement of 171 dollars in exchange for a single vote.

We estimate that 50.1% of candidates are altruistic. This proportion mostly reflects the difference in funds that region 1 receives from deputies who run for reelection relative to the other regions. In the data, region 1 receives about 30% of the funds, whereas the other three regions receive on average around 23%. If this difference had been larger then, conditional on our estimates of β_L and β_H, the model would have required a larger fraction of egoistic types in the candidate pool. With these parameter estimates, we match the U-shaped pattern in the data quite well. The largest difference between the simulated and actual data is only 0.8 percentage points for region 4.

Voting Parameters: γ_0, $\gamma_{1,1}$, $\gamma_{1,2}$, $\gamma_{1,3}$, $\gamma_{1,4}$, σ_θ, δ, π_δ. On average, incumbents enjoy a 32.9 percentage point advantage over challengers in the probability of being elected. Our model can match this difference reasonably well with $\gamma_0 = 0.023$. We can use this estimate to compute the incumbency advantage of a deputy by calculating the average probability that an incumbent wins reelection over all possible allocations in our model and compare it to the same probability when we set $\gamma_0 = 0$. We estimate an incumbency advantage of 11.8%.

Our estimates of the effect of public funds on votes by region are: $\gamma_{1,1} = 0.123$, $\gamma_{1,2} = 0.325$, $\gamma_{1,3} = 0.000$, and $\gamma_{1,4} = 0.010$. The mapping between these estimates and the moments we used to identify them is intuitive. In the data, the incumbents that allocate most of their budget to region 3, while ignoring region 1, are never re-elected; hence, an estimate for $\gamma_{1,3}$ that is statistically equal to zero.

A similar argument applies to our estimates of $\gamma_{1,1}, \gamma_{1,2}$, and $\gamma_{1,4}$ once we consider the number of voters in each of the regions. Region 1 has the highest number of voters. Thus, the model does not require a large coefficient for $\gamma_{1,1}$ to explain the high electoral return for funds allocated to that region. In contrast, regions 2 has only 1/8 of the population of region 1. And yet, if deputies allocate at least 2/3 of their budget to that region without allocating any resources to region 1, they still have a 25% chance of winning, which is slightly below 1/3 of the probability of winning if they transfer all their resources to region 1. To match those moments, our model requires $\gamma_{1,2}$ to be almost 3 times the size of $\gamma_{1,1}$. Region 4 also has only 1/8 of the population of region 1. But there, the incentive to target the region for welfare reasons are much higher. Given these incentives and the reelection rates of deputies who allocate most of their budget to that region, the parameter $\gamma_{1,4}$ does not need to be large to match the moments we use for the voting function.

To provide an economic interpretation of the parameters $\gamma_{1,1}$, $\gamma_{1,2}$, $\gamma_{1,3}$, and $\gamma_{1,4}$, we compute how the probability of being elected changes as an incumbent shifts resources from one region to another. Region 1 is the most attractive in terms of electoral returns: if a deputy were to shift all of her or his resources from Region 3 (the least attractive) to Region 1, the probability of reelection would increase by 93.5 percentage points. By comparison, transferring those resources to Region 2 or Region 4 would increase the likelihood of reelection by 56.5 and 1.2 percentage points, respectively.

We estimate the upper bound of the district-level shock σ_θ to be 0.048. In the data, there is an 85.7 percentage point difference in the probability of winning between allocating at least 2/3 of one's budget to region 1 versus allocating at least 2/3 of the budget to region 3 and nothing to region 1. Because differences in allocations can have such large effects on electoral outcomes, the model does not need large district-level shocks to explain the observed patterns. Our estimates imply that politicians lose at most 4.8% of their votes because of the arrival of district-level shocks.

We estimate the electoral appeal parameter δ to be 0.05. This is consistent with the fact that the probability of winning is relatively high (20%) even if a deputy gives zero resources to region 1. Our estimate of δ implies that politicians with high electoral appeal have a 10.2 percentage point advantage over low-appeal types in the probability of winning, all else equal. We also estimate that approximately 47.6% of politicians have high electoral appeal, which enables us to match the fraction of deputies who choose to transfer zero funds to region 1 and run for reelection.

To see how electoral appeal and altruism affect the allocation of public funds in our model, we plot in Figure 4 the simulated share of funds allocated to each region by our four types of politicians, conditional on running. There is a stark difference between altruistic and egoistic politicians in the allocation of funds. Egoistic deputies allocate approximately 36% of their funds to region 1 compared to only 21% for altruistic politicians. In contrast, altruistic politicians give significantly more to regions 3 and 4 where the electoral returns are the lowest. These differences in allocations across deputies with different degrees of altruism allow the model to explain the U-shaped pattern we observe in Panel (b) of Figure 3. The distinction between high and low electoral appeal types is greatest in regions 1 and 2. Given their electoral advantage, high appeal types can afford to allocate a smaller fraction of their funds to region 1 and shift some of their resources to other regions. This pattern is larger for egoistic politicians, who care relatively more about reelection.

Rivalry and Diversion Parameters: φ, φ_{NR}. We identify the rivalry parameter φ using the covariance between two variables among deputies who run for reelection: (i) the difference in per-capita GDP between region 1 and any other region and (ii) the corresponding difference in allocated funds. In the data, this moment is negative and equal to -0.19, indicating that incumbents who run transfer more funds to the region with the highest per-capita GDP. We expected this because these deputies care about reelection and the region with the highest per-capita GDP also has the most votes. However, this number is substantially closer to zero than it would be absent welfare considerations. To match this number, our model requires a $\varphi = 0.010$, which implies a high degree of non-rivalry: for funded projects to be a private good, φ would have to be 0.000036 ($1/27,817), given that the average population in Roraima is 27,817.

To identify the diversion parameter φ_{NR}, we employ the same variation used to identify φ, but for deputies who forgo reelection. In the data, this moment is equal to 0.158. A positive moment indicates that the region with the highest per-capita GDP receives fewer funds than poorer regions, which is what we would expect if, as we assume in the model, incumbents who do not run have no or only limited electoral incentives. With this moment, we estimate $\varphi_{NR} = 0.611$, which implies that incumbents who forgo reelection divert 38.9% more funds than those who run. Overall, the estimated φ_{NR} highlights an important tradeoff: although deputies who do not run place more weight on welfare considerations, they also divert more resources. We explore this tradeoff further when we consider the policy of a one-term limit.

With our estimates of φ and φ_{NR}, we are able to match the sign of the targeted data moments and relatively well the size of the moment for deputies who forgo reelection. The size of the moment for deputies who run for reelection is noticeably more negative in the data than in the simulations. To match this moment better, the model would have to increase the share of resources allocated to the region with high GDP, region 1, for those deputies. But these politicians already allocate a large share of funds to this region, which matches well what is observed in the data.

Cost of Running and Variances: C_R, σ_ν, $\sigma_{\varepsilon,R}$, $\sigma_{\varepsilon,NR}$. We estimate the cost of running parameter and the variance parameters as a fraction of the value of running to simplify their interpretation. We identify the cost of running parameter using the fraction of deputies who run for reelection. In the data, 71% of incumbents run for reelection. We match this moment well with an estimated cost of running parameter equal to about 0.9% of the total utility value of running. The estimated variances are all relatively small, with the largest one being equal to 3.7% of the utility value of running. This suggests that the shocks in our model do not play an important role in explaining the variation we see in politicians' decisions.

Specification Tests. In Figure 5, we test how well our model matches data moments not used in the estimation. We consider these 11 moments: the probability of running conditional on allocating at least 1/3 of the budget to region m, for $m = 1, \dots, 4$; the probability of winning conditional on allocating at least 1/3 of the budget to region m, for $m = 1, \dots, 4$; the share of funds allocated to region m by deputies who ran but did not win, for $m = 1, 2, 3$.[19]

The model matches these additional moments quite well, given its parsimony. There are only two cases, in which the difference between the simulated and data moments is larger than 5 percentage points: the probability of running conditional on allocating at least 1/3 to region 4 (too high with a difference of 5.4) and the probability of winning for region 2 (too high with a difference of 6.1). Nevertheless, even for these two moments, the difference is relatively small and the model is still able to match the ranking across regions.

5.2. Political Distortions

How much do political incentives affect the allocation of public funds? To answer this question, we compare the distribution of public funds generated by the estimated model to the social planner's allocation, as defined in problem (7). Figure 6, which plots the difference between the two allocations, indicates that deputies distort approximately 26.8% of their allocations relative to the social optimum. The distortions arise because deputies underprovide to region 4, which has low electoral returns but the highest welfare gains.

To understand what determines these distortions, we simulate the model holding constant different factors. We find that we can decompose these distortions into three channels. The first is electoral incentives. If one were to eliminate electoral incentives by equating electoral gains across regions, we would reduce overall distortions by 28.5%. We can also prevent deputies from stealing public funds. If we were to shut down this channel, distortions would decrease by an additional 13.5%. The remaining 58% of the distortions stems from incomplete information. Deputies allocate their funds in response to what they believe other deputies will do, but without knowing their types. Even if we equate electoral gains and eliminate corruption, deputies will still distort their allocations by transferring a smaller fraction of resources to the region with the highest welfare gains relative to the social planner, due to the uncertainty

19. Because of the budget constraint, only 3 out of the 4 moments are independent.

surrounding the actions of the other deputies. As shown in Mazzocco (2020), this is a general result that applies to all Hyperbolic Absolute Risk Aversion welfare functions and not only to our estimated model.

General Insights. Even though the distortions we have estimated are specific to the state of Roraima, the economic and political forces in our model apply broadly. Therefore, we can use the model to provide several general insights. The first insight is that smaller disparities in electoral gains significantly reduce the size of the distortions among regions with similar welfare gains. As the disparities decline, politicians with electoral incentives allocate less resources to regions where the electoral returns are highest, thus reducing the distortions. We can document this general result by simulating a model under three scenarios in which all regions have welfare equal to their average: (i) the region with the most voters (region 1) has the electoral gains observed in the data; (ii) region 1 has half the observed electoral gains; (iii) region 1 has twice the observed electoral gains. The results reported in the first part of Table 4 indicate that reducing the electoral gains by half decreases the distortions by 45.7 %, whereas doubling them increases the distortions by 22.7%. The asymmetry is explained by the fact that the disparity in electoral gains observed in the data is already large, with region 1 having about eight times the number of voters in regions 2-4.

Another insight from our model is that larger disparities in welfare gains increase the size of the distortions among regions with similar electoral gains. To understand this result, it is important to recall that in our model deputies make decisions without knowing the type of the other incumbents. Given this uncertainty about the other incumbents' type and the concavity of the welfare function, deputies who care about welfare will want to insure the regions with lower welfare gains against the possibility of other incumbents providing very few transfers. Thus, if the regions have identical electoral gains, then the deputies will misallocate their funds relative the social planner's allocation by transferring relatively more resources to the regions with lower welfare gains. As these disparities in welfare gains increase, so does the size of the distortions. We quantify this effect by simulating our model for three different settings, under the assumption that all regions have the same average electoral gains and regions 1-3 have the same average welfare gains: (i) region 4, the most productive region, has the observed welfare gains; (ii) region 4 has half the observed welfare gains; (iii) region 4 has twice the observed welfare gains. Table 4 reports that doubling the disparities in welfare gains increases the distortion by 52.5%, whereas reducing the welfare gains of region 4 by half lowers distortions by 91.7%. We observe the asymmetry in results because when we halve region 4's welfare gains, region 4's welfare is very close to the average.

A third general insight involves corruption. Distortions increase with the amount stolen by deputies who do not run. To see why, it is sufficient to consider the social planner's problem. The social planner allocates relatively more to regions with higher welfare gains. As the amount of resources decreases due to corruption, the concavity of the welfare function implies that these regions receive a relatively higher share of them, which increases the distortions. When we simulate these effects, we find that doubling the amount of corruption increases the distortions by 39.9%, whereas reducing corruption lowers distortions by 55.2%.

5.3. Policy Evaluations

In Section 5.2, we document that both electoral incentives and incomplete information play important roles in the distortions we estimate. In this section, we consider two types of policies targeted at these sources of distortions. The first set of policies involve electoral rules. If electoral incentives drive part of the distortions then electoral rules that weaken these incentives might help reduce these distortions. We focus on two alternative electoral rules: approval voting and a one-term-limit policy. The second type of policies aim to lower the information frictions in the model. With incomplete information, incumbents decide where to allocate their funds under uncertainty about the type of all other deputies. One possible way to reduce the uncertainty inherent in their maximization problem is to decrease the number of deputies that compete within a district. We could easily achieve this by introducing a policy that redistricts the state into multiple districts. At the extreme, one could even create single-member districts, as is the case in the U.S. and other countries.

For each policy simulation, we re-compute the deputies' beliefs to be consistent with their choices under the new environment. We do this by iterating over the beliefs until convergence.

Approval Voting. Brazil's currently uses plurality voting: residents can only vote for a single candidate. Approval voting allows citizens to vote for multiple candidates. This alternative system has been proposed by economists and political scientists as an improvement over plurality voting (see Laslier and Sanver (2010) for a series of papers discussing the properties and virtues of approval voting). For example, approval voting encourages voters to vote sincerely and produces electoral outcomes that better represent voters' preferences. Moreover, approval voting generally assures victory to candidates with the greatest overall support, which is not necessarily the case with standard plurality voting (Brams and Fishburn 2005).[20] Our paper is the first attempt to evaluate whether it also has the additional benefit of reducing the distortions created by electoral incentives.

20. Ahn and Oliveros (2016) show that in elections with common values, approval voting outperforms other scoring rules including plurality voting and Borda count in aggregating information.

To formally describe approval voting, consider J candidates who compete in the elections over S seats. Suppose residents can vote for K candidates. We can then define approval voting as a vector of scores for each voter i, $(s^i_1, \ldots, s^i_J)$, with $s^i_j = 1$ if resident i votes for candidate j and $s^i_j = 0$ otherwise, and $\sum_j s^i_j \leq K$. The S seats are won by the politicians with the $S_j = \sum_i s^i_j$ highest total scores. Plurality voting is a special case of approval voting in which $\sum_j s^i_j \leq 1$. In our simulations, we compare plurality voting to an approval voting system in which residents can vote for K candidates with $2 \leq K \leq 5$.[21]

We report the simulation results in Figure 7. We plot the allocation of public funds across the four regions generated by the different approval-voting rules relative to the social planner's allocation. We find that as we allow voters to rank more candidates, distortions do decrease but the reductions are relatively small. For instance, if the government adopted a 5-person rule, the distortions in the allocation of public funds would decrease by only 0.5%.

Given this finding, a natural question to ask is why are the effects of approval voting so small? The reason is that there are two countervailing forces. On the one hand, approval voting incentivizes politicians to reallocate their funds away from regions with higher electoral gains because they no longer need to be ranked first to receive votes. This enables politicians to allocate funds to other regions.

On the other hand, this policy makes the elections more competitive. As we increase the number of candidates voters can rank, the probability that weaker candidates – challengers and those with lower valence – win a seat increases substantially. Under the current system, the probability that a challenger wins an election is 22.5% and a low-valence type 31.8%. If we adopted a 5-person rule, these probabilities would increase by around 9% (see Figure 8). As the elections become more competitive, the incentive to target regions with higher electoral gains increases. This second effect limits the efficacy of approval voting in reducing distortions.

The results depicted in Figure 8 illustrate another important effect of approval voting. In our model, voters believe that challengers will distribute public funds similarly to the current incumbents. Because of this consistency, the major difference between incumbents and challengers in the probability of winning is the incumbency advantage. The fact that challengers win more, as we increase the number of candidates voters can vote for, indicates that approval voting can be effective in reducing the incumbency advantage. It does so, however, at the unexpected cost of increasing electoral distortions.

21. In addition to finding a fixed point over beliefs, approval voting requires the computation of an exploded logit for each simulation to determine the total number of votes each candidate receives. As we increase the number of candidates a voter can list, the computation of the exploded logit becomes exponentially more burdensome. Due to these computational constraints, we are only able to simulate the model up to a 5-person rule.

Term Limits. The second electoral rule we simulate limits incumbents to a single term. Currently, deputies are allowed to be elected indefinitely. However, several countries have argued for, and in some cases implemented, term limits as a way to improve representation. It is noteworthy that the Chamber of Deputies recently voted in favor of this policy for some of Brazil's executive branches.[22]

In standard political economy models, single term limits are never optimal compared to either multiple term limits or no term limits (e.g. Smart and Sturm (2013)). In these models, politicians only care about private gains and the possibility of reelection incentivizes them to choose policies in accordance with voters' preferences. In our model, incumbents also care about aggregate welfare, which creates an interesting tradeoff between having no term limits and a limit of a single term. On the one hand, a single term limit should reduce distortions because electoral incentives would no longer influence the allocation of public funds. On the other hand, deputies who do not run divert significantly more funds than deputies who still face reelection incentives. The ability to determine which of these two effects dominates is an important contribution of our model. When we compare the results of our model to a counterfactual situation in which deputies cannot run for reelection, we find that political distortions decrease from 26.8 to 17.8%. However, because of the increase in corruption, welfare as a whole goes down by 4.5%, suggesting that voters would be worse off with a one-term-limit policy.

Redistricting. In other countries, such as in the U.S., federal legislators represent single-member districts. This eliminates the strategic considerations that incumbent deputies from Brazil must account for when allocating their funds. In this section, we simulate the effects of reducing the number of deputies that have to compete within a single district. We believe that these results would speak to a policy that either reduces proportionally the number of deputies across states or redistricts states into several sub-districts while maintaining the same number of representatives. In fact, in 2009 a deputy (Antonio Carlos Mendes Thame - PSDB) from the state of São Paulo proposed such a bill (PLP 545/2009). The idea of the reform was to divide a state into subdistricts with a minimum of 3 incumbents. Under this proposal, the state of Roraima would consist of two subdistricts with four representatives.

In Figure 9, we plot the distortions associated with three separate simulations that gradually reduce the number of deputies from eight to two. The level of distortions decreases significantly and monotonically as we reduce the number of deputies. For example, when we go from eight to four deputies,

22. On 5/27/2015, the Chamber of Deputies voted to eliminate reelection for the office of President, Governor, and Mayor. For more information see: http://noticias.uol.com.br/politica/ultimas-noticias/2015/05/27/camara-vota-o-fim-da-reeleicao.htm

distortions decrease by 24%. The reason for this decline becomes clear when we decompose the sources of the distortions for the case of only four deputies. Incomplete information accounts for 32% of the distortions compared to our previous estimate of 58% with 8 deputies. Electoral incentives now account for the majority of the distortions at 54.5%.

5.4. Robustness

Selection. A key assumption underlying our simulated policies is that the parameters of the model are invariant to our electoral reforms. Because we do not model the decision to become a politician, one might be concerned that our policies also affect the proportion of types in the candidate pool. For example, we have shown that approval voting improves the electoral chances of politicians with lower ex-ante probabilities of winning a seat. Thus, we also expect approval voting to increase the proportion of politicians who are altruistic or have low electoral appeal.

In Appendix Table A.2, we evaluate the robustness of our policy simulations to increases in the proportion of candidates who are either altruistic or of low electoral appeal. We focus on the 5-person scoring rule and the 4-person redistricting policy. For each policy, we measure the distortion in public funds as we increase, by 2 percentage points at a time, the proportion of altruistic and low-appeal types in the candidate pool.[23] For approval voting, we find that political distortions remain virtually unchanged. For the redistricting policy, For redistricting policy, the distortion decline slightly as we increase the proportion of low probability types, but the effects are quite small. Moreover, because the policy makes elections more competitive, we would expect it to decrease the proportion of low probability types.

Strategic Voting. We have also assumed that citizens vote sincerely for the candidates from whom they expect to receive the most public funds. This is a reasonable assumption given our reduced-form finding that voters only reward politicians who transfer the largest or second largest amount of resources to their municipality (see Figure 1). If strategic voting were important, we would expect to see a significant share of votes for candidates who do not transfer a substantial fraction of resources to the municipality. Nevertheless, using the estimated model, we can assess the robustness of our policies to the possibility that some residents vote strategically. In Appendix Table A.2, we also re-simulate the 5-person scoring rule and the 4-person redistricting policy under the assumption that residents vote strategically. In particular, we assume that residents do not vote for candidates with less than a 10% probability of winning the election and instead cast their votes in favor of one of the remaining

23. We do not simulate these effects for the term limit policy because a change in politician type does not affect the allocation decisions of politicians who do not run.

candidates.[24] The results in the table suggest that our policies are robust to strategic voting.

6. Conclusions

This paper develops and estimates a model of how politicians allocate public funds in an environment in which incumbents play a strategic game. We apply the model to the Brazilian context, where federal legislators are entitled to approximately R$1.5 million per year to fund projects within their state. Based on the budgetary allocation decisions of federal legislators representing the state of Roraima, our estimates suggest that deputies misallocate approximate 26.8% of these public funds relative to a social planner's allocation. To reduce these distortions, it would be worth considering a policy that reduces the numbers of deputies that represent a single district, as was proposed to the chamber in 2009. A policy that limited deputies to a single term would also be effective at reducing distortions, but the policy would have to come with additional safeguards against corruption for it to be in the voters' best interests.

Overall, our study highlights the importance of political institutions for the allocation of public expenditures and the type of distortions that can arise when the incentives that affect politicians' decisions are different from those that characterize a social planner. Our paper is also one of the first attempts to determine whether changes to the electoral rule can help align these incentives. Our results suggest that some emphasis should also be placed on attracting better types of politicians (Ferraz and Finan 2009; Dal Bó et al. 2013).

Although our model fits the data relatively well, it is quite parsimonious and can be extended and generalized in several interesting directions. One possible extension would be to make the game dynamic. As Diermeier et al. (2005) correctly emphasize, politicians are forward-looking agents whose career choices are dynamic in nature. We model these intertemporal aspects to some degree through the decision to run for reelection, but it would be interesting to model the dynamic dimension more explicitly, for instance by taking into consideration the fact that politicians can run for reelection for multiple terms. Other directions of future research will ultimately depend on the collection of new data. For example, with data on campaign spending, one could easily extend our model to examine whether budgetary amendments complement or substitute campaigning (Da Silveira and De Mello 2011; Kang 2016). One could

24. As is common in the political science literature, we define strategic voting as the act of choosing a less-preferred candidate when one's most preferred option has little to no chance of winning (e.g. Riker and Ordeshook (1968). The assumption of a 10% threshold is not critical for the results. We have experimented with various thresholds ranging from 5%-20% and have found similar results.

then investigate the impact of campaign financing laws on not only electoral performance, but also public funds allocation.

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7. Tables and Figures

	Brazil		Roraima		
	1996-1999		1996-1999	1996-2013	
	Mean	S.D.	Mean	Mean	S.D.
Budgetary Amendments					
Amount per deputy per year (millions)	1.32	2.47	1.21	1.17	1.16
Number issued per deputy per year	16.13	5.32	12.93	8.98	6.69[a]
Number of candidates per seat	6.35	1.71	4.75	8.25	2.41
Number of parties per seat	1.44	0.71	1.75	2.50	5.40
Elected by the party	0.13	0.33	0.25	0.25	0.44
Ran for reelection	0.68	0.18	1.00	0.71	0.23
Reelection rate	0.72	0.15	0.75	0.47	0.21
Municipal GDP per capita	3,643.76	4,030.76	4,335.06	4,752.32	1048.71
Population	30,111.42	180,597.60	21,418.00	24,560.00	56117.58

Notes: Columns 1 and 2 report summary statistics averaged across states for the period 1996-1999 (i.e. 50^{th} legislature). Column 3 reports summary statistics for the state Roraima during the same period. Column 4 reports summary statistics for the state of Roraima during the period 1996-2013, which is the period used in the estimation. Four legislative elections took place during this period: 1998, 2002, 2006, 2010. [a] In column 4, we compute the average number of amendments issued for the period 1996-2003. After 2003, we only have data on the total amount issued by deputy per year.

Table 1. Summary Statistics

	Parameter	Estimate	Std Err
Welfare Function			
Productivity in region 1	ρ_1	0.016	0.006
Productivity in region 2	ρ_2	0.243	0.043
Productivity in region 3	ρ_3	0.160	0.042
Productivity in region 4	ρ_4	0.581	0.046
Altruism			
Egoistic type	β_H	0.070	0.017
Altruistic type	β_L	0.133	0.050
Proportion of altruistic politicians	π_β	0.501	0.299
Voting Decisions			
Incumbency advantage	γ_0	0.023	0.008
Effects of public funds on vote shares, region 1	$\gamma_{1,1}$	0.123	0.044
Effects of public funds on vote shares, region 2	$\gamma_{1,2}$	0.325	0.096
Effects of public funds on vote shares, region 3	$\gamma_{1,3}$	0.00001	0.0001
Effects of public funds on vote shares, region 4	$\gamma_{1,4}$	0.010	0.458
High electoral appeal	δ	0.050	0.015
Proportion of high appeal politicians	π_δ	0.476	0.116
Upper bound of support of state-level shock	σ_θ	0.048	0.011
Rivalry and Diversion			
Degree of rivalry	φ	0.010	0.005
Diversion of funds if not running	φ_{NR}	0.611	0.211
Decision to Run and Shocks			
Cost of running	C_R	0.005	0.003
Std. dev. of shocks on decision to run	σ_ν	0.051	0.022
Std. dev. of preference shocks if running	$\sigma_{\varepsilon,R}$	0.020	0.004
Std. dev. of preference shocks if not running	$\sigma_{\varepsilon,NR}$	0.013	0.002

Notes: This table presents the model's parameter estimates. The standard errors are computed using the asymptotic distribution of the estimated parameters.

TABLE 2. Parameter Estimates

Moments	Model (1)	Data (2)
Welfare Function		
Share allocated to region 1 if not running	0.112	0.104
Share allocated to region 2 if not running	0.223	0.229
Share allocated to region 3 if not running	0.243	0.250
Share allocated to region 1 if running	0.296	0.300
Share allocated to region 2 if running	0.240	0.242
Share allocated to region 3 if running	0.164	0.167
Voting Function		
Pr(Winning) for incumbents - Pr(Winning) for challengers	0.306	0.329
Pr(Winning) if share allocated $\geq 2/3$ to region 1	0.885	0.857
Pr(Winning) if share allocated $\geq 2/3$ to region 2 & 0 to region 1	0.461	0.250
Pr(Winning) if share allocated $\geq 2/3$ to region 4 & 0 to region 1	0.102	0.333
Difference between winners and losers in share allocated to region 1	0.180	0.200
Difference between winners and losers in share allocated to region 3	-0.044	-0.033
Difference between winners and losers in share allocated to region 4	-0.095	-0.025
Covariance between difference in allocations and GDP if running	-0.100	-0.190
Covariance between difference in allocations and GDP if not running	0.117	0.158
Pr(Winning) if share allocated to region 1 = 0	0.219	0.200
Pr(Running) if share allocated to region 1 = 0	0.607	0.556
Decision to Run and Shocks		
Pr(Running)	0.730	0.714
Pr(Running) if share allocated $\geq 2/3$ to region 1	0.926	0.875
Pr(Running) if share allocated $\geq 2/3$ to region 2 & 0 to region 1	0.707	0.571
Pr(Running) if share allocated $\geq 2/3$ to region 3 & 0 to region 1	0.534	0.333
Variance of allocation shocks if running	0.003	0.003
Variance of allocation shocks if not running	0.012	0.012

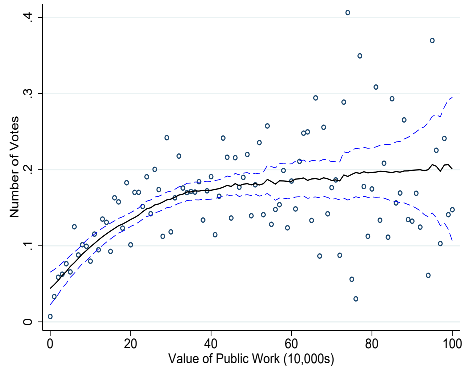
Notes: This table presents the moments used to estimate the model's parameter. Column 1 reports simulated moments based on 10,000 simulations. Column 2 reports the data moments.

TABLE 3. Moments Used in the Estimation

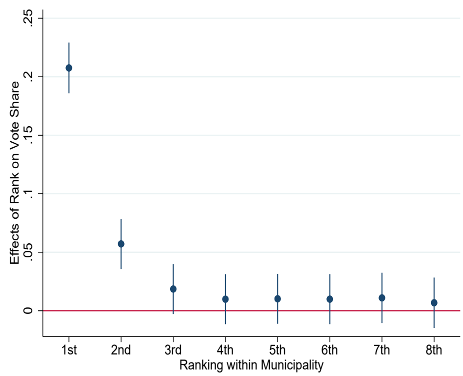
	% Change in Distortions
Panel A: Changes in Political Gains with Identical Welfare	
Region 1 Has Half the Political Gains	−45.7%
Region 1 Has Twice the Political Gains	22.7%
Panel B: Changes in Welfare with Identical Political Gains	
Region 4 Has Half the Weight on Welfare	−91.7%
Region 4 Has Twice the Weight on Welfare	52.5%
Panel C: Changes in the Fraction Stolen by Non-Running Deputies	
Half the Estimated Fraction Stolen	−55.2%
One and Half the Estimated Fraction Stolen	39.9%

Notes: This table presents percent changes in the size of the distortions from simulating the model under the conditions specified in each row. In panel A, all regions have welfare equal to their average. In Panel B, all regions have the same average electoral gains and regions 1-3 have the same average welfare gains. We compute these distortions based on 10,000 simulations.

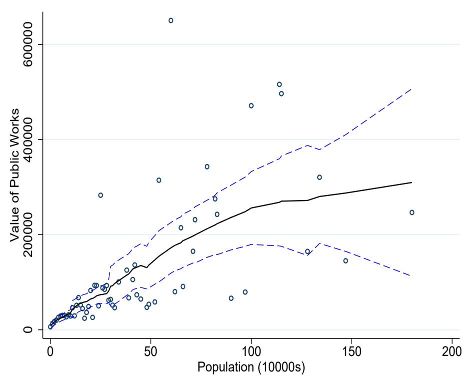
TABLE 4. Changes in Distortions with Changes in Welfare and Political Gain Disparity



(A) Public Funds and Share of Votes



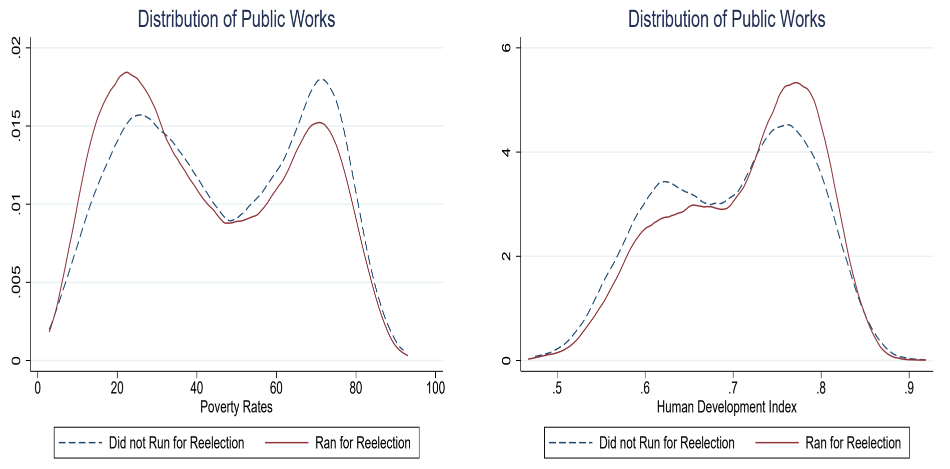
(B) Vote Share and Rank by Amount of Public Funds



(C) Public Funds and Population Size

FIGURE 1. Public Expenditures and Outcomes

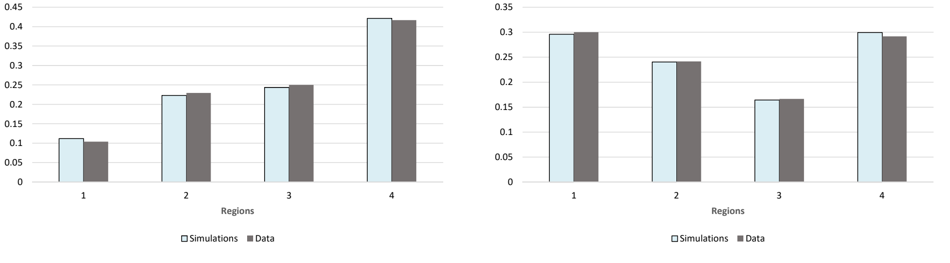
Notes: Panel A depicts the association between the amount of public funds deputies allocated to the municipality during the 1996-1999 term and the share of votes they received in 1998 elections. Panel B plots coefficient estimates from a regression of the share of votes an incumbent received in a municipality and a set of dummies indicating the incumbent's rank within the municipality based on spending. Incumbents ranked above 22 are the excluded category. Panel C depicts the association between population size and the amount of public funds a deputy allocated to the municipality during the 1996-1999 term. The solid line was computed using lowess. The dashed lines are the corresponding 95 percent confidence intervals. Each dot represents the mean of the dependent variable computed based on equally-sized bins.



(A) (B)

FIGURE 2. Distribution of Public Funds by Welfare Levels

Notes: Panel A depicts kernel density plots of the allocation public funds by poverty level of the municipality. Panel B depicts kernel density plots of the allocation public funds by the municipality's Human Development Index. These density plots are estimated separately for incumbents who ran for reelection and those that did not. These figures were computed based on a sample of 5,550 municipalities.



(A) Did not run for reelection (B) Ran for reelection

FIGURE 3. Comparison Between Model's Prediction and Actual Allocations

Notes: This figure compares the allocation of public funds as predicted from our model to the actual data. Panel A plots by region, the share of public funds allocated by incumbents who did not run for reelection. Panel B plots by region, the share of public funds allocated by incumbents who ran reelection. The simulated allocations are based on 10,000 simulations.

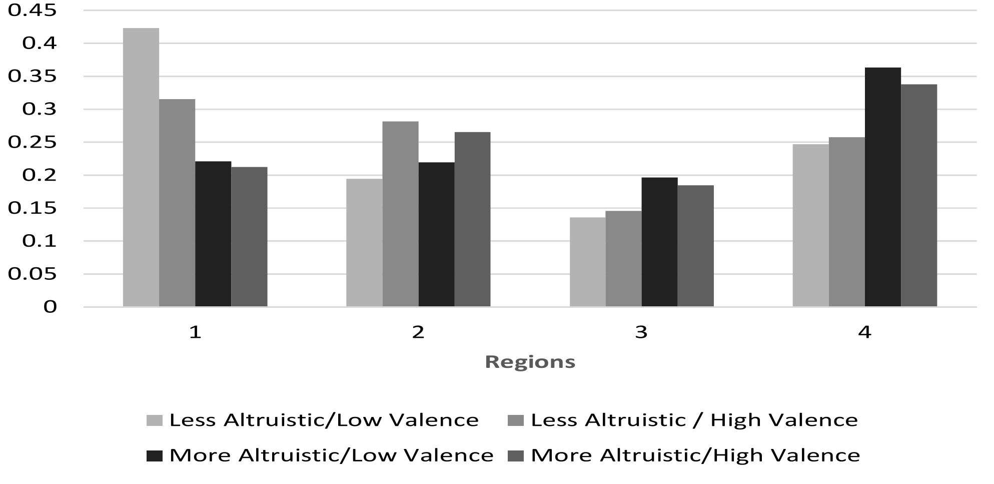
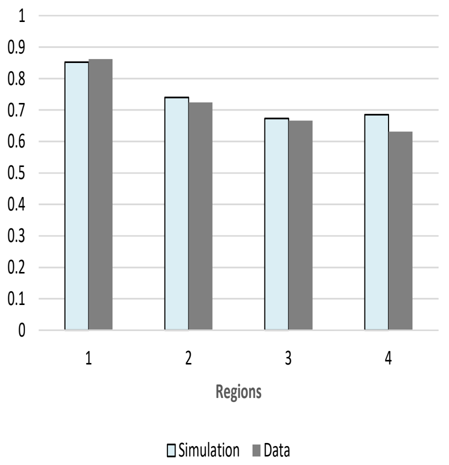
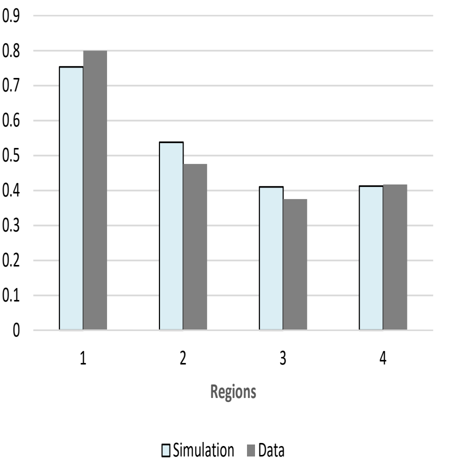


Figure 4. Distribution of Allocations By Politician Type

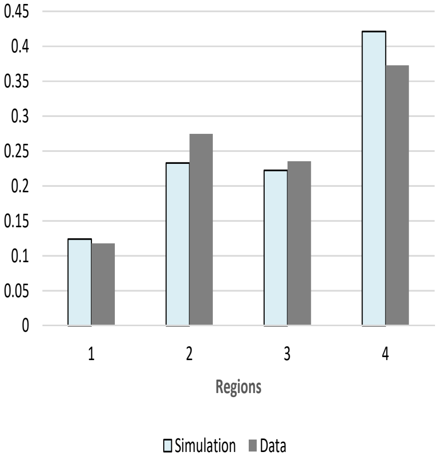
Notes: This figure plots the share of public funds allocated by incumbents who ran reelection, by politician type. The simulated allocations are based on 10,000 simulations.



(A) $Pr(\text{Running}|\text{Allocation} \geq 1/3)$



(B) $Pr(\text{Winning}|\text{Allocation} \geq 1/3)$



(C) Allocations among those who ran but were not elected

FIGURE 5. Comparison Between Model's Prediction and Actual Allocations - Moments not Matched

Notes: Panel A plots the probability of running conditional on allocating more than 1/3 of one's budget to a particular region. Panel B plots the probability of winning conditional on allocating more than 1/3 of one's budget to a particular region. Panel C plots the allocation decisions of incumbent who ran but were not elected.

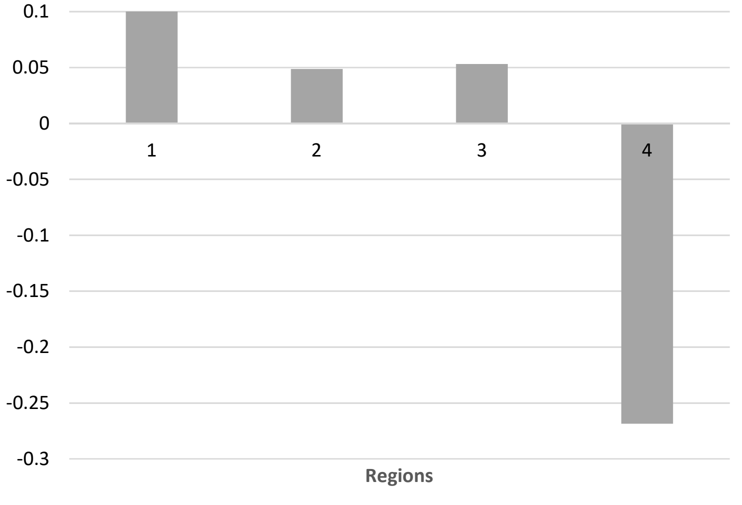


FIGURE 6. Deviation from the Social Planner

Notes: This figure plots the allocation of public funds relative to the social planner allocation for each of the policy simulations. The share of public funds is computed by region and averaged over 10,000 simulations.

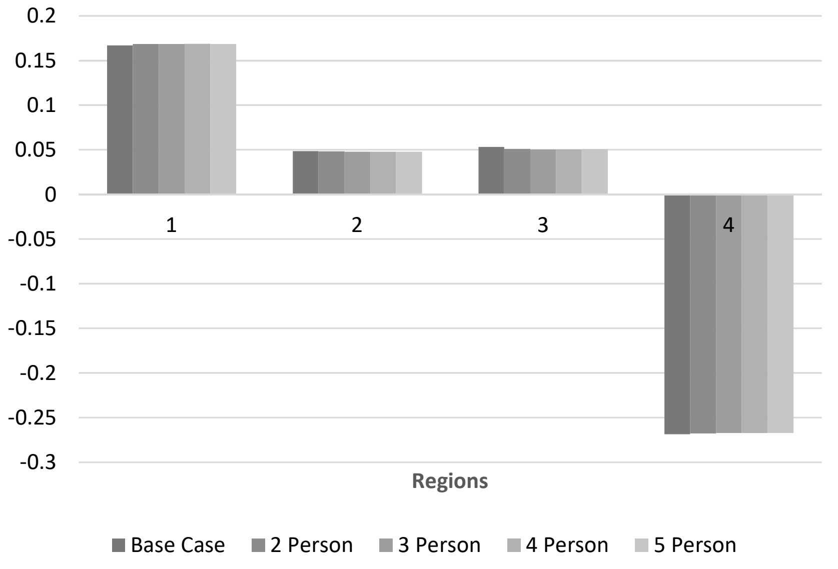


Figure 7. Policy Simulations: Approval Voting

Notes: This figure plots the allocation of public funds relative to the social planner allocation for each of the policy simulations. The share of public funds is computed by region and averaged over 10,000 simulations. The 2-person policy refers to the scoring rule: $(1, 1, 0, \ldots, 0)$. The 3-person policy refers to the scoring rule: $(1, 1, 1, 0, 0, \ldots, 0)$, etc. The base case policy refers to our original results.

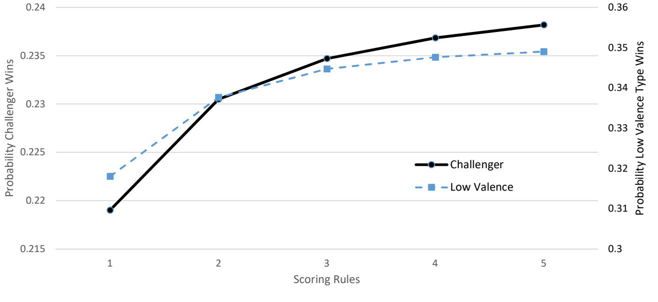


Figure 8. Probability a Challenger Wins

Notes: See note in Figure 7.

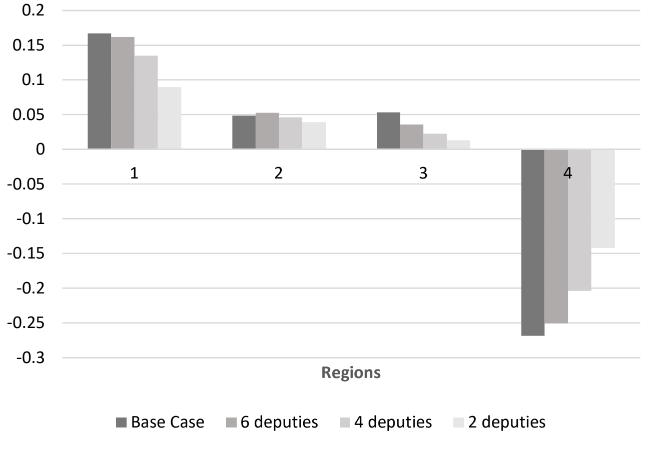


FIGURE 9. Policy Simulations: Redistricting

Notes: This figure plots the allocation of public funds relative to the social planner allocation for each of the policy simulations. The share of public funds is computed by region and averaged over 10,000 simulations. The n-deputy policy refers to a policy in which there are only n deputies in the district, for $n = 2, 4, 6$. The base case policy refers to our original results.

Appendix A: Appendix: Tables and Figures

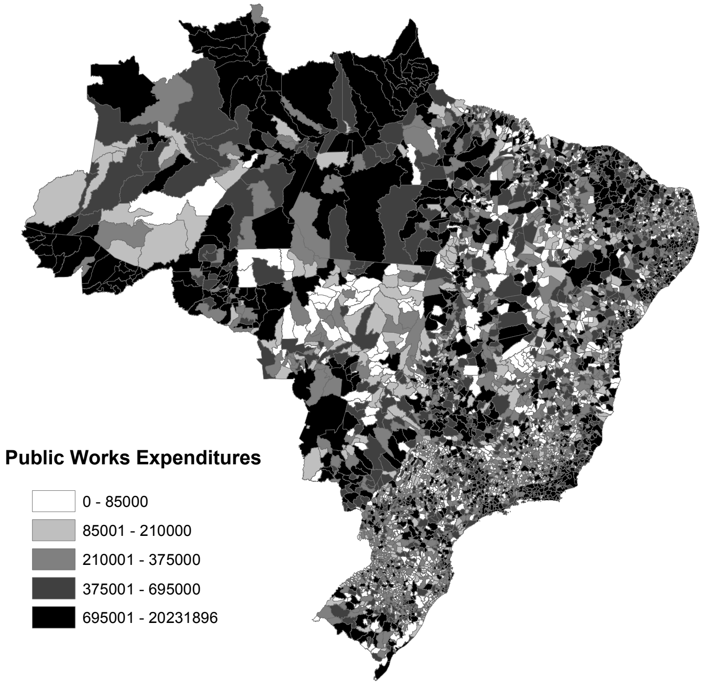
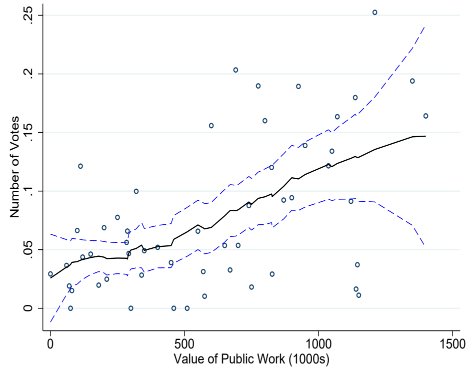
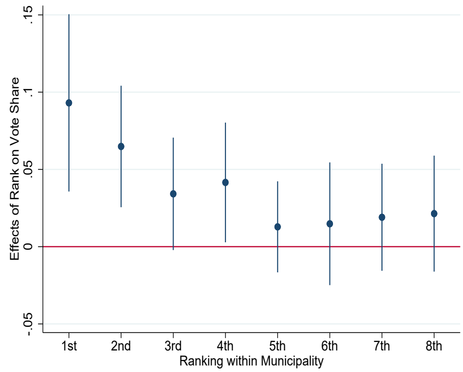


Figure A.1. Distribution of Budgetary Amendments

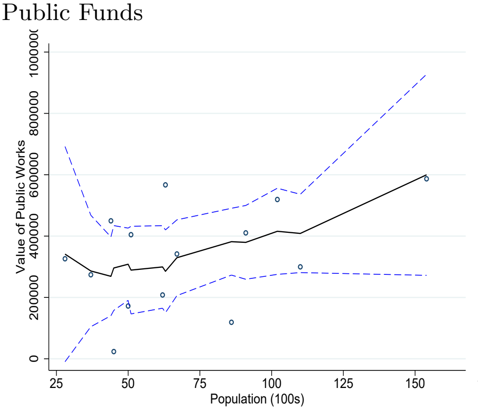
Notes: The map depicts the distribution of public funds during the 1996-1999 term by municipality.



(A) Public Funds and Share of Votes



(B) Vote Share and Rank by Amount of Public Funds



(C) Public Funds and Population Size

FIGURE A.2. Public Expenditures and Outcomes for the state of Roraima

Notes: Panel A depicts the association between the amount of public funds deputies allocated to the municipality during the 1996-1999 term and the share of votes they received in 1998 elections. Panel B plots coefficient estimates from a regression of the share of votes an incumbent received in a municipality and a set of dummies indicating the incumbent's rank within the municipality based on spending. Incumbents ranked above 8 are the excluded category. Panel C depicts the association between population size and the amount of public funds a deputy allocated to the municipality during the 1996-1999 term. The solid line was computed using lowess. The dashed lines are the corresponding 95 percent confidence intervals. Each dot represents the mean of the dependent variable computed based on equally-sized bins.

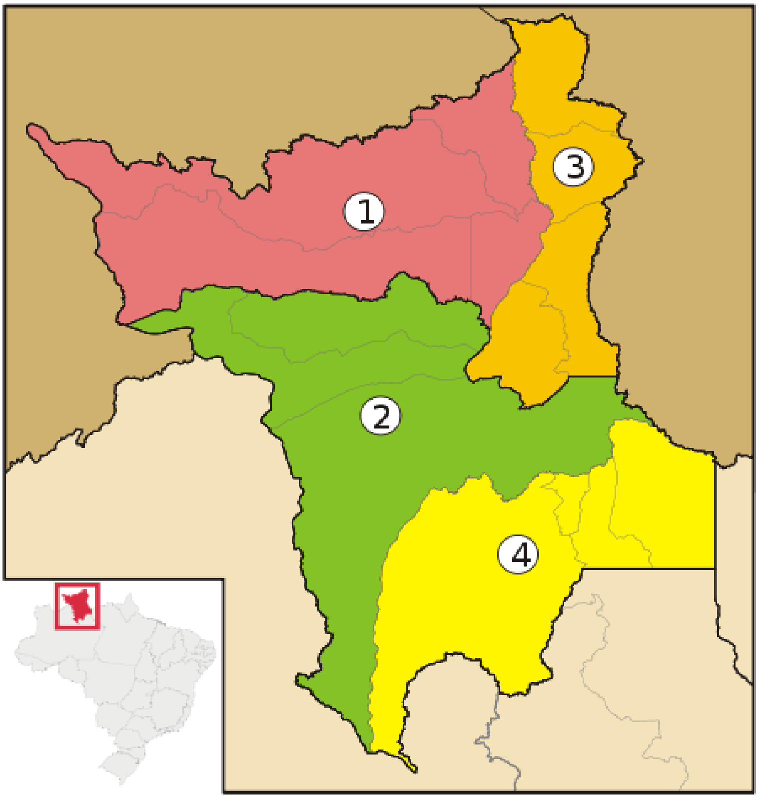


Figure A.3. State of Roraima

Notes: The map depicts the state of Roraima and its macro-regions.

	Vote Share				Amount of Funds Allocated
	(1)	(2)	(3)	(4)	(5)
Panel A: Brazil					
Public funds ($100,000s)	0.027***	0.028***		0.028***	
	(0.002)	(0.002)		(0.002)	
Rank within the municipality			-0.027***		
			(0.002)		
Public funds from other party members				-0.001***	0.002
				(0.000)	(0.004)
Total votes of other party members				-0.000**	
				(0.000)	
Public funds from other parties					0.007***
					(0.001)
Number of observations	154139	154139	154139	151489	154097
R^2	0.13	0.25	0.21	0.25	0.06
Panel B: State of Roraima					
Public funds ($100,000s)	0.007***	0.007***		0.007***	
	(0.001)	(0.001)		(0.001)	
Rank within the municipality			-0.012***		
			(0.002)		
Public funds from other party members				-0.000	-0.087
				(0.001)	(0.078)
Total votes of other party members				-0.000**	
				(0.000)	
Public funds from other parties					0.007
					(0.024)
Number of observations	120	120	120	120	120
R^2	0.31	0.56	0.46	0.56	0.05
Municipality Intercepts		✓	✓	✓	
Deputy Intercepts		✓	✓	✓	✓

Notes: Column 1 reports the unadjusted correlation between the amount of public funds a deputy allocated to the municipality during the 1996-1999 term and the number of votes she or he received from the municipality during the subsequent election. Column 2 reports the same relationship as in Column 1 but adjusts for both deputy and municipal fixed-effects. Column 3 reports the relationship between the number of votes a deputy received and her or his ranking in the municipality with respect to the amount of public goods she or he provided. Columns 4-6 replicate the regressions in columns 1-3 but use the deputy's vote share in the municipality as the dependent variable. The estimation has been restricted to only those incumbents that ran for reelection. Clustered standard errors by deputy and municipality in parentheses.

Table A.1. Relationship between Electoral Performance and Allocation of Public Funds

	Base Case	Strategic Voting	Increase in Low Probability Types				
			2%	4%	6%	8%	10%
Approval Voting (5-person)	0.269	0.268	0.267	0.267	0.267	0.267	0.267
Redistricting (4-person)	0.204	0.204	0.203	0.201	0.201	0.200	0.199

Notes: This table reports the distortions in public fund allocations associated with the policies listed in each row under different modelling assumption. In column 1, we report the distortions under the base case. In column 2, we assume that voters vote strategically. In columns 3-8, we increase the proportion of high altruism and low valence types in the candidate pool by the amount indicated in each column.

Table A.2. Effects of the Policy on Distortions: Robustness

Appendix B: Appendix: Proof of Proposition 1

The proof is based on the existence results established in Milgrom and Weber (1985). To use their results, it is helpful to rewrite our model in the following way. Remember that $q = \left\{q^1, \ldots, q^J\right\}$ denotes the allocations of all deputies, $d = \left\{d^1, \ldots, d^J\right\}$ their decisions to run for reelection, and $\theta_j = \left(\beta_j, \delta_j, \varepsilon_q^j, \nu_R^j\right)$ describes deputy j's type. Let

$$U_R^j\left(q^j, d^j, q^{-j}, d^{-j}; \theta_j, \theta_{-j}\right) = \left(1-\beta_j\right) p^j\left(q, d\right) + \beta_j \sum_{m=1}^{M} W^m\left(q\right) - C_R^j + \varepsilon_{q,R}^j + \nu_R^j$$

and

$$U_{NR}^j\left(q^j, d^j, q^{-j}, d^{-j}; \theta_j, \theta_{-j}\right) = \bar{v}_{np}^j + \beta_j \sum_{m=1}^{M} W^m\left(q\right) + \varepsilon_{q,NR}^j.$$

Then, deputy j chooses the optimal allocation and whether to run according to the following problem:

$$\max_{d^j}\left\{\max_{q^j}\int U_R^j\left(q^j, d^j, q^{-j}, d^{-j}; \theta_j, \theta_{-j}\right)\sigma\left(d\theta_{-j}\right),\right.$$
$$\left.\max_{q^j}\int U_{NR}^j\left(q^j, d^j, q^{-j}, d^{-j}; \theta_j, \theta_{-j}\right)\sigma\left(d\theta_{-j}\right)\right\}.$$

The problem can alternatively be written in the following form:

$$\max_{q^j, d^j} d^j \int U_R^j\left(q^j, d^j, q^{-j}, d^{-j}; \theta_j, \theta_{-j}\right)\sigma\left(d\theta_{-j}\right) +$$
$$\left(1-d^j\right)\int U_{NR}^j\left(q^j, d^j, q^{-j}, d^{-j}; \theta_j, \theta_{-j}\right)\sigma\left(d\theta_{-j}\right).$$

Or equivalently,

$$\max_{q^j, d^j}\int\left[d^j U_R^j\left(q^j, d^j, q^{-j}, d^{-j}; \theta_j, \theta_{-j}\right) + \right.$$
$$\left.\left(1-d^j\right)U_{NR}^j\left(q^j, d^j, q^{-j}, d^{-j}; \theta_j, \theta_{-j}\right)\right] d\sigma\left(\theta_{-j}\right).$$

We can therefore redefine the utility of deputy j as

$$U^j\left(q^j, d^j, q^{-j}, d^{-j}; \theta_j, \theta_{-j}\right) = d^j U_R^j\left(q^j, d^j, q^{-j}, d^{-j}; \theta_j, \theta_{-j}\right) +$$
$$(1-d^j)U_{NR}^j\left(q^j, d^j, q^{-j}, d^{-j}; \theta_j, \theta_{-j}\right).$$

Deputy j's problem can then be written as follows:

$$\max_{q^j, d^j}\int U^j\left(q^j, d^j, q^{-j}, d^{-j}; \theta_j, \theta_{-j}\right)\sigma\left(d\theta_{-j}\right).$$

Let $s\left(\cdot\right)$ be a strategy profile of the game, i.e. a function mapping any value of the vector θ to the pair of vectors $\left(q\left(\theta\right), d\left(\theta\right)\right)$. We can now define a pure-strategy and a mixed-strategy Bayesian Nash equilibrium for this setting.

Definition B.1. A pure-strategy Bayesian Nash equilibrium is a strategy profile $s(\cdot)$, such that for every deputy $j \in J$ and type $\theta_j \in \Theta$:

$$s^j(\theta_j) = \arg\max_{s^j} \int U^j\left(s^j, s^{-j}(\theta_{-j}); \theta_j, \theta_{-j}\right) \sigma(d\theta_{-j}).$$

To define a mixed-strategy Bayesian Nash equilibrium, for every type θ_j, let $m^j\left(s^j; \theta_j\right)$ be a probability measure over the strategy space S^j, and M^j player j's set of such mixed strategies. Then, we can extend the deputy j's utility to the set of mixed strategies by an expected utility calculation:

$$U^j\left(m^j, m^{-j}; \theta_j, \theta_{-j}\right) = \int_{S_1} \cdots \int_{S_J} U^j\left(s^j, s^{-j}; \theta_j, \theta_{-j}\right) m^1\left(ds^1; \theta_1\right) \ldots \\ \ldots m^J\left(ds^J; \theta_J\right) \sigma(d\theta_{-j}).$$

We can now introduce the mixed extension of the initial game in pure strategy $G = \left(S^j, U^j\right)_{j=1}^{J}$ as $\bar{G} = \left(M^j, U^j\right)_{j=1}^{J}$.

Definition B.2. A mixed-strategy m^* is a mixed-strategy Bayesian Nash equilibrium of the initial game G if m^* is a pure-strategy Bayesian Nash equilibrium of the extended game $\bar{G}$.

Theorem 1, Propostion 1, and Proposition 3 in Milgrom and Weber (1985) establish that a game of incomplete information of the type considered in this paper has a mixed-strategy Bayesian Nash equilibrium if two conditions are satisfied: (i) the set of actions available to each player S^j is finite and (ii) the types of the players, $\theta_1, \ldots, \theta_J$, are drawn from independent distributions. In the model we estimate, each player has a finite set of actions since she can choose among four possible allocations of resources, and types are drawn from independent distributions. Hence, a mixed-strategy Bayesian Nash equilibrium exists.