Economics 202A
Suggested Solutions to the Monetary Economics Problem Set

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1 The tradeoff between low inflation and policy flexibility

This is a version of the Barro-Gordon (or Keydland-Prescott) model with the standard Phillips curve

$$y = y^* + b(\pi - \pi^e)$$

and the social welfare function

$$SWF = \gamma y - \frac{a\pi^2}{2}.$$

However now we assume that Central Bank has a different objective function that assigns a different weight to the output

$$U_{CB} = c\gamma y - \frac{a\pi^2}{2}, \quad c > 0.$$

(a) Given π^e and parameters, CB will maximize its utility subject to the Phillips curve. We can substitute Phillips curve into the objective function to get

$$\max_{\pi} U_{CB} = c\gamma(y^* + b(\pi - \pi^e)) - \frac{a\pi^2}{2}$$

and the FOC is

$$\begin{aligned} c\gamma b &= a\pi, \\ \pi &= \frac{c\gamma b}{a}, \end{aligned}$$

which does not depend on the expected inflation because the utility of the central banker is linear in output.

(b) Central bank observes γ but the public does not when it sets its expectations. Therefore rational expectations assumptions will give us

$$\pi^e = \mathrm{E}(\pi) = \mathrm{E}(\frac{c\gamma b}{a}) = \frac{c\gamma^* b}{a}.$$

(c) We can write $\gamma = \gamma^* + \epsilon_\gamma$, where ϵ_γ is the deviation of γ from its mean and thus have mean zero and variance σ^2_γ. Then $\pi - \pi^e = c\epsilon_\gamma b/a$, We now can substitute these results to evaluate the expected value of the SWF

$$\begin{aligned}
\mathrm{E}SWF &= \mathrm{E}[\gamma y - \frac{a\pi^2}{2}] = \mathrm{E}[\gamma(y^* + b(\pi - \pi^e)) - \frac{a\pi^2}{2}] = \\
&= \mathrm{E}[\gamma(y^* + \frac{b^2 c\epsilon_\gamma}{a}) - \frac{b^2c^2\gamma^2}{2a}] = \\
&= \mathrm{E}[\gamma y^* + \frac{b^2 c\gamma\epsilon_\gamma}{a} - \frac{b^2c^2\gamma^2}{2a}] = \\
&= y^*\mathrm{E}\gamma + \frac{b^2c}{a}\mathrm{E}(\gamma^*\epsilon_\gamma + \epsilon^2_\gamma) - \frac{b^2c^2}{2a}\mathrm{E}\gamma^2 = \\
&= y^*\gamma^* + \frac{b^2c}{a}\sigma^2_\gamma - \frac{b^2c^2}{2a}(\sigma^2_\gamma + \gamma^{*2}).
\end{aligned}$$

(d) Now we decide how different should be the central banker's utility function from the social welfare function in order to maximize SWF given the policy of the central banker. We maximize the result from (c) with respect to c.

$$\begin{aligned}
\frac{\partial SWF}{\partial c} &= \frac{b^2}{a}\sigma^2_\gamma - \frac{b^2c}{a}(\sigma^2_\gamma + \gamma^{*2}) = 0, \\
c &= \frac{\sigma^2_\gamma}{\sigma^2_\gamma + \gamma^{*2}}
\end{aligned}$$

There are two goals central banker is trying to achieve - adjust the output to the peoples variable taste and keep inflation low. How much weight the central banker attaches to these goals depends on his c. It turns out that there is an optimal level of c for the public. It depends negatively on the average level of γ^*, because the higher us the γ on average, the higher is the unexpected inflation on average (from the Phillips curve), the more we want central banker to concentrate on the offsetting this effect (thus lower c). The c is increasing in the variance of the shock. If the shocks to our preferences are very variable, we want central banker to be able to adjust (be flexible) and put less weight on the inflation (thus lower c).

2 Money versus interest rate targeting

See answers to problem 3 in the New Keynesian problem set suggested solutions.

3 Uncertainty and policy

This problem suggest you to analyze the model with the uncertainty about policy effects.

The output is given by the following expression

$$y = x + (\kappa + \epsilon_\kappa)z + \nu,$$

where z is the policy instrument and it is affected by the random multiplier $k = \kappa + \epsilon_\kappa$. The only random variable that is known to the policymaker is x. What shall the government do in this case?

Because government knows x when it sets its policy z, it can set z as a function of x. However, because the multiplier on z is also random it is not optimal to do so in a discretionary way, because then the shocks to multiplier will move y as a result of z all over the place. Therefore we can conclude that there should be a rule that government follows.

Another justification for the rule is that if it is optimal to set z as some particular function of x at time t, it should be optimal to do the same thing at time $t+i$, since there is no expectations and dynamic inconsistency problem here. Therefore we can restrict our attention to the rule.

What is the rule? Both z and x enter the equation for y in a linear way, therefore it makes sense to restrict attention to the linear function $z = a + bx$. Then

$$\begin{aligned} y &= x + k(a + bx) + \nu = (1 + bk)x + ak + \nu = \\ &= (1 + b\kappa)x + b\epsilon_\kappa x + a\kappa + a\epsilon_\kappa + \nu. \end{aligned}$$

(a) What is the variance of the output if government follows the rule described above?

$$\begin{aligned} \mathrm{Var}(y) &= (1 + b\kappa)^2\mathrm{Var}(x) + b^2\mathrm{Var}(\epsilon_\kappa x) + a^2\sigma_\epsilon^2 + \sigma_\nu^2 = \\ &= (1 + b\kappa)^2\mathrm{Var}(x) + b^2\mathrm{Var}(x)\sigma_\epsilon^2 + a^2\sigma_\epsilon^2 + \sigma_\nu^2, \end{aligned}$$

where we used the fact that $\mathrm{Var}(XY) = \mathrm{Var}(X)\mathrm{Var}(Y)$ if X and Y are independent and mean zero.

(b) Now we find a and b that minimize this variance. Take then FOCs of the equation derived in (a) with respect to a and b.

$$\begin{aligned} \frac{\partial \mathrm{Var}(y)}{\partial a} &= 2a\sigma_\epsilon^2 = 0 \implies a = 0 \\ \frac{\partial \mathrm{Var}(y)}{\partial b} &= 2(1 + b\kappa)\mathrm{Var}(x)\kappa + 2b\sigma_\epsilon^2\mathrm{Var}(x) = 0 \implies \\ b &= -\frac{\kappa}{\kappa^2 + \sigma_\epsilon^2}. \end{aligned}$$

Therefore the optimal policy rule is

$$z = -\frac{\kappa}{\kappa^2 + \sigma_\epsilon^2}x.$$

Note that it is optimal for the government to offset the shocks. Also, if ϵ_κ would not be there, it would be optimal to completely offset the shock by setting $z = x/\kappa$, but because there is an uncertainty about the multiplier, it is dangerous to do so because of the risk to create too much noise to the output. Therefore, the more noisy is the multiplier, the more careful is the government in using its policy instrument, because it can have unexpectedly large effect on the economy.

(c) We can see from our solution to (b) that z does not depend on the variance of ν. This should not be a surprise, since ν is completely unobserved, policy can not depend on it directly and thus can not offset its effect. Thus the uncertainty about the current state of the economy should not affect the case for government intervention.

(d) See the discussion in the end of part (b). The uncertainty about the policy instrument makes government less able to counteract the shock. And because of the risk to "over-do" government should not try to offset all the shock.