

Consumption Commitments: Neoclassical Foundations for Habit Formation*

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Abstract

This paper studies a model where consumption has two components, one that is costlessly adjustable and one that involves an adjustment cost. Agents have time-separable neoclassical preferences over the two components of consumption. We show that aggregating over a heterogeneous population of such agents implies dynamics identical to those of a representative consumer economy with habit formation utility. In particular, aggregate consumption is a slow-moving average of past consumption levels, and risk aversion is amplified because the marginal utility of wealth is determined by excess consumption over the prior commitment level. In certain cases, the representative consumer's preferences and consumption dynamics coincide exactly with the exponential form of habit proposed by Constantinides (1990). Hence, consumption commitments provide simple, non-psychological micro-foundations for "habit."

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1 Introduction

Representative consumer habit formation models have attracted interest in many areas of economics. In asset pricing, Sundaresan (1989), Constantinides (1990) and Campbell and Cochrane (1999) use habit formation to understand the mean equity premium and the joint dynamics of equity returns and aggregate consumption. In monetary economics, a growing literature makes use of habit preferences to match the hump-shaped response of consumption to interest-rate shocks (Fuhrer (2000), Christiano, Eichenbaum, and Evans (2003), McCallum and Nelson (1999)). In other applications, Boldrin, Christiano and Fisher (2001) demonstrate that habit preferences can help match a variety of business cycle facts, including the excess sensitivity of consumption. Carroll, Overland and Weil (2000) show that habit sheds light on the joint behavior of savings and growth. Ljungqvist and Uhlig (2000) show that optimal fiscal policies are very different with habit-forming consumers.

Habit formation has traditionally been modelled as a direct assumption about the structure of preferences, motivated by the intuition that psychological factors make agents accustomed to consuming certain goods. In this paper, we provide a non-psychological motivation for habit-like behavior that arises from technological constraints in consumption rather than an assumption about preferences.

The basic observation that motivates our model is that most consumption goods have a component that is difficult to adjust in the short run. As noted by Grossman and Laroque (1990), who demonstrated the importance of adjustment costs for consumption and savings behavior, frictions in adjustment are widespread. For example, changing consumption of illiquid durables such as housing, cars, and furniture typically requires payment of a transaction cost either implicitly or explicitly. Consumption of services such as insurance, education, or cellular phones often involves signing a contract with large penalties for early termination.¹ Similarly, significant portions of nondurable consumption (e.g., food, clothing, fuels) are also difficult to adjust. If one lives in an area with few restaurants, it may be difficult to increase food consumption by going to more expensive restaurants in the short run.

These observations suggest a model of “consumption commitments” where consumption has two components, one that is freely adjustable at all times, and another that can only be adjusted infrequently, perhaps by payment of a transaction cost. We analyze a model where agents have conventional time-separable preferences over these two components of consumption. Our main

¹Late payments do not eliminate consumption commitments. In the event of an income shock, the bill must still be paid. The ability to make late payments only acts as an additional credit channel.

result is that aggregate dynamics in an economy with heterogeneous households of this type coincide precisely with those of existing representative-consumer habit models. In this sense, consumption commitments provide neoclassical micro-foundations for habit formation.²

The intuition underlying the connection between commitments and habit can be seen with an example. Consider an individual who consumes commitment and non-commitment components of a consumption good in equal shares. After making his commitment, suppose the individual faces a shock that necessitates a 10% reduction in expenditure. If the agent cannot adjust his commitments immediately, he must cut discretionary consumption by 20%, raising his marginal utility of wealth more sharply in the short run. Hence, commitments, like habit, magnify the impact of shocks on marginal utility and effectively make the consumer more risk averse. In the long run, the individual may choose to adjust the committed component as well. The committed portion of consumption thus acts as a state variable that adjusts with a lag, making shocks have lasting impacts on consumption. This example illustrates that the commitment model shares two basic features with habit models: amplified risk aversion and sluggish adjustment to shocks.

We formalize the connection between the commitment and habit theories by first modelling the portfolio choice and consumption decision of an individual who has neoclassical preferences over the two components of consumption. To make the model analytically tractable, we assume that consumers have separable, constant elasticity of substitution preferences. We consider two types of adjustment rules for the commitment component: time-dependent and state-dependent. As we discuss below, the time-dependent framework is a better description of the adjustment process for some consumption goods, while the state-dependent framework is more suitable for others. The main aggregation results are similar in both models.

In the time dependent model, agents are permitted to reset commitment consumption costlessly at certain (possibly stochastic) times, but cannot change commitments at any other date. Agents can invest in two publicly traded assets (risky and safe). The risky asset follows a general exponential Brownian motion that permits time-varying returns, and nests standard cases such as i.i.d. returns and mean-reverting returns. We assume that markets are complete, which requires that agents can make arrangements to insure the idiosyncratic risk associated with uncertainty in adjustment dates. We consider a continuum of such agents with staggered adjustment dates. The dynamics of total consumption in this economy are identical to those that arise from representative-consumer habit formation utility, with a habit stock that depends on a slow-moving time average of past consumption levels. In the special case where adjustment dates follow a Pois-

²The qualitative similarity between commitments and habit has been observed in prior studies; see below for further details.

son process, the habit specification coincides exactly with the exponential form of Constantinides (1990), with the exception that we have external rather than internal habit formation.

In the state dependent model, agents can adjust consumption of the commitment component at any time, but must pay a transaction cost to do so. Aggregating models of consumers who follow state-dependent adjustment rules derived from utility maximization is generally quite difficult. The most tractable specification for aggregation, developed by Caballero (1993) in the context of durables purchases, typically cannot be obtained from maximization of power utility. As a result, previous studies of aggregate dynamics with adjustment costs and optimizing agents use numerical techniques (Marshall and Parekh, 1999), time-dependent adjustment (Lynch, 1996, Gabaix and Laibson, 2001, Reis, 2004) or approximate solutions (Caballero and Engel, 1993). These studies focus on a model with a single illiquid good, as in Grossman and Laroque (1990).

We make progress on the aggregation problem here by showing that the introduction of a second, adjustable component of consumption makes aggregation *easier*.³ Coupled with an appropriate specification of idiosyncratic risk across households to smooth aggregate dynamics, the second component enables us to use tools developed by Caballero (1993) to characterize the cross-sectional distribution of commitment consumption using a stochastic partial differential equation. This differential equation can in turn be used to characterize the dynamics of aggregate commitments. These dynamics are shown to coincide with the dynamics of the habit stock for a representative consumer with habit formation utility. In the state-dependent model, the habit equivalence result is obtained with incomplete markets, but requires i.i.d. returns for the publicly traded risky asset. In addition, the weights on past consumption are more complicated than in standard specifications because they depend on the entire history of past shocks. However, in the limit as aggregate uncertainty becomes small relative to idiosyncratic risk, the habit stock becomes a time-average with deterministic weights that decay exponentially.

Both aggregation results stem from the fact that in habit formation models, consumer well-being is determined by surplus consumption over current habit, which is a slow-moving time average of past consumption levels. In our models, commitments are slow-moving at the microeconomic level in that they are changed infrequently. When an adjustment takes place, the new commitment level reflects current prosperity, and therefore recent consumption levels. Consequently, the aggregate commitment stock is a slow-moving function of past aggregate consumption levels. Summing the individual Euler equations yields an aggregate Euler equation where utility is determined by surplus consumption over the level of commitments, exactly as in the habit model.

³Beaulieu (1993) also notes that the presence of an adjustable good simplifies the household problem, but does not study aggregate dynamics.

The habit representations extend to an environment with multiple consumption goods, each of which has a commitment and adjustable component. Since all types of consumption involve frictions to some degree, the model can in principle rationalize habit-like behavior for all consumption categories. However, the model predicts that “habit” should be stronger in consumption categories that involve larger commitment components. For example, consumption of housing should adjust more sluggishly to macroeconomic shocks than consumption of food or clothing. We present some suggestive evidence regarding this prediction using the methodology of Parker and Julliard (2005). Using data on consumption from national accounts, we find that excess stock returns generate larger responses in the short run for lower-commitment consumption categories, as the model predicts. Several recent empirical studies of consumption behavior (summarized in section 4) also find that more discretionary components of consumption are more responsive to shocks. While other psychological factors could also be important, these results suggest that commitments are a source of habit-like behavior in practice.

An important caveat in evaluating the empirical content of the model is that the covariance between consumption and returns is not large even in lower-commitment categories (e.g., food or nondurables). There are two potential explanations for this low covariance given the high observed equity premium: (1) All categories, including food consumption, have a substantial commitment feature; or (2) there are other non-commitment factors which also contribute to the sluggishness of adjustment. Exploring the empirical implications of commitment effects for equity premia is an important issue left for future research. In this paper, we focus on establishing the equivalence of consumption commitments at the microeconomic level with habit formation in the aggregate.

This paper builds on and contributes to a few other strands of the literature in macroeconomics in addition to the literature on habit formation described above. The qualitative similarity between the commitment and habit models has been pointed out in several recent papers that study adjustment costs in consumption. Dybvig (1995) examines ratcheting consumption demand in a model with extreme habit persistence, and notes that such preferences could be motivated by pre-commitment in consumption. Flavin and Nakagawa (2003) analyze asset pricing in a two-good adjustment cost model, and observe that consumption adjusts slowly in response to shocks, as in habit formation models. Postlewaite, Samuelson and Silverman (2004), Fratantoni (2001), and Li (2003) analyze two-good models with adjustment costs in different contexts and also note their similarity to habit. Our analysis contributes to this literature by presenting a formal aggregation theorem showing that adjustment cost and habit models are not merely similar, but observationally identical at the aggregate level.

This paper is also related to a growing literature that examines the implications of commitments in many other contexts. Postlewaite, Samuelson and Silverman (2004) show that commitments generate incentives to bunch uninsured risks together, potentially explaining real wage rigidities. Chetty (2003) shows that agents with consumption commitments exhibit significantly higher degrees of risk-aversion to moderate-stake wealth fluctuation than they do to large-stake wealth fluctuations using data from labor markets. Shore and Sinai (2004) show that amplified moderate-stake risk aversion makes households who are subject to exogenously higher income risk optimally undertake larger housing commitments. Olney (1999) gives historical evidence of the macroeconomic importance of commitments by arguing that large exposure to installment finance forced households to cut back on other consumption and was therefore responsible for a significant share of the welfare loss during the Great Depression.

Several studies have examined the importance of durable goods for consumption and asset pricing in other contexts. Dunn and Singleton (1986), Eichenbaum and Hansen (1990), and Heaton (1993), Heaton (1995) carefully model the service flows from durable consumption and combine durability with time non-separable preferences. More recently, Lustig and Nieuwerburgh (2004), Piazzesi, Schneider and Tuzel (2005), and Yogo (2006) explore additional channels through which durables affect asset pricing such as composition risk and collateral concerns. These papers do not incorporate the adjustment cost feature of durable goods. Hence, their ideas are relevant independent of commitment effects.

The remainder of the paper is organized as follows. The model and aggregation results with the time-dependent adjustment rule are presented in the next section. Section 3 considers state-dependent adjustment. Section 4 discusses suggestive evidence. Section 5 concludes.

2 Time-Dependent Adjustment

2.1 Setup

Consider an economy inhabited by a continuum of consumers with identical neoclassical preferences over two components of a consumption good: commitment (x) and non-commitment, or adjustable, consumption (a). Each consumer maximizes expected lifetime utility given by

$$\mathbb{E} \int_0^{\infty} e^{-\rho t} \left(\frac{a_t^{1-\gamma}}{1-\gamma} + \mu \frac{x_t^{1-\gamma}}{1-\gamma} \right) dt \tag{1}$$

where ρ is the discount rate and μ measures the relative preference for commitments. Because utility is time-separable, γ measures the elasticity of intertemporal substitution as well as relative risk aversion for an individual who is free to adjust both x and a . Because utility is separable

across the two components, the intertemporal and across-good elasticities of substitution are also equal. These simple preferences make the model analytically tractable.⁴

There are two financial assets traded in the economy.⁵ The first is a riskless bond, with a price process given by

$$\frac{dB_t}{B_t} = r dt$$

where r is the riskfree rate, which is assumed to be constant. The second is a risky asset with dynamics given by an exponential Brownian motion

$$\frac{dS_t}{S_t} = (r + \pi_t)dt + \sigma_t dz_t$$

where z_t is a standard Brownian motion that generates a filtration $\{\mathcal{F}_t, 0 \leq t < \infty\}$, π_t is the expected excess return (equity premium), and σ_t is the standard deviation of asset returns. We assume that π_t and σ_t are measurable, adapted to the filtration $\{\mathcal{F}_t\}$, σ_t is uniformly bounded, $\sigma_t > \varepsilon$ almost surely for some $\varepsilon > 0$, and π_t satisfies Novikov's condition:

$$\mathbb{E} \left[\exp \left(\frac{1}{2} \int_0^t \pi_s^2 ds \right) \right] < \infty \quad \text{for all } t \geq 0. \quad (2)$$

This specification of dynamics for the risky asset nests several special cases of interest. For example, i.i.d. returns are obtained when $\pi_t = \pi$ and $\sigma_t = \sigma$ are constant. When π_t/σ_t follows an Orstein-Uhlenbeck process, returns are mean-reverting (as in Kim and Omberg, 1996). When π_t/σ_t follows a square-root process, the Campbell and Cochrane (1999) specification of expected returns is obtained.

Let w_t denote the wealth of the consumer in period t and α_t the share of wealth held in the risky asset. Normalizing the relative price of the commitment and adjustable components at one, define total consumption as $c = a + x$. The dynamic budget constraint is

$$dw_t = [(r + \alpha_t \pi_t) w_t - c_t] dt + \alpha_t w_t \sigma_t dz_t. \quad (3)$$

Note that the total mean return on the wealth portfolio is $r + \alpha_t \pi$, and a wealth share of α_t in risky assets gives a standard deviation of $\alpha_t \sigma_t$ in the growth rate of wealth.

In this section, we model commitments using a time-dependent adjustment rule. Agents are able to costlessly change consumption of x at certain ‘‘adjustment dates,’’ but cannot change

⁴Cobb-Douglas preferences also permit a habit representation result. In that case, the representative consumer has proportional habit utility (as in Abel, 1990) over adjustable consumption, with a habit stock that is a slow-moving function of past levels of both a and x consumption. We focus on the CES case here in the interest of space.

⁵In practice, certain commitments may also be assets (e.g., housing). With complete markets, introducing such assets does not affect the analysis.

consumption of x at any other time. The adjustment opportunities arrive at random dates, which are independently and identically distributed across the population. This model of commitment is motivated by the observation that changes in many types of consumption are often prompted by exogenous shocks. For instance, empirical studies of mobility find that a large fraction of housing sales are prompted by changes in family composition or employment (e.g., Merrill, 1984, Venti and Wise, 1989, Venti and Wise, 2000). Similarly, individuals typically have an opportunity to update insurance choices when events such as marriage or job changes occur. Of course, the time dependent framework is not a suitable description of the adjustment process in all circumstances. In the next section, we consider a model where individuals can adjust consumption at any time by paying a transaction cost, and show that this specification generates similar behavior in the aggregate economy.

Let $G(s)$ denote the unconditional probability distribution function of the time s that elapses between two adjustment dates. We assume that $G(\cdot)$ is the same for all households, and for technical reasons, make the following “asymptotic smallness” assumption about $G(\cdot)$.

Assumption 1 *There exist $K, m > 0$ such that for all s and t where $G(t) < 1$, $(1 - G(s + t))/(1 - G(t)) \leq Ke^{-ms}$.*

This assumption ensures in particular that the conditional expected waiting time after t periods have elapsed without adjustment is at most K/m and hence finite.

Our general setup admits two important special cases. The first is the case of deterministic adjustment dates, where each household is free to adjust x only every T periods; between these periods, x is fixed and no adjustment is possible. The adjustment dates themselves may be staggered across households, but the length of time between adjustment dates is T for all households. This case corresponds to models of Taylor pricing. A second special case is when adjustment dates arrive at an exponential rate: during a short interval dt , a consumer can adjust x with probability λdt . This specification is similar to the Calvo-pricing rule common in macroeconomics.⁶ Our formulation is more general than either of these cases because we allow adjustment dates to be probabilistic and their arrival rate to depend on the length of time since the last adjustment occurred.

In addition to the financial assets introduced above, we assume that other securities exist such that markets are dynamically complete. In the special case with deterministic adjustment dates, this assumption holds by design, since the only source of uncertainty is the risky asset. With

⁶Assumption 1 is satisfied for both of these special cases. In the deterministic adjustment case, choose any K, m pair with $K > e^{mT}$. In the exponential arrivals case, set $m = \lambda$ and $K = 1$.

stochastic adjustment, market completeness requires introducing additional securities (or informal arrangements) whose payoff is related to the uncertainty in the random arrival of adjustment dates. The existence of arrangements that allow households to share risks associated with adjustment dates is not unrealistic. For instance, young adults sometimes live with their family until they get married, effectively being insured against uncertainty in when they buy a new house. Similarly, friends and families often provide support and gifts after shocks that induce new commitments, such as wedding or childbirth. Nonetheless, market completeness for adjustment dates is in general a non-trivial requirement. In the next section, we show that the qualitative results derived below hold with incomplete markets and adjustment costs, although simple analytic expressions for habit dynamics are no longer available in this case.

The model can be interpreted as an economy in general equilibrium, in the spirit of Constantinides (1990) or Cox, Ingersoll, and Ross (1985). In the equilibrium interpretation, the stock S and the bond B correspond to two exogenous, constant returns to scale production technologies, both of which use the adjustable component as input and produce it as output. The portfolio decisions of the households can be interpreted as investments into these technologies, and total wealth is equal to the capital stock. In addition, there is an irreversible production technology that transforms one unit of the adjustable component into one unit of the commitment component.⁷ Finally, due to technological constraints, commitment consumption can only be adjusted on reset dates. With this specification of technology and our earlier description of preferences, the aggregate economy is in general equilibrium.

2.2 Household Optimization

Each household's problem is to maximize (1) subject to the reset rule for consumption commitments and the budget constraint (3). Let Δ denote the time elapsed since the consumer last adjusted x . Note that between reset dates, $\Delta = \Delta(t)$ depends linearly on calendar time, that is, $d\Delta = dt$. Denote the history of stock market shocks in the economy by $\mathcal{H}_t = \{z_s | 0 \leq s \leq t\}$. When μ_t and σ_t vary over time, \mathcal{H}_t summarizes all information available at t about the conditional distribution of μ_u and σ_u for $u \geq t$. The following result characterizes the solution to the household's problem.

Proposition 1 *Between adjustment dates, the optimal policy of a household with wealth w_t , commitments x_t and time elapsed since last adjustment Δ , is fully determined by $w_t - \Lambda(\Delta)x_t$,*

⁷By scaling units, the model can accommodate any linear production function transforming adjustables into commitments.

Δ , and \mathcal{H}_t where

$$\Lambda(\Delta) = \int_0^\infty e^{-rs} \frac{1 - G(s + \Delta)}{1 - G(\Delta)} ds$$

is the expected present value of outstanding commitments.

Relative risk aversion over wealth is

$$\gamma \frac{w_t}{w_t - \Lambda(\Delta)x_t}.$$

At a reset date, the consumer sets $x_t = \kappa \cdot a_t$ for an appropriate constant κ .

The dynamic of adjustable consumption on all dates is characterized by

$$\frac{da_t}{a_t} = \left\{ \frac{\pi_t^2}{2\gamma^2\sigma_t^2} + \frac{1}{\gamma} \left(\frac{\pi_t^2}{2\sigma_t^2} + r - \rho \right) \right\} dt + \frac{\pi_t}{\gamma\sigma_t} dz \quad (4)$$

Proof. See the Appendix. ■

The consumer's behavior can be easily understood as a two-step budgeting process. At any time between adjustment dates, the agent first sets aside $\Lambda(\Delta)x_t$, which is used to finance outstanding commitment payments until the next adjustment date. Because markets are complete, the distribution of outstanding commitments only influences the agent's choices through its expected value. Expected outstanding commitments equal $\Lambda(\Delta)x_t$ because the probability that a household who last adjusted Δ periods ago will not adjust for the next s periods is $(1 - G(s + \Delta))/(1 - G(\Delta))$. For example, when adjustment dates arrive deterministically every T periods, outstanding commitments are $\Lambda(\Delta)x_t = x_t \{1 - e^{-r(T-\Delta)}\} / r$. For small r , this present value expression is approximately $x_t(T - \Delta)$, which is simply the sum of commitment consumption for the next $T - \Delta$ periods.

After setting aside money for commitment consumption, the agent is left with "net wealth" of $w_t^{\text{net}} = w_t - \Lambda(\Delta)x_t$. She then decides how much of net wealth to spend on adjustable consumption, and how much to invest in stocks and bonds. The expression w_t^{net} thus determines the coefficient of relative risk aversion of the induced indirect utility. This two-step budgeting is sensible: it reflects the common practice of first paying fixed bills such as a mortgage payment, utilities, service contracts, etc. out of a monthly paycheck, and then deciding how to allocate what is left on discretionary consumption or savings.⁸

An interesting comparative static that follows from the proposition is that when the value of outstanding commitments $\Lambda(\Delta)x$ is higher, the consumer is more risk averse. The value of $\Lambda(\Delta)x$ can be high for two reasons. The first, loosely speaking, is when the expected adjustment date is

⁸With complete markets, a labor income stream can be introduced simply by changing the definition of wealth to include the present value of labor income.

far away – that is, when $\Lambda(\Delta)$ is high. Second, $\Lambda(\Delta)x$ is high when the commitment level x is high, i.e., when a large fraction of consumption cannot be adjusted in the short run.

One manifestation of amplified risk aversion is that agents with more commitments hold safer portfolios. With i.i.d. returns, it is easy to show that the optimal portfolio holding of stocks is

$$\alpha_t w_t = \frac{\pi}{\gamma \sigma^2} \cdot (w_t - \Lambda(\Delta)x_t). \quad (5)$$

With a more general Ito process for returns, the optimal portfolio rule is more complex, and also depends on intertemporal hedging demand, as in Merton (1971) and Campbell and Viceira (1999). However, commitments continue to induce agents to hold safer portfolios in the general case.

On an adjustment date, the optimal choice of x_t is proportional to current adjustables a_t because of a first order condition that equates marginal utility across the two components of consumption. In this first order condition, the expected marginal utility of commitments is $x_t^{-\gamma}$ times a function of the probability distribution of the next adjustment date. Since this probability distribution $G(\cdot)$ is exogenous in the time-dependent model, the factor of proportionality κ is exogenous.⁹

Formula (4) implies that the trajectory of adjustable consumption is fully determined by the rate of return of the stock market. This observation is central to the subsequent aggregation results, since it guarantees that the trajectory of each household’s adjustable consumption does not depend on how far a consumer is from her adjustment date (i.e., on Δ). To understand why this is true, note that by market completeness, the dynamics of returns pin down the value of the intertemporal marginal rate of substitution (IMRS) for the household. Since commitments are fixed in the short run, the IMRS in turn pins down the growth rate of adjustable consumption.

2.3 Aggregate dynamics

In the aggregate analysis, the distribution of net wealth, adjustable, and commitment consumption across individual households with the same value of Δ becomes irrelevant. Therefore, with a slight abuse of notation, we henceforth use $w_t^{\text{net}}(\Delta)$, $a_t(\Delta)$, and $x_t(\Delta)$ to represent the total values of these variables across households with a given Δ . Let capital variables denote unconditional aggregates, so that X_t , A_t , and C_t stand for aggregate commitment, adjustable, and total consumption at time t . For example, since $x_t(\Delta)$ equals total commitment consumption by households who last adjusted Δ periods ago,

$$X_t = \int_0^\infty x_t(\Delta) d\Delta.$$

⁹In the state-dependent model of Section 3, the probability distribution of the next adjustment date depends on the history of shocks unless returns are i.i.d.

We begin by introducing a distribution function for the adjustment dates in the aggregate economy. Let $A_t(s) = \int_0^s a_t(\Delta) d\Delta$ denote total adjustable consumption at time t by agents who adjusted their commitment consumption x less than s periods ago. Define $H_t(s) = A_t(s)/A_t(\infty) = A_t(s)/A_t$ as the share of aggregate adjustable consumption accounted for by these households. The function $H_t(s)$ can be thought of as the cross-sectional distribution of past adjustment dates, where households are measured in terms of their contribution to total adjustable consumption. This representation of the distribution of adjustment dates will prove convenient because agents have perfectly correlated adjustable consumption. Hence, any fixed subset of households consumes a constant fraction of aggregate adjustable consumption over time.

To complete the model, we specify the initial conditions from which the economy begins at $t = 0$. The only initial conditions that matter in the subsequent analysis are the initial values of commitment consumption as a function of Δ and the initial values of net wealth as a function of Δ . Let $x_0(\Delta)$ represent an exogenous function measuring total commitment consumption at $t = 0$ by households who last adjusted consumption Δ periods ago. To specify the initial values of net wealth, recall that adjustable consumption is fully determined at the household level by net wealth and Δ (Proposition 1). As a result, specifying the initial values of adjustable consumption as a function of Δ is equivalent to specifying the initial values of net wealth as a function of Δ . We therefore specify the initial net wealth values by assuming that the economy starts from a cross-sectional distribution $H_0(s)$ of past adjustment dates (measured as above by contributions to total adjustable consumption), with total adjustable consumption of A_0 .

In general, the cross-sectional distribution $H_t(s)$ will vary over time. The following characterizes aggregate dynamics of commitment consumption for an economy with an ergodic (steady-state) distribution $H(s)$.¹⁰

Theorem 1 *An ergodic cross-sectional distribution $H(s)$ exists. When the population is started from this distribution,*

$$X_t = k(t)X_0 + b \int_0^t (1 - G(t - s)) \cdot A_s ds \quad (6)$$

where the weights satisfy $0 \leq k(s)$, $1 - G(s) \leq Ke^{-ms}$, and $b > 0$. X_t can also be written in terms of past total consumption as

$$X_t = o(t)X_0 + \int_0^t \zeta(u)C_{t-u} du \quad (7)$$

¹⁰In the working-paper version of this manuscript (Chetty and Szeidl, 2004), we show that the qualitative results extend to an arbitrary absolutely continuous initial distribution in the case when adjustment dates are deterministic. A similar extension is possible in the stochastic adjustment case as well.

where the functions $o(\cdot)$ and $\zeta(\cdot)$ satisfy

$$|\zeta(u)| \leq bK e^{-(m-bK)u} \quad \text{and} \quad |o(u)| \leq bK e^{-(m-bK)u}$$

Hence, $bK < m$ is a sufficient condition for the weights $\zeta(u)$ and $o(u)$ to go to zero exponentially. The weights $\zeta(u)$ and $o(u)$ always go to zero exponentially if waiting times are exponentially distributed. In the deterministic adjustment case, $\zeta(u)e^{m'u}$, $o(u)e^{m'u} \in L^2(\mathbb{R})$ for some $m' > 0$ and the weights always go to zero exponentially on average.

Proof. The dynamics of the cross-sectional distribution $H_t(s)$ is characterized by

$$\frac{d(1 - H_t(s))}{dt} = h_t(s) - \int_s^\infty \frac{h_t(u)}{1 - G(u)} dG(u) \quad (8)$$

assuming that $H_t(\cdot)$ is differentiable with density $h_t(\cdot)$. The two terms of this expression arise as follows: During a short dt time period, the share of adjustable consumption that adjusted more than s periods ago, $(1 - H_t(s))$, changes because (1) households who adjusted $s - dt$ periods ago join this group and (2) consumers who adjusted u periods ago adjust with probability $dG(u)/(1 - G(u))$ and leave this group.

An ergodic distribution requires $dH_t(s)/dt = 0$. Let

$$h_t(s) = \frac{1 - G(s)}{\int_0^\infty 1 - G(u) du}. \quad (9)$$

By Assumption 1, the denominator is well defined because it equals the finite expectation of a random variable distributed according to $G(\cdot)$. Since the function $h_t(\cdot)$ in (9) integrates to one, it is a probability density function. The associated distribution function $H_t(\cdot)$ satisfies (8) as well as $dH_t(s)/dt = 0$. Hence, if the economy is started from the initial distribution (9), it will always remain in this distribution.

When the economy is in steady state, the share of households (measured in terms of adjustable consumption) who adjust on any date s equals $h(0)ds$ at all times. By Proposition 1, these households set $x_t = \kappa a_t$. As a result, commitments made on any date s are a constant proportion of aggregate adjustable consumption A_s , and can be written as $bA_s ds$ where $b = h(0)\kappa$. Of these commitments, a share $1 - G(t - s)$ remain at date t . Therefore

$$X_t = \int_0^t b A_s \cdot (1 - G(t - s)) ds + X_0 k(t)$$

where $k(t)$ is the share of X_0 that has not adjusted since date zero, which can be written as

$$k(t) = \int_0^\infty \frac{1 - G(\Delta + t)}{1 - G(\Delta)} \frac{x_0(\Delta)}{X_0} d\Delta.$$

Because $(1 - G(\Delta + t)) / (1 - G(\Delta)) \leq Ke^{-mt}$, we also have $k(t) \leq Ke^{-mt}$.

The total consumption representation in (7) follows immediately from the lemma below, which is proved in the Appendix.

Lemma 1 *If commitment consumption is of the form*

$$X_t = \int_0^t j(t-s)A_s ds + k(t)X_0$$

for some functions $j(\cdot)$ and $k(\cdot)$ then we can write

$$X_t = o(t)X_0 + \int_0^t \zeta(t-s)C_s ds$$

where the functions $\zeta(\cdot)$ and $o(\cdot)$ uniquely solve the Volterra integral equations

$$\zeta(u) = j(u) - \int_0^u \zeta(v)j(u-v)dv \quad (10)$$

$$o(t) = k(t) - \int_0^t \zeta(t-s)k(s)ds \quad (11)$$

with initial conditions $\zeta(0) = j(0)$, $o(0) = k(0)$.

In the total consumption representation (7), the weights can be bounded by noting that (10) implies

$$|\zeta(u)| \leq j(u) + \int_0^u |\zeta(v)|j(u-v)dv$$

where $j(u) = b(1 - G(u)) \leq bKe^{-mu}$ is a positive function by assumption. Direct substitution yields

$$|\zeta(u)|e^{mu} \leq bK + bK \int_0^u |\zeta(v)|e^{mv} dv.$$

Since this is true for all $u \geq 0$, we can apply Gronwall's lemma to bound the function $|\zeta(u)|e^{hu}$

$$|\zeta(u)|e^{mu} \leq bK \exp\left(bK \int_0^u 1 du\right) = bK \cdot e^{bKu}$$

which implies that

$$|\zeta(u)| \leq bK \cdot e^{(bK-m)u}$$

as claimed. Furthermore,

$$o(t) \leq k(t) + \int_0^t |\zeta(t-s)|k(s)ds \leq Ke^{-mt} + \int_0^t bK \cdot e^{(bK-m)(t-s)}Ke^{-ms} ds = Ke^{(bK-m)t}.$$

The parameter b is derived from the underlying parameters of the model. We now establish that there are underlying parameters for which $bK < m$ holds. When $\mu = 0$, $x = 0$, and therefore $b = 0$. By continuity, for μ sufficiently small, $bK < m$ continues to hold.

In the special case when adjustment dates arrive at an exponential rate, $G(s) = 1 - e^{-\lambda s}$, and $j(u) = be^{-\lambda u}$. The integral equations (10) and (11) are satisfied with $\zeta(u) = b \cdot e^{-(\lambda+b)u}$ and $o(u) = e^{-(\lambda+b)u}$. Note that in this special case, the adjustment process has no memory, and the representation holds for any initial distribution $H_0(s)$.

The proof for the deterministic adjustment case is more complicated, and is given in the appendix. ■

The content of this theorem is that we can express aggregate commitment consumption as a time average of past aggregate adjustable consumption or a time average of past aggregate total consumption. Because such time averages are continuous, the theorem implies that aggregate commitments follow a continuous path. The representation also shows that aggregate commitments are a process with bounded variation. Intuitively, the discontinuous adjustment dynamics at the household level created by commitments are “smoothed out” in the aggregate because adjustment dates are staggered across households.

The weights in the representation (6) using adjustable consumption go to zero exponentially, reflecting the intuition that commitments made in the distant past have little weight in total commitments today, because households are likely to have updated since then. Showing that the weights in the representation (7) using total consumption go to zero is more difficult, because it requires characterizing the solution to the Volterra integral equation (10). A first approach is to bound the solution in (10) by taking absolute values of both sides and applying Gronwall’s lemma. This argument verifies that the weights asymptote to zero when $bK < m$.

This approach cannot be extended to the case when $bK > m$. However, an alternate set of conditions which guarantee exponential decay of the weights in this case can be derived. When $bK > m$, the solution to the integral equation obtained after taking absolute values in (10) can asymptote to infinity. To make progress, we do not take absolute values and instead study the Fourier transform of equation (10). As we show in the appendix, this is an algebraic equation that can be solved directly for the Fourier transform of $\zeta(u)$ in terms of the Fourier transform of $1 - G(u)$. We can then express $\zeta(u)$ using the inverse Fourier transform as an integral on the complex plane. Using contour integration to compute this integral, Lemma 4 in the appendix shows that when the Fourier transform of $1 - G(u)$ satisfies certain regularity conditions, the inverse Fourier transform $\zeta(u)$ will be a valid solution to (10) and also satisfies $e^{m'u}\zeta(u) \in L^2(\mathbb{R})$ for some $m' > 0$. Hence $\zeta(u)$ asymptotes to zero exponentially in a well-defined sense provided that the regularity conditions hold even when $bK > m$. For the special case with deterministic waiting times, the regularity conditions for the Fourier transform of $1 - G(u)$ are easy to verify, and the exponential decay of the weights follows immediately.

The theorem leads to the following aggregation result.

Theorem 2 *The aggregate dynamics of consumption are the optimal policy of a representative consumer with external habit formation utility function*

$$E \int_0^{\infty} e^{-\rho t} \frac{(C_t - X_t)^{1-\gamma}}{1-\gamma} dt$$

where habit X_t is a time average of past levels of surplus or total consumption, as in (6) and (7). In the special case of exponentially distributed arrival times, habit dynamics can be written explicitly as

$$X_t = X_0 e^{-d \cdot t} + D \int_0^t e^{-d \cdot u} C_{t-u} du \quad (12)$$

with suitable positive constants d and D .

Proof. The proof follows immediately from the results in Theorem 1. Because adjustable consumption is perfectly correlated across households, aggregate adjustable consumption follows the same dynamics as household level adjustable consumption

$$\frac{dA_t}{A_t} = \left\{ \frac{\pi^2}{2\gamma^2\sigma^2} + \frac{1}{\gamma} \left(\frac{\pi^2}{2\sigma^2} + r - \rho \right) \right\} dt + \frac{\pi}{\gamma\sigma} dz$$

and hence satisfies the Euler equation

$$A_s^{-\gamma} = e^{-\rho(t-s)} \cdot E_s \left[A_t^{-\gamma} R_{s,t} \right]$$

for any traded financial return $R_{s,t}$ between t and s . Equivalently,

$$(C_s - X_s)^{-\gamma} = e^{-\rho(t-s)} \cdot E_s \left[(C_t - X_t)^{-\gamma} R_{s,t} \right]$$

which is just the optimality condition of a consumer with external habit process X_t and habit preferences over total consumption C_t . Habit is external because it does not enter in the first order condition. This proves the existence of the representative consumer. The representation of X_t as a slow-moving time average follows from the previous theorem. With exponentially distributed arrival dates, we can choose $d = \lambda + b$ and $D = b$ to get equation (12). ■

This theorem shows that consumption commitments provide neoclassical microfoundations for habit formation. It states that an economy of households who make infrequent adjustments in commitment consumption looks identical in the aggregate to a single individual with a smooth, slow-moving habit stock that depends on past consumption levels. The habit of the representative agent is a special case of the specification advocated in Chen and Ludvigson (2004):

$$X_t = f(C_t, C_{t-1}, \dots, C_{t-L}).$$

When the arrival dates of adjustment opportunities are unpredictable, the habit rule can be written as a simple exponential time average, regardless of the initial distribution of relative adjustable consumption $H_0(s)$. This result underscores the strong connection between the commitments model and habit formation. In the exponential arrival case, the representative consumer's preferences and the habit dynamics of our aggregated model are *exactly the same* as those in Constantinides (1990). The only difference is that Constantinides assumes internal habit formation, whereas aggregating the commitments model yields external habit.

2.4 Multiple consumption goods

We now investigate the robustness of the habit representation result to the introduction of multiple consumption goods that differ in degree of commitment (e.g., housing, food, clothing). In particular, we show that aggregate total consumption and aggregate consumption of each good can be viewed as the optimal choice of a representative agent with habit formation preferences. This result is useful when considering the empirical implications of the model (see section 4).¹¹

Consider an economy with two consumption categories (1 and 2), each of which have adjustable and commitment components.¹² Total consumption of category i is $c_i = a_i + x_i$. Households solve

$$\max \mathbb{E} \int_0^\infty e^{-\rho t} \left[\left(\mu_1^a \cdot \frac{a_{1,t}^{1-\gamma}}{1-\gamma} + \mu_1^x \cdot \frac{x_{1,t}^{1-\gamma}}{1-\gamma} \right) + \left(\mu_2^a \cdot \frac{a_{2,t}^{1-\gamma}}{1-\gamma} + \mu_2^x \cdot \frac{x_{2,t}^{1-\gamma}}{1-\gamma} \right) \right] dt \quad (13)$$

where the μ parameters measure the relative preference for each of the consumption goods. There is a separate distribution of waiting times associated with the commitment component of each consumption type. Waiting times may be correlated with each other within a household. However, adjustment dates are i.i.d. across households and over time, and complete insurance markets exist.

Because both a_1 and a_2 are immediately adjustable and utility is scalable, the consumer chooses a_1 and a_2 such that $a_1/a_2 = (\mu_1^a/\mu_2^a)^{1/\gamma}$ always holds. Therefore, the above maximization problem is equivalent to

$$\max \mathbb{E} \int_0^\infty e^{-\rho t} \left[\mu^a \cdot \frac{a_t^{1-\gamma}}{1-\gamma} + \mu^x \cdot \frac{x_{1,t}^{1-\gamma}}{1-\gamma} + \mu^x \cdot \frac{x_{2,t}^{1-\gamma}}{1-\gamma} \right] dt \quad (14)$$

where $\mu^a = [(\mu_1^a)^{1/\gamma} + (\mu_2^a)^{1/\gamma}]^\gamma$ and $a = a_1 + a_2$. As in the basic model with one consumption category, the dynamics of a are completely pinned down by the financial asset returns. Hence, a

¹¹We show in our working paper (Chetty and Szeidl, 2004) that heterogeneity across households in the relative preference for commitments μ can also be introduced without changing the aggregation results.

¹²The analysis extends with minor modifications to N categories.

representative consumer with preferences over $C - (X_1 + X_2)$ exists. Because A_1 is proportional to A , there is also a representative consumer with preferences over $C_1 - X_1$.

To complete the habit representation, it remains to show that X_1 and X_2 can be written as slow-moving time averages of past consumption. Note that the household's optimization problem can be solved in two stages. First, fix a feasible time- and state-contingent plan of x_2 consumption. The state-contingent payment stream associated with this plan can be swapped for a fixed payment at date zero, because markets are dynamically complete. The reduced problem of choosing optimally over a and x_1 with remaining wealth after making the fixed payment for x_2 is identical to that analyzed in the basic case, because utility is separable over all goods. It follows from Theorem 1 that X_1 (and, by analogous arguments, X_2) can be represented as a time average of A . The linearity of the representations implies that $X = X_1 + X_2$ can also be expressed as a time average A . Invoking Lemma 1, X can also be represented as a time average of C , and X_1 can be represented as a time average of C_1 . The weights asymptote to zero in all these representations under conditions equivalent to those in Theorem 1. Hence, both total consumption C and consumption of each category (C_1 and C_2) follow dynamics that are the optimal choice of representative consumers with habit preferences, where habit equals total commitment consumption or commitment consumption within a particular category, respectively.

3 State Dependent Adjustment

3.1 Basic Setup and Household Optimization

Consider a model where the adjustment process for the committed portion of consumption x is determined by a state-dependent instead of time-dependent rule. Preferences are the same as above. For tractability reasons explained below, we assume that returns are i.i.d., so $\pi_t = \pi$ and $\sigma_t = \sigma$ are constant. Unlike the time-dependent case, no additional insurance markets are assumed.

In the state-dependent model, consumers can change consumption of x_t at any time, but must pay a transaction cost $q \cdot x_t$ to do so. This model of commitments captures the transaction costs inherent in changing consumption of illiquid durables such as cars or appliances, or the fixed costs involved in changing plans or contracts. Such models have a long tradition in the literature on consumption and investment, starting with work by Grossman and Laroque (1990) and Bertola and Caballero (1990). More recently, several studies have examined state-dependent models with two consumption components, one freely adjustable and one that is costly to adjust (Flavin and Nakagawa 2003, Fratantoni 2003, Li 2003). The household-level model we study below is a

special case of Flavin and Nakagawa’s model. Our contribution lies in aggregating this model, and characterizing the relation of aggregate dynamics to habit formation.

Aggregation of the adjustment cost model with i.i.d. returns is facilitated by introducing an adjustable component of consumption for two basic reasons. First, one can solve for the dynamics of the adjustable consumption component explicitly from returns using the first order condition of the household because adjustable consumption follows an exponential Brownian motion. Second, the household’s choices of x can be expressed in terms of an (S,s) band over x/a , the ratio of commitment to optimal adjustable consumption. The dynamics of $\log(x/a)$ are given by a Brownian motion with constant drift and diffusion within the no-adjustment region, as in the tractable specification studied by Caballero (1993) and others. Note that if returns are not i.i.d., the (S,s) band for x/a will depend on the history of returns, which makes aggregation difficult.

We begin by analyzing the individual household’s problem. As shown by Flavin and Nakagawa (2003), the optimal adjustment policy of each household in this type of model can be written in terms of an (S,s) band over a state variable x/w . To aggregate, we describe the household’s policy for x in terms of an (S,s) band over x/a instead of x/w . Given that the optimal choice of a follows a known Brownian motion, such an (S,s) band can be used to fully describe a household’s behavior as a function of exogenous parameters and shocks. To obtain an (S,s) band for x/a , observe that adjustable consumption a_t is a strictly increasing function of current wealth w_t conditional on current x , because with separable preferences a_t is a normal good. The optimal consumption function $a_t(w_t, x)$ can be used to map the (S,s) band over wealth into an (S,s) band over adjustable consumption for any given x . Since utility is homogenous of degree $1 - \gamma$, the (S,s) band can be written in terms of x/a .

Define $y = \log(x/a)$. Then each household’s optimal policy for the committed portion of consumption can be described by three numbers, $\{L, U, M\}$. For $y \in (L, U)$, the household does not adjust x from its prior level; as soon as y reaches L or U , the household resets x so that $y = M$. Since households have identical preferences and the model is scalable in wealth, the numbers $\{L, U, M\}$ do not vary across households in the economy.

If there is no heterogeneity across households and the only risk is aggregate risk due to stock market fluctuations, all households would cross the edges of their (S,s) bands simultaneously, generating jumps in the process of aggregate commitment consumption. To generate a smooth process for aggregate consumption, we introduce heterogeneity across households by allowing for an additional, household-specific risky investment opportunity. This background risk can be thought of as entrepreneurial investment or labor income risk (where “investment” corresponds to investment in human capital). The value of household i ’s idiosyncratic investment follows an

exponential Brownian motion

$$\frac{dS_t^i}{S_t^i} = (r + \pi_I)dt + \sigma_I dz_t^i$$

where the z^i 's are standard Brownian motions uncorrelated across households. Each household is free to invest or disinvest an arbitrary amount into her private asset at any time.

Importantly, even with this idiosyncratic risk, the choices of x for all households in the economy can still be characterized using three numbers $\{L, U, M\}$ as above. The dynamics of x at the household level can be described by a simple (S,s) rule because returns on the idiosyncratic asset are unpredictable, eliminating the need for an additional state variable. The numbers $\{L, U, M\}$ do not vary across households because all households have access to assets with identical joint distributions. However, because of the idiosyncratic noise, households have different values of y in general – that is, they will be in different locations within their (S,s) bands. The non-degenerate cross-sectional distribution of y at each time t yields a smooth path for aggregate consumption because only a small fraction of households adjust x in response to a shock.

3.2 Aggregation

The distribution of wealth and consumption across individual households with the same value of y becomes irrelevant in the aggregate analysis. Therefore, as in the time-dependent model, we henceforth use x , a , and w to denote totals for all households with a given value of y . As above, we use capital letters to denote unconditional aggregates.

Let $f(y, t) = a_t(y)/A_t$ denote the share of aggregate adjustable consumption at time t accounted for by households whose log adjustable-to-commitment ratio equals y . The density $f(y, t)$ can be thought of as the cross-sectional distribution of y measured in terms of shares of A_t . With this measure of the distribution of y , X_t can be written as a function of A_t by simply integrating over $f(y, t)$. Assume that at $t = 0$, all households have adjustable consumption equal to A_0 , and the initial cross-sectional distribution of commitments is such that $f(y, 0)$ is a smooth density with $f(y, U) = f(y, L) = 0$. This distribution pins down the cross-sectional distribution of wealth as well.

Define

$$\eta = \frac{1}{\gamma} \left(\frac{\pi^2}{2\sigma^2} + \frac{\pi_I^2}{2\sigma_I^2} \left(1 + \frac{1}{\gamma} \right) + r - \rho \right)$$

and

$$\sigma_T^2 = \left(\frac{\pi_I}{\gamma\sigma_I} \right)^2 + \left(\frac{\pi}{\gamma\sigma} \right)^2.$$

The next result, which is based on Propositions 1 and 2 in Caballero (1993), characterizes the evolution of the cross-sectional distribution $f(y, t)$ and the dynamics of aggregate commitments.

Proposition 2 $f(y, t)$ satisfies the stochastic partial differential equation

$$df(y, t) = \left[\left(\eta + \frac{\pi_I^2}{2\gamma^2\sigma_I^2} \right) \frac{\partial f(y, t)}{\partial y} + \frac{\sigma_T^2}{2} \frac{\partial^2 f(y, t)}{\partial y^2} \right] dt + \frac{\pi}{\gamma\sigma} \frac{\partial f(y, t)}{\partial y} dz$$

together with the following boundary conditions:

$$\begin{aligned} f(U, t) &= f(L, t) = 0 \\ \frac{\partial f(M, t)^+}{\partial y} - \frac{\partial f(M, t)^-}{\partial y} &= \frac{\partial f(U, t)^-}{\partial y} - \frac{\partial f(L, t)^+}{\partial y} \\ f(M, t)^+ &= f(M, t)^-. \end{aligned}$$

Aggregate commitments follow the dynamics

$$dX_t = A_t \frac{\sigma_T^2}{2} \cdot (f_y(L, t)(e^M - e^L) + f_y(U, t)(e^U - e^M)) dt. \quad (15)$$

We adapt Caballero's proofs to establish this result (see the appendix for details). The proposition shows that aggregate commitments are "smooth" in the sense that they follow a bounded variation process; loosely speaking, the share of adjustable consumption A_t adjusting on date t is proportional to dt . When there is no aggregate uncertainty ($\pi = \sigma = 0$), the dz term in the stochastic partial differential equation that describes the evolution of the cross-sectional distribution disappears. In this case, the differential equation has a time-invariant solution, corresponding to the steady-state of an economy with only idiosyncratic risk. In that steady state, aggregate quantities follow a deterministic path. With aggregate risk, the cross-sectional distribution is constantly perturbed relative to its steady state value, and aggregate variables follow stochastic dynamics.

By integrating over the cross sectional distribution, we obtain expression (15) for the dynamics of aggregate commitments dX_t . The intuition for this expression is as follows: A measure of households proportional to $f_y(L, t)$ adjust their commitments because they have reached the lower bound of the (L, U) region. When they adjust, they reset x at e^M times their current adjustable consumption, instead of the prior ratio of e^L . Analogous adjustments occur for the households who hit the upper boundary. Since households are measured in terms of their adjustable consumption relative to A_t , the expressions are multiplied by A_t to recover the change in the level of X_t . Finally, the change in aggregate commitments is proportional to total volatility σ_T^2 because this term determines the frequency with which households close to L or U drift to the thresholds and adjust.

The next result establishes the habit representation in the economy with state-dependent adjustment.

Theorem 3 *The aggregate dynamics of consumption are the optimal policy of a representative consumer with external habit formation utility*

$$E \int_0^\infty e^{-\delta t} \frac{(C_t - X_t)^{1-\gamma}}{1-\gamma} dt$$

as long as the discount rate $\delta = \rho - \frac{\pi_I^2}{2\sigma_I^2} \left(1 + \frac{1}{\gamma}\right) > 0$. Habit X_t is given by

$$X_t = \int_0^t \zeta(t, s) C_s ds + X_0 \cdot o(t)$$

where the weights $\zeta(t, s) = \zeta(t, s, \mathcal{H}_t)$ and $o(t, s) = o(t, s, \mathcal{H}_t)$ depend on the history of aggregate shocks in the economy. Consumption from the distant past has an exponentially vanishing role in determining current habit:

$$X_0 \cdot o(t) + \int_0^u \zeta(t, s) C_s ds \leq K_5 \exp\{-K_3(t-u)\} C_u$$

for positive constants K_5 and K_3 .

If $\pi = \sigma = 0$, the economy has a steady-state distribution in which $\zeta(\cdot)$ and $o(\cdot)$ are deterministic, and habit dynamics are given by

$$X_t = \int_0^t \zeta(t-s) C_s ds + X_0 \cdot o(t) \quad (16)$$

where $\zeta(u)$ and $o(u)$ go to zero exponentially in u .

Proof. Since the only risky assets for household i are S and S^i , there exists a unique stochastic discount factor in the payoff space associated with the household-specific private market. The following dynamics for adjustable consumption generates a state price density that prices both risky assets as well as the safe asset

$$a_t^i = a_0^i \exp \left\{ \frac{1}{\gamma} \left(\frac{\pi^2}{2\sigma^2} + \frac{\pi_I^2}{2\sigma_I^2} + r - \rho \right) t + \frac{\pi}{\gamma\sigma} z_t + \frac{\pi_I}{\gamma\sigma_I} z_t^i \right\}$$

and hence must describe the optimal choice of household i . Because $a_0^i = A_0$ for all i , aggregating across i yields, by the strong law of large numbers for a continuum of agents (Sun, 1998)

$$\begin{aligned} A_t &= A_0 \exp \left\{ \frac{1}{\gamma} \left(\frac{\pi^2}{2\sigma^2} + \frac{\pi_I^2}{2\sigma_I^2} + r - \rho \right) t + \frac{\pi}{\gamma\sigma} z_t \right\} \int_i \exp \left\{ \frac{\pi_I}{\gamma\sigma_I} z_t^i \right\} di \\ &= A_0 \exp \left\{ \frac{1}{\gamma} \left(\frac{\pi^2}{2\sigma^2} + \frac{\pi_I^2}{2\sigma_I^2} \left(1 + \frac{1}{\gamma}\right) + r - \rho \right) t + \frac{\pi}{\gamma\sigma} z_t \right\}. \end{aligned}$$

Define a new discount rate $\delta = \rho - \left(1 + \frac{1}{\gamma}\right) \frac{\pi_I^2}{2\sigma_I^2}$. Then the dynamics of aggregate adjustable consumption is given by

$$A_t = A_0 \exp \left\{ \frac{1}{\gamma} \left(\frac{\pi^2}{2\sigma^2} + r - \delta \right) t + \frac{\pi}{\gamma\sigma} z_t \right\}.$$

This is exactly the dynamics of adjustable consumption that would obtain for a representative consumer with power utility over A_t and discount rate δ who can invest in the publicly traded risky and safe assets. The existence of a representative consumer obtains even though markets are incomplete because idiosyncratic shocks cancel out in the aggregation, as in Grossman and Shiller (1982). However, the presence of idiosyncratic risk increases both the mean and the variance of household consumption growth. To compensate for the increase in mean consumption growth in the aggregate, the representative consumer must be more patient than the individual households.

Since $A_t = C_t - X_t$, this representative consumer can be thought of as having habit preferences over total consumption C_t , where habit is external and follows the dynamics of X_t . To characterize the dynamics of X_t , note that according to Proposition 2, the share of agents who adjust during a short period ds is $g(s)ds = (\sigma_T^2/2) \cdot (f_y(L, s) + f_y(U, s)) \cdot ds$. Therefore, new commitments made on date s equal $e^M g(s)A_s ds$. Of the households who adjust on s , some fraction $\vartheta(t, s)$ do not adjust between s and t . Hence

$$X_t = \int_0^t e^M g(s) \vartheta(t, s) A_s ds + o(t) X_0$$

where $o(t)$ is the share of commitments in place on date zero that have not been adjusted since. On any given date s , the ratio C_s/A_s is bounded both from above and below by functions of U and L . Introducing the bounded function $C_s/A_s = e(s)$ and defining $g(s)h(t, s)e(s) = \zeta(t, s)$ we get the desired representation (16).

Now turn to bound the value of commitments that have not been adjusted since a given date s . For agent i not to update, it must be that y^i remains within the (L, U) band between dates s and t . The probability of this event depends on the history of aggregate shocks between s and t , captured by the path of aggregate adjustable consumption. The following lemma, proved in the appendix, shows that this probability goes to zero exponentially uniformly across all paths of aggregate adjustable consumption.

Lemma 2 *There exist positive constants K_3 and K_4 such that*

$$\max_{y_s^i, \{A_v | s \leq v \leq t\}} E [1 \{ y_v^i \in (L, U) \text{ for } s \leq v \leq t \} | \{ A_v | s \leq v \leq t \}] \leq K_4 \exp(-K_3(t-s)). \quad (17)$$

The left hand side in (17) is the expectation, conditional on the path of aggregate adjustables between s and t , of an indicator variable that assumes the value of 1 if and only if y^i stays within the (L, U) band between s and t . The strong law of large numbers implies that this expectation also measures the share of households in the cross-section who do not adjust their commitments during the period between s and t . The maximization is over the initial condition y_s and over all possible paths of aggregate adjustable consumption.

The Lemma implies that the total amount of commitments on date t that were made on date s or before satisfies

$$X_0 \cdot o(t) + \int_0^s e^M g(v) h(t, v) A_v ds \leq A_s e^U K_4 \exp(-K_3(t-s)) = K_5 \exp(-K_3(t-s))$$

because total commitments on date s were at most $A_s e^U$, and of these, at most a share $K_4 \exp(-K_3(t-s))$ remains on date t .

When $\pi = \sigma = 0$, by Proposition 2, if a steady-state cross-sectional distribution $f(y)$ exists, it must satisfy

$$0 = \left(\mu + \frac{\pi_I^2}{2\gamma^2\sigma_I^2} \right) f'(y) + \frac{\sigma_T^2}{2} f''(y).$$

This equation can be solved explicitly to give a piecewise exponential cross-sectional distribution for y , showing that a steady-state distribution exists. Given this steady state distribution, the share of households who adjust on any date s is $(\sigma_T^2/2) \cdot (f_y(L) + f_y(U)) ds$ a constant, and these households all choose commitments to be $x_t = e^M a_t$. Hence, new commitments on date s are $b A_s ds$ where $b = e^M (\sigma_T^2/2) \cdot (f_y(L) + f_y(U))$. Because all randomness is idiosyncratic, the share of commitments still in place at t , $\vartheta(t, s)$, only depends on the time elapsed between s and t , $t-s$. Thus aggregate commitments can be written as

$$X_t = \int_0^t b \vartheta(t-s) A_s ds + X_0 \cdot o(t)$$

where $o(t)$ is defined as usual. Finally, note that the ratio $X_t/A_t = \int_L^U e^y f(y) dy$ is constant over time. As a result, $C_t/A_t = K_6$ is also constant over time, and defining $\zeta(s) = K_6 b \vartheta(s)$ leads to the desired representation. The exponential decay for $\zeta(t)$ follows from the argument above. ■

This result is the analog of Theorem 2 in the time-dependent case. There are two differences between the aggregation results in the time and state dependent models. First, in the state-dependent model, the weight on total consumption from the distant past in the habit stock approaches zero exponentially without any additional technical conditions. Second, the habit stock in the state-dependent model depends on a time average of past consumption levels with *state-dependent* weights. The weight placed on consumption at date s in determining X_t depends on the entire history of shocks, and not just the time that has elapsed since that date ($t-s$).

In the special case with no aggregate uncertainty (where the only risky asset is household-specific), aggregate dynamics become deterministic in the state-dependent model. This yields the more familiar representation of habit preferences where the weight on consumption that occurred on date s is deterministic and depends only on $t-s$. This result suggests that when aggregate uncertainty is small relative to idiosyncratic risk, the widely-used habit specification

with deterministic weights (as in Constantinides, 1990) is a good approximation of aggregate dynamics in the state-dependent commitments model.

The habit representation extends to an environment with multiple consumption categories as in Section 2.4. Assume that consumers have preferences given by (13) where adjustment of x_1 has a cost q_1x_1 and adjustment of x_2 has a cost q_2x_2 . A representative consumer with power utility over $A = C - X$ exists as before. The dynamics of C_1 can be described with a similar representation over $A_1 = C_1 - X_1$. To characterize the dynamics of X and X_1 , note that the private market of household i is dynamically spanned by the private asset S^i and the publicly traded financial asset. Therefore, there is a portfolio that finances any given consumption path of x_2 which is adapted to the filtration generated by the Brownian motions z and z_i . Remaining wealth is used to finance the optimal consumption of a and x_1 , which simplifies the multiple good setup to the two-component version studied above. The optimal path of x_1 can therefore be described with an (S,s) band over the ratio x_1/a . The optimal path of x_2 is described by a second (S,s) band over x_2/a . Invoking Theorem 3, it follows that X_1 and X_2 , and hence X as well, can be represented as linear time averages of A . Since A/C is bounded, X can also be expressed as a linear time-average of C where consumption from the distant past has an exponentially vanishing weight. Analogously, X_1 can be written as a slow-moving time-average of past levels of C_1 .

4 Suggestive Evidence

In this section, we derive some testable predictions of the model, and discuss existing studies as well as new evidence related to these predictions. Our objective here is not to structurally estimate the model, but to summarize some stylized facts that cast light on the hypothesis that commitments induce habit-like behavior. We focus on two basic predictions regarding the impulse response of consumption to shocks: (1) Consumption should respond slowly to shocks; and (2) Consumption categories that involve a higher degree of commitment should respond more sluggishly than categories that involve less commitment.

Both of these predictions can be easily derived from the multiple-good time and state dependent models with i.i.d. returns. We sketch the derivation here; details are given in the appendix. Suppose the economy is in a resting state in the sense that no aggregate shocks have taken place ($dz = 0$) for a long time period. Consider a consumption category j where total consumption at time t is $C_{j,t} = A_{j,t} + X_{j,t}$, the sum of the adjustable and commitment components. By the habit representation result, the growth rates of C_j and X_j will be identical in this resting state. Hence, there exists χ_j such that $X_j = \chi_j C_j$ and $A_j = (1 - \chi_j)C_j$. Now suppose there is an aggregate

shock on date t_0 , i.e. a change Δz_{t_0} in the Brownian motion driving the stock market. After t_0 , suppose $dz = 0$.¹³ The impact of this shock on consumption growth rates can be characterized by taking a log-linearization of the equation for total consumption. In both the time and state dependent models, the growth in consumption of category j over a horizon of S periods (between t_0 and a subsequent time $t_0 + S$) is given by

$$\Delta \log C_{j,t_0+S} = \log C_{j,t_0+S} - \log C_{j,t_0-} \approx vS + \frac{\pi}{\gamma\sigma} [(1 - \chi_j) + \chi_j\theta(S)] \cdot \Delta z_{t_0} \quad (18)$$

where v is the trend growth rate in consumption and $\theta(\cdot)$ is a deterministic increasing function that satisfies $\theta(0) < 1$ (with $\theta(0) = 0$ in the time dependent model) and $\lim_{S \rightarrow \infty} \theta(S) = 1$. This equation captures a general intuition that does not rely on particular assumptions about preferences and returns. Because the adjustable component of consumption can be changed freely at all times, it jumps immediately to the new level $A_{j,t_0+} = A_{j,t_0} \exp(\Delta z_{t_0} \cdot \pi/(\gamma\sigma))$. In the time dependent case, the immediate response of X_j to the shock is zero ($\theta(0) = 0$) because the set of households that adjusts exactly on t_0 has measure zero. In the state dependent case, the initial response of X_j to a small shock, though not necessarily zero, is still limited ($\theta(0) < 1$), because only a subset of agents adjust. Over time, more households adjust to the shock, and in the limit when all households have adjusted, the ratio $X_{j,t}/A_{j,t}$ converges to its resting state value of $\chi_j/(1 - \chi_j)$.

Equation (18) yields several testable predictions. The first is that $\partial(\Delta \log C_{j,t_0+S})/\partial(\Delta z_{t_0})$ is rising in S , i.e., shocks have a larger impact on consumption growth over longer horizons. There is considerable existing evidence for this prediction. At the microeconomic level, empirical studies of vehicle and housing consumption using panel data (e.g., Attanasio, 2000, Eberly, 1994, Martin, 2003) find substantial inertia in responses to wealth fluctuations.¹⁴ At the macroeconomic level, several studies have documented “excess sensitivity” of consumption to past shocks by showing that lagged income growth is highly correlated with consumption growth (Flavin 1981, Campbell and Mankiw 1989, Attanasio and Weber 1993). Similarly, Parker and Julliard (2005) find that S&P500 quarterly excess returns predict consumption growth over longer horizons (3-4 years) more strongly than over short horizons (less than 1 year).

This existing evidence does not shed light on whether sluggish adjustment arises from commitment effects. We explore this issue by exploiting the variation in χ_j across consumption categories. Let $\beta_{j,S} = \frac{\pi}{\gamma\sigma} [(1 - \chi_j) + \chi_j\theta(S)]$ denote the coefficient on Δz_{t_0} , and introduce the

¹³Strictly speaking, this (as well as any other) path for the driving Brownian motion has zero probability. However, as the optimal policies for all households are well defined along this path, characterizing aggregate quantities along this trajectory is a well-defined problem.

¹⁴For instance, Attanasio’s estimates imply that households allow the share of car consumption in total consumption to fall to 20% of the desired level before adjusting.

error term ε_{jtS} which captures deviations of consumption growth from that predicted by the model. Then (18) implies the following estimating equation:

$$\log(c_{j,t+S}) - \log(c_{j,t-1}) = \alpha_{j,S} + \beta_{j,S}\Delta z_t + \varepsilon_{jtS}. \quad (19)$$

In this equation, which is a log-linearized version of the equation estimated by Parker and Julliard, the $\beta_{j,S}$ coefficients capture the impulse response of consumption of type j to shocks.¹⁵ The comparative statics of $\beta_{j,S}$ with respect to χ_j generate two predictions: (1) The intercept $\beta_{j,S}|_{S=0}$ is decreasing in χ_j , i.e., the initial response of consumption to shocks is smaller for categories that involve more commitment; and (2) the slope $\partial\beta_{j,S}/\partial S$ is increasing in χ_j , i.e., the consumption response grows more rapidly with the horizon S for categories that involve more commitment. This comparative static analysis is heuristic in that both χ_j and $\theta(\cdot)$ are endogenous to the underlying parameters of the model. We justify this approach more rigorously in the appendix by showing that varying the relative preference weights μ_j across consumption categories appropriately generates variation in χ_j while holding $\theta(\cdot)$ fixed.

We provide some suggestive evidence regarding the two predictions by estimating (19) at various horizons S for sub-categories of consumption within NIPA nondurables and services, the most commonly used consumption measure in macroeconomic empirical studies. While no single category consists completely of adjustable or commitment consumption, there are plausible differences in commitment shares across categories. We examine three categories: (1) Housing services, which is the rental value of the housing stock; (2) Housing services + household operations, which adds housing-related expenses such as utility bills and maintenance to (1); and (3) Nondurable goods, which include food, clothing, non-house gas and oil consumption, and other nondurable items. Intuition suggests that χ_j falls from category (1) to (3): It is easier to change expenditures on energy bills and maintenance than to move houses; and it is easier to adjust spending on nondurables than to change housing-related expenses.

Following Parker and Julliard, we measure shocks (Δz_t) using quarterly S&P 500 returns. Let R_t^e denote the excess return on the S&P500 in each quarter t , defined as the difference between the return on the S&P500 from quarter $t - 1$ to quarter t and the return on 3 month US Treasury bills over the same period. We use the “end-of-period” consumption convention and compute the real growth rate of consumption for each consumption category between quarter $t - 1$ and quarter $t + S$, for $S \in \{0, 1, \dots, 15\}$. The growth rate of consumption for $S = 0$ reflects consumption

¹⁵The approximate equation (18) was derived in a resting state of the model with no aggregate shocks before t_0 . If aggregate shocks are i.i.d., one can show that the regression equation (19) remains valid in approximation even when the economy is not in such a resting state.

growth contemporaneous to the stock market shock, while growth rates for larger values of S capture responses over longer horizons.

Figure 1 plots the $\{\beta_{j,S}\}_{S=0}^{15}$ coefficients when (19) is estimated using OLS for each of the three categories ($j = 1, 2, 3$). In panel A, we use the full sample of available returns and consumption (from 1947Q1-2004Q4). In panel B, we examine the robustness of these results to outliers and sample specification by removing the top and bottom 5% of the S&P returns (quarterly returns that exceed 11.5% in absolute value). In the full-sample regressions, these observations are the most influential (measured by DF-Beta statistics) and it is therefore useful to check whether the results are robust to the exclusion of these observations.

Both panels of Figure 1 indicate that the initial response to innovations in asset prices of the more committed categories is substantially smaller than for nondurables. The estimates in Panel B show that four quarters after a shock, the growth rate of nondurables is 2.85 times that of total housing-related expenses, and 5.85 times that of housing rental values. The β_S coefficients rise more rapidly with the horizon S for the more committed categories. After four years have elapsed, the magnitude of consumption responses across the three categories is similar. Hence, the consumption growth patterns support both predictions described above: higher-commitment categories respond more sluggishly to shocks, but eventually revert to steady-state values.

Consistent with these results, other recent studies have found that consumption of more discretionary components is more responsive to shocks using different methods and datasets. Ait-Sahalia, Parker, and Yogo (2004) find that the consumption of luxury goods is an order of magnitude more responsive to returns than consumption of the basic necessities that comprise most of NIPA nondurables and services. Jagannathan and Wang (2004) show that consumption growth from one December to the next are more successful in explaining asset returns, with the rationale that a larger fraction of purchases at the end of the year (holiday season) tend to be discretionary. Flavin and Nakagawa (2003) estimate a structural model of housing and food consumption using household-level panel data and find stronger evidence of sluggish adjustment in housing consumption than food.

5 Conclusion

This paper has shown that two leading explanations of macroeconomic behavior, habit formation and lumpy adjustments in consumption, are closely related. In particular, aggregating a model in which agents consume goods that have two components – one of which involves a rigidity in adjustment – yields aggregate dynamics that match those of a representative consumer habit

formation model. Hence, habit-formation preferences can be viewed as a convenient representation of the frictions inherent in consumption rather than a non-standard model of decision making.

To be clear, we do not claim that psychological habit does not exist or that commitments explain all habit-like aspects of consumption patterns. Our point is simply that psychology-based habit effects could be significantly amplified and reinforced by the commitments mechanism proposed here. An interesting direction for future research would be to estimate the fraction of aggregate habit that can be accounted for by microeconomic commitments.

Appendix

Proof of Proposition 1

With complete markets there is a unique stochastic discount factor that pins down the dynamics of adjustable consumption for all households (Harrison and Kreps, 1979). When Novikov's condition (2) is satisfied, the following process for adjustable consumption

$$\frac{da_t}{a_t} = \left\{ \frac{\pi_t^2}{2\gamma^2\sigma_t^2} + \frac{1}{\gamma} \left(\frac{\pi_t^2}{2\sigma_t^2} + r - \rho \right) \right\} dt + \frac{\pi_t}{\gamma\sigma_t} dz$$

gives rise to a state price deflator $a_t^{-\gamma}$ that prices both financial assets in the economy, as well as all other contingent claims that depend on adjustment dates at expected value discounted by the riskfree rate. As a result, this equation must govern the adjustable consumption of all households in the economy.

Assume that the household adjusts on date t , and denote the next adjustment date after t by τ . The optimal choice of x_t is governed by the first order condition

$$a_t^{-\gamma} = \frac{1}{\Lambda(0)} E_t \left[\int_t^\tau e^{-\rho s} ds \right] \mu \cdot x^{-\gamma}. \quad (20)$$

The left hand side is the marginal utility loss of reducing adjustable consumption by \$1 on t . The effective relative price between adjustable and commitment consumption is $\Lambda(0)$, because, by complete markets, increasing commitment consumption by \$1 during the period $[t, \tau]$ has a cost of $\Lambda(0)$. The right hand side expresses the gain in expected utility from a $1/\Lambda(0)$ increase in commitment consumption during $[t, \tau]$. Because the expectation on the right hand side is fully determined by the distribution function $G(\cdot)$, equation (20) implies that $x_t = \kappa \cdot a_t$ with $\kappa = \left(E_t \left[\int_t^\tau e^{-\rho s} ds \right] \mu / \Lambda(0) \right)^{1/\gamma}$ a constant.

Between reset dates, the consumer's problem depends on the history of shocks \mathcal{H}_t that determine the conditional distribution of π_t and σ_t in the future, as well as three state variables: w_t , x_t , and Δ . Let $V_\Delta(w_t, x_t, \mathcal{H}_t)$ denote the consumer's value function.

Lemma 3 *The value function can be written as*

$$V_{\Delta}(w_t, x_t, \mathcal{H}_t) = \tilde{V}_{\Delta}(w_t - \Lambda(\Delta)x_t, \mathcal{H}_t) + \mu \frac{x_t^{1-\gamma}}{1-\gamma} \cdot \int_0^{\infty} e^{-\rho s} \frac{1 - G(s + \Delta)}{1 - G(\Delta)} ds \quad (21)$$

where

$$\tilde{V}_{\Delta}(w_t - \Lambda(\Delta)x_t, \mathcal{H}_t) = \varphi(\Delta, \mathcal{H}_t) \cdot \frac{(w_t - \Lambda(\Delta)x_t)^{1-\gamma}}{1-\gamma} \quad (22)$$

for a suitable function $\varphi(\Delta, \mathcal{H}_t)$.

Proof. Consider two consumers with identical Δ and w_t^{net} at date t (but possibly different x_t). Given complete markets, the stream of commitment payments can be swapped for a single sure payment of $\Lambda(\Delta)x_t$ at t . After making this transaction, the two consumers both have remaining (net) wealth $w_t - \Lambda(\Delta)x_t$ and the same conditional distribution of future adjustment dates. As a result, they make identical consumption and investment decisions, and their expected discounted utility from current and future adjustable consumption and future commitment consumption (after the next adjustment date) is equal. This yields (21), where \tilde{V} only depends on Δ and w_t^{net} and the second term on the right hand side is the expected utility of commitments currently in place.

Denote the arrival time of the next adjustment date by τ . Since period utility is homogenous of degree $1 - \gamma$, if we multiply by $\varpi > 0$ the net wealth of a consumer with a given Δ and time- and state-contingent consumption plan $\{\{a_s | s \geq t\}, \{x_s | s \geq \tau\}\}$, his new contingent plan will be $\{\{\varpi a_s | s \geq t\}, \{\varpi x_s | s \geq \tau\}\}$. Hence \tilde{V} , his present discounted utility from adjustables and future (post τ) commitments is multiplied by $\varpi^{1-\gamma}$. Hence \tilde{V} is homogenous of degree $1 - \gamma$ conditional on Δ ; since it is also continuous, it must admit the power functional form with an appropriate multiplier $\varphi(\Delta, \mathcal{H}_t)$.¹⁶ ■

The coefficient of relative risk aversion over wealth is then:

$$\frac{-V_{ww} \cdot w}{V_w} = \gamma \cdot \frac{w_t}{w_t - \Lambda(\Delta)x_t}.$$

To derive the optimal portfolio choice, observe that by (22), given Δ and \mathcal{H}_t the household problem is homogenous of degree $1 - \gamma$ in w_t^{net} , and hence the optimal portfolio holding of stocks can be written as $\tilde{\alpha}(\Delta, \mathcal{H}_t) \cdot w_t^{\text{net}}$ for some function $\tilde{\alpha}$. When returns are i.i.d., we can explicitly solve for $\tilde{\alpha}$ by first noting that the value function no longer depends on \mathcal{H}_t because that contains no information about future returns. Market completeness requires the existence, on each date

¹⁶In our working paper, Chetty and Szeidl (2004), we explicitly characterize the function $\varphi(\Delta, \mathcal{H}_t)$ when returns are i.i.d.

t , of an additional asset for each household i that pays off if and only if i gets to adjust on date t . Such “immediate insurance” assets are sufficient to dynamically complete the market. In the optimum, i invests in a portfolio of these additional assets. Because the adjustment dates of other households are independent and the associated assets are priced at expected value, those assets will be held by i only in a diversified pool where they constitute a riskfree investment. However, i may hold additional shares of the asset that pays off if i himself gets to adjust immediately. Denote by ω_t the number of shares i buys, on date t , from that asset that pays \$1 in the event when i gets to adjust at date t , and zero otherwise. Note that ω_t is chosen endogenously and depends on all state variables. For all values of Δ where $G(\Delta)$ is differentiable (that is, almost everywhere), the cost of this insurance on date t is $dt \cdot \omega_t G'(\Delta)/(1 - G(\Delta))$ because $dt \cdot G'(\Delta)/(1 - G(\Delta))$ is likelihood that adjustment happens within the next dt time period. Therefore, the Bellman equation of i on dates t where $G(\Delta(t))$ is differentiable is

$$\begin{aligned} \rho \tilde{V}_\Delta(w^{\text{net}}) &= \max_{a, \alpha, \omega} \frac{a^{1-\gamma}}{1-\gamma} + \frac{d\tilde{V}_\Delta}{d\Delta} + \frac{d\tilde{V}_\Delta}{dw^{\text{net}}} \left[(r + \tilde{\alpha}\pi)w^{\text{net}} - a - \omega_t \frac{G'(\Delta)}{1 - G(\Delta)} \right] + \\ &\quad \frac{1}{2} \frac{d^2 \tilde{V}_\Delta}{d^2 w^{\text{net}}} \tilde{\alpha}^2 \sigma^2 (w^{\text{net}})^2 + \frac{G'(\Delta)}{1 - G(\Delta)} \left[\tilde{V}_0(w^{\text{new}} - \Lambda(0)x^{\text{new}}) - \tilde{V}_\Delta(w^{\text{net}}) \right] \end{aligned}$$

where w^{new} and x^{new} are the new levels of wealth and commitment consumption after an adjustment. Taking the first order condition in the portfolio share and substituting in the functional form from Lemma 3 gives $\tilde{\alpha} = \pi/(\gamma\sigma^2)$, so the optimal portfolio holding of stocks is

$$\frac{\pi}{\gamma\sigma^2} \cdot (w_t - \Lambda(\Delta)x_t)$$

for almost all dates t . But changing stockholdings on a measure zero set of dates does not affect allocations; as a result, this expression describes the optimal policy on all dates.

Proof of Lemma 1

Consider the process

$$\tilde{X}_t = o(t)X_0 + \int_0^t \zeta(t-s)C_s ds.$$

We will show that $\tilde{X}_t = X_t$ for all $t \geq 0$. First note that

$$\begin{aligned} \tilde{X}_t &= o(t)X_0 + \int_0^t \zeta(t-s) [A_s + X_s] ds \\ &= o(t)X_0 + \int_0^t \zeta(t-s)A_s + \zeta(t-s) \left[\int_0^s j(s-u)A_u du + k(s)X_0 \right] ds \\ &= o(t)X_0 + \int_0^t A_s \left[\zeta(t-s) + \int_0^{t-s} j(u)\zeta(t-s-u) du \right] ds + X_0 \int_0^t \zeta(t-s)k(s) ds. \end{aligned}$$

Equating coefficients, $X_t = \tilde{X}_t$ holds if

$$j(t-s) = \zeta(t-s) + \int_0^{t-s} j(u)\zeta(t-s-u) du$$

or, with $t-s = u$,

$$\zeta(u) = j(u) - \int_0^u \zeta(v)j(u-v)dv$$

and

$$o(u) = k(u) - \int_0^u \zeta(u-v)k(v)dv.$$

Substituting in $u = 0$ gives $\zeta(0) = j(0)$ and $o(0) = k(0)$. The integral equation for $\zeta(u)$ then yields a unique solution, which can be used to determine $o(\cdot)$. By the above argument, a pair of functions that solve these equations also give $X_t = \tilde{X}_t$, which is the desired representation.

Proof of special case in Theorem 1

We need to characterize the solution to the integral equation (10). By defining $j(u) = \zeta(u) = 0$ for $u < 0$, we can write (10) equivalently as

$$\zeta(u) = j(u) - \int_{\mathbb{R}} \zeta(v)j(u-v)dv. \quad (23)$$

By Assumption 1, $j(u) \in L^2(\mathbb{R})$. Because the Fourier-transform is an L^2 isometry, provided that $\zeta(u) \in L^2(\mathbb{R})$ holds, we can take the Fourier-transform of this equation. Using the fact that the Fourier-transform of a convolution is the product of the Fourier-transforms, we have

$$\mathcal{F}(\zeta) = \mathcal{F}(j) - \mathcal{F}(j) \cdot \mathcal{F}(\zeta)$$

where $\mathcal{F}(\cdot)$ denotes the Fourier-transform of a function. The resulting equation can be solved out for $\mathcal{F}(\zeta)$ and yields

$$\mathcal{F}(\zeta) = \frac{\mathcal{F}(j)}{1 + \mathcal{F}(j)}.$$

Define

$$l(z) = \frac{\mathcal{F}(j)(z)}{1 + \mathcal{F}(j)(z)}.$$

Our goal is to show that 1) $\mathcal{F}^{-1}(l)(u)$ is a valid solution of (10), and 2) $\mathcal{F}^{-1}(l)$ goes to zero exponentially.

To show 1), note that if $l(z)$ is square-integrable, then its inverse Fourier-transform exists and by construction satisfies (23). However, it is possible that $\mathcal{F}^{-1}(l)(u)$ is nonzero for $u < 0$, in which case (23) is not equivalent to (10). A solution $\zeta(u)$ that is not zero for $u < 0$ would imply that in the habit representation (7) X_t depends on future consumption. To rule this out, we must show that $\mathcal{F}^{-1}(l)(u) = 0$ for $u < 0$.

To establish 2), our approach is to note for any m' , one can define $j_1(u) = j(u)e^{m'u}$ and $\zeta_1(u) = \zeta(u)e^{m'u}$. By construction, $j_1(\cdot)$ and $\zeta_1(\cdot)$ together satisfy (10) and hence (23) as well. If we can perform the Fourier inversion argument with $j_1(\cdot)$, then we obtain $\zeta_1(u) = \zeta(u)e^{m'u} \in L^2(\mathbb{R})$. If $m' > 0$, this result suffices to show that $\zeta(u)$ goes to zero exponentially in a well-defined sense.

The following lemma shows that when $l(z)$ satisfies sufficient regularity conditions, these arguments can be made precise.

Lemma 4 *If for some $0 \leq m' < m$ the function $l(z)$*

- *extends to a holomorphic function in an open neighborhood of the half-plane $\text{Im } z \leq m'$,*
- *has no singularities in $\text{Im } z \leq m'$,*
- *is square-integrable on the line $\text{Im } z = m'$,*
- *satisfies $\lim_{|\text{Re } z| \rightarrow \infty} |l(z)| \rightarrow 0$*
- *satisfies $\lim_{n \rightarrow \infty} e^{-ns} \cdot \int_{\text{Im } z = -n} |l(z)| |dz| = 0$ for all $s > 0$,*

then $\zeta = \mathcal{F}^{-1}(l)$ is a valid solution to the integral equation (10) and $\zeta(u)e^{m'u} \in L^2(\mathbb{R})$.

Defining $o(u)$ with $\zeta = \mathcal{F}^{-1}(l)$ using (11), we also have $o(u)e^{m'u} \in L^2(\mathbb{R})$. In particular for some $K_7 > 0$

$$\int_t^\infty |\zeta(u)|^2 du \leq K_7 e^{-m't} \quad \text{and} \quad \int_t^\infty |o(u)|^2 du \leq K_7 e^{-m't}.$$

Proof. Let $\zeta_1(u) = \zeta(u)e^{m'u}$ and $j_1(u) = j(u)e^{m'u}$. Then $j_1(u) \in L^2(\mathbb{R})$ by Assumption 1 because $m' < m$, and it is easy to verify that $\zeta_1(u)$ and $j_1(u)$ also satisfy the Volterra equation (10). Let

$$l_1(z) = \frac{\mathcal{F}(j_1)(z)}{1 + \mathcal{F}(j_1)(z)}.$$

Multiplying by $e^{m'u}$ in the time domain is the same as shifting the frequency domain by $m'i$, thus $l_1(z) = l(z + m'i)$. Hence our assumptions guarantee that $l_1(z)$ is square integrable on the real line, and its inverse-Fourier transform is a well-defined function in $L^2(\mathbb{R})$. We now show that $\mathcal{F}^{-1}(l_1)(u)$, our candidate for $\zeta_1(u)$, equals zero if $u < 0$; this also proves that $\zeta(u) = e^{-m'u}\zeta_1(u) = 0$ if $u < 0$. By the definition of the inverse Fourier transform,

$$\mathcal{F}^{-1}(l_1)(u) = \lim_{B \rightarrow \infty} \frac{1}{2\pi} \int_{-B}^B e^{izu} l_1(z) dz.$$

Because the integrand is a holomorphic function in a neighborhood of $\text{Im } z \leq 0$, we can compute the integral using contour integration. Since $l(\cdot)$ has no singularities when $\text{Im } z \leq m'$, $l_1(\cdot)$ will have

no singularities when $\text{Im } z \leq 0$, and the integral above equals the integral over $[-ni - B, -ni + B]$ plus the integrals over $[-B, -B - ni]$ and $[B - ni, B]$ for any $n > 0$. Hence

$$\begin{aligned} \left| \int_{-B}^B e^{izu} l_1(z) dz \right| &\leq \left| \int_{-B-ni}^{B-ni} e^{izu} l_1(z) dz \right| + 2n \max_{|\text{Re } z|=B} |l_1(z)| \\ &\leq e^{nu} \int_{\text{Im } z=-n} |l_1(z)| |dz| + 2n \max_{|\text{Re } z|=B} |l_1(z)|. \end{aligned}$$

Since $u < 0$, by the assumptions of the lemma, we can choose n large to make the first term as small as we like. Then we can choose B for n such that the second term will be as small as desired. It follows that $\zeta_1(u)$, and therefore $\zeta(u)$, defined using the inverse Fourier transform, equal zero for $u < 0$. Hence $\zeta(u)$ is a valid solution to (10).

To conclude, note that $\zeta(u)e^{m'u} = \zeta_1(u) = \mathcal{F}^{-1}(l_1)(u)$ is in $L^2(\mathbb{R})$ since so was $l_1(z)$. Because a convolution of two square-integrable functions is square-integrable,

$$o_1(u) = e^{m'u} k(u) - \int_0^u \zeta_1(u-v) k(v) e^{m'v} dv$$

is also an element of $L^2(\mathbb{R})$, and $o(u) = e^{-m'u} o_1(u)$ solves (11) by construction. Since both $o_1(\cdot)$ and $\zeta_1(\cdot)$ are square-integrable,

$$\lim_{t \rightarrow \infty} \int_t^\infty |\zeta(u)|^2 e^{m'u} du = \lim_{t \rightarrow \infty} \int_t^\infty |o(u)|^2 e^{m'u} du = 0$$

and hence

$$\int_t^\infty |o(u)|^2 e^{m'u} du + \int_t^\infty |\zeta(u)|^2 du \leq K_7 e^{-m't}$$

for appropriate K_7 . ■

We now apply this result for the setup with deterministic arrival dates. There, one can compute

$$l(z) = \frac{b [e^{-iTz} - 1]}{b [e^{-iTz} - 1] - iz}.$$

If $l(z)$ has a singularity in z_0 , it must be that

$$e^{-iTz_0} - 1 = \frac{iz_0}{b} \tag{24}$$

is satisfied. For $\text{Im } z_0 \leq 0$ this only holds when $z_0 = 0$, because otherwise the two sides of this equation constitute disjoint sets in the complex plain. For $\text{Im } z \leq 0$, the right hand side will be in the closed half-plane to the right of the imaginary axis, while the left-hand side is a circle with radius less than or equal to 1 around the point -1 . The only potential overlap between these two regions obtains when $z_0 = 0$. It is easy to verify that this is not a singularity.

For $\text{Im } z_0 > 0$, singularities are possible. However, in any domain \mathcal{D} where $\text{Im } z_0$ is bounded, the left hand side of (24) is bounded; given the functional form on the right hand side, the set of z_0 in \mathcal{D} satisfying (24) must lie in a bounded region. Because both sides are entire functions, there can be only a finite number of solutions in a bounded region, but then there can be only a finite number of solutions in \mathcal{D} as well. This shows that there exists a solution z_0 that has the smallest nonzero imaginary value. Choose $0 < m' < \text{Im } z_0$ for that z_0 , then $l(z)$ has no singularities in $\text{Im } z \leq m'$ and holomorphic in an open neighborhood of this half-space. It is easy to see that $l(z)$ satisfies the other conditions of Lemma 4. Hence, in the special case with deterministic arrival times, the weights truly vanish at an exponential rate.

Proof of Proposition 2

The dynamics of the log commit-to-adjustable ratio of agent i , y^i can be written as

$$dy^i = \frac{\pi_I^2}{2\gamma^2\sigma_I^2}dt - \frac{\pi_I}{\gamma\sigma_I}dz^i - d\log A_t.$$

Define P_t by

$$dP_t = d\log A_t - \frac{\pi_I^2}{2\gamma^2\sigma_I^2}dt,$$

then

$$dy^i = dP_t - \frac{\pi_I}{\gamma\sigma_I}dz^i.$$

Introduce the notation $\pi/(\gamma\sigma) = \sigma_A$ and $\pi_I/(\gamma\sigma_I) = \sigma_H$ (for aggregate and household). We follow Caballero (1993) in providing a heuristic derivation of the stochastic partial differential equation (SPDE) characterizing the dynamics of the cross-sectional distribution using a discrete state/time analogue of the driving processes. The reader is referred to Krylov and Rozovski (1977, 1978) for more formal derivations of the Kolmogorov equation. Let

$$\Delta P_t = \begin{cases} \sigma_A\sqrt{\Delta t} & \text{with prob. } \frac{1}{2} \left(1 + \frac{\eta - \sigma_H^2/2}{\sigma_A}\right) \\ -\sigma_A\sqrt{\Delta t} & \text{with prob. } \frac{1}{2} \left(1 + \frac{\eta - \sigma_H^2/2}{\sigma_A}\right) \end{cases}$$

Define $\xi = -\sigma_A/\sigma_T$. If $\Delta P_t = \sigma_A\sqrt{\Delta t}$ then let

$$\Delta y_t = \begin{cases} \sigma_T\sqrt{\Delta t} & \text{with prob. } \frac{1}{2}(1 + \xi) \\ -\sigma_T\sqrt{\Delta t} & \text{with prob. } \frac{1}{2}(1 - \xi) \end{cases}$$

and if $\Delta P_t = -\sigma_A\sqrt{\Delta t}$ then let

$$\Delta y_t = \begin{cases} \sigma_T\sqrt{\Delta t} & \text{with prob. } \frac{1}{2}(1 - \xi) \\ -\sigma_T\sqrt{\Delta t} & \text{with prob. } \frac{1}{2}(1 + \xi). \end{cases}$$

We need to compute how probabilities and shares of adjustable consumption are communicated across time and space. To do this, we write the Kolmogorov equation, adjusted for the fact that idiosyncratic and aggregate shocks affect not only the cross-sectional probability distribution but also the share of individual adjustable consumption in aggregate adjustables (both through the numerator and the denominator).

$$\begin{aligned}
f(y, t) = & \\
& 1 \{ \Delta P_t > 0 \} \left[f(y - \Delta y, t - \Delta t) \frac{1 + \xi}{2} \exp \left\{ -\Delta y - \Delta P - \frac{\sigma_H^2}{2} \Delta t \right\} + \right. \\
& \left. + f(y + \Delta y, t - \Delta t) \frac{1 - \xi}{2} \exp \left\{ \Delta y - \Delta P - \frac{\sigma_H^2}{2} \Delta t \right\} \right] \\
& + 1 \{ \Delta P_t < 0 \} \left[f(y - \Delta y, t - \Delta t) \frac{1 - \xi}{2} \exp \left\{ -\Delta y + \Delta P + \frac{\sigma_H^2}{2} \Delta t \right\} + \right. \\
& \left. + f(y + \Delta y, t - \Delta t) \frac{1 + \xi}{2} \exp \left\{ \Delta y + \Delta P + \frac{\sigma_H^2}{2} \Delta t \right\} \right].
\end{aligned}$$

Rearranging and approximating $f(y - \Delta y, t - \Delta t)$ by a second-order Taylor approximation around $f(y, t - \Delta t)$, approximating the exponential function to the second order around the point zero, making use of the fact that $(\Delta y)^2 = \sigma_I^2 \Delta t$, and taking the limit as Δt goes to dt , we get

$$df(y, t) = \left[\left(\eta + \frac{\pi_I^2}{2\gamma^2\sigma_I^2} \right) \frac{\partial f(y, t)}{\partial y} + \frac{\sigma_T^2}{2} \frac{\partial^2 f(y, t)}{\partial y^2} \right] dt + \frac{\pi}{\gamma\sigma} \frac{\partial f(y, t)}{\partial y} dz$$

as desired.

The boundary conditions follow exactly as in Caballero (1993). For the first boundary condition, evaluate the discretized Kolmogorov equation at L , note that $f(L - \Delta y, t - \Delta t)$ equals zero because it is outside the band, and then take the limit as Δt goes to zero to obtain $f(L, t) = 0$. A similar argument shows that $f(U, t) = 0$. For the other two boundary conditions, integrate both sides of the stochastic partial differential equation over (L, U) and use the fact that $\int f(y, t) dy = 1$ to obtain

$$0 = \int_L^U \left[\left(\mu + \frac{\pi_I^2}{2\gamma^2\sigma_I^2} \right) \frac{\partial f(y, t)}{\partial y} + \frac{\sigma_T^2}{2} \frac{\partial^2 f(y, t)}{\partial y^2} \right] dy \cdot dt + \left[\frac{\pi}{\gamma\sigma} \int_L^U \frac{\partial f(y, t)}{\partial y} dy \right] \cdot dz$$

where we exchanged the integral and differential operators. In order for the process on the right hand side to be identically zero, both the drift and diffusion term must vanish, which implies

$$\begin{aligned}
\int_L^U \frac{\partial f(y, t)}{\partial y} dy &= 0 \\
\int_L^U \frac{\partial^2 f(y, t)}{\partial y^2} dy &= 0.
\end{aligned}$$

Integrating by parts these equations and making use of $f(L, t) = f(U, t) = 0$ gives the remaining two boundary conditions.

To derive the dynamics of aggregate commitments, note that

$$X_t = \int_L^U \exp(y) f(y, t) dy \cdot A_t$$

and we can use Ito's lemma to write

$$dX_t = A_t \int_L^U \exp(y) \cdot df(y, t) \cdot dy + dA_t \cdot \int_L^U \exp(y) f(y, t) dy + \left\langle \int_L^U \exp(y) \cdot df(y, t) \cdot dy, dA_t \right\rangle.$$

We now evaluate each term on the right hand side. The first term is

$$A_t \int_L^U \exp(y) \cdot \frac{\partial f(y, t)}{\partial y} \left\{ \left(\mu + \frac{\pi_I^2}{2\gamma^2\sigma_I^2} \right) dt + \frac{\pi}{\gamma\sigma} dz \right\} dy + F_t \int_L^U \exp(y) \cdot \frac{\partial^2 f(y, t)}{\partial y^2} \frac{\sigma_T^2}{2} dt \cdot dy.$$

Integrating by parts, and making use of the boundary conditions above allows us to express this term as

$$-X_t \left(\left(\mu + \frac{\pi_I^2}{2\gamma^2\sigma_I^2} \right) dt + \frac{\pi}{\gamma\sigma} dz \right) + A_t \frac{\sigma_T^2}{2} \cdot (f_y(L, t)(e^M - e^L) + f_y(U, t)(e^U - e^M)) dt + \frac{\sigma_T^2}{2} X_t dt.$$

The second term is

$$X_t \cdot \frac{dA_t}{A_t} = X_t \left(\left(\mu + \frac{\pi_I^2}{2\gamma^2\sigma_I^2} \right) dt + \frac{\pi}{\gamma\sigma} dz \right)$$

and the third term is

$$- \left(\frac{\pi}{\gamma\sigma} \right)^2 X_t dt.$$

Collecting terms,

$$dX_t = A_t \frac{\sigma_T^2}{2} \cdot (f_y(L, t)(e^M - e^L) + f_y(U, t)(e^U - e^M)) dt.$$

Proof of Lemma 2

Write

$$dy^i = \frac{\pi_I^2}{2\gamma^2\sigma_I^2} dt - \frac{\pi_I}{\gamma\sigma_I} dz^i - d \log A_t = d\tilde{y}^i - d \log A_t$$

where

$$d\tilde{y}^i = \frac{\pi_I^2}{2\gamma^2\sigma_I^2} dt - \frac{\pi_I}{\gamma\sigma_I} dz^i$$

is idiosyncratic while $d \log A_t$ represents aggregate risk.

Our goal is to compute an upper bound for the expectation in (17). By the Markov property of Brownian motion, we can assume without loss of generality that $s = 0$; this simplifies notation.

For y^i to remain in the (L, U) band during the $[0, t]$ time period, it must remain there during $[0, t_1]$ as well as during $[t_1, t]$:

$$\begin{aligned} & \max_{y_{i_0}, \{A_v | 0 \leq v \leq t\}} \mathbb{E} \left[1 \{y_v^i \in (L, U) \text{ for } 0 \leq v \leq t\} \mid \{A_v | 0 \leq v \leq t\} \right] \leq \\ & \leq \max_{y_0^i, \{A_v | 0 \leq v \leq t_1\}} \mathbb{E} \left[1 \{y_v^i \in (L, U) \text{ for } 0 \leq v \leq t_1\} \mid \{A_v | 0 \leq v \leq t\} \right] \times \\ & \quad \times \max_{y_{t_1}^i, \{A_v | t_1 \leq v \leq t\}} \mathbb{E} \left[1 \{y_v^i \in (L, U) \text{ for } t_1 \leq v \leq t\} \mid \{A_v | t_1 \leq v \leq t\} \right]. \end{aligned}$$

This inequality suggests that we can bound (17) by splitting the $[0, t]$ interval into several pieces as above and obtaining bounds for each piece. Fix a path of aggregate adjustables A_v . In order for $y_v^i = A_v + \tilde{y}_v^i$ to be within the (L, U) band during $[0, t_1]$, it must be in particular that $\tilde{y}_{t_1}^i \in (L - A_{t_1}, U - A_{t_1})$. Hence we can bound the first term on the right hand side of the previous equation as

$$\max_{y_0^i, \{A_v | 0 \leq v \leq t_1\}} \mathbb{E} \left[1 \{y_v^i \in (L, U) \text{ for } 0 \leq v \leq t_1\} \mid \{A_v | 0 \leq v \leq t\} \right] \leq \max_a \mathbb{E} \left[1 \{\tilde{y}_{t_1}^i \in (L - a, U - a)\} \right].$$

Because \tilde{y}_t^i is a Brownian motion with a drift, the variance of $\tilde{y}_{t_1}^i$ is proportional to t_1 , and hence the right hand side is at most $K_2/\sqrt{t_1}$ for a suitable positive constant K_2 . Now choose n to be a positive integer, split the interval $(0, t)$ into n equal sub-intervals. By the above logic, we can bound the probability that y remains within the (U, L) band during each sub-interval by $K_2/\sqrt{t/n}$, to obtain

$$\max_{y_0^i, \{A_v | 0 \leq v \leq t\}} \mathbb{E} \left[1 \{y_v^i \in (L, U) \text{ for } 0 \leq v \leq t\} \mid \{A_v | 0 \leq v \leq t\} \right] \leq \left(\frac{K_2}{\sqrt{t/n}} \right)^n.$$

There exists $t_0 > 0$ such that for $t > t_0$, there is always an integer n between $t/4K_2^2$ and $t/9K_2^2$. Choosing such an n implies that the bound on the right hand side is at most $(1/2)^{t/9K_2^2}$ which exponentially goes to zero in t . For $t < t_0$, we can bound (17) by 1. This shows that the share of households who have not adjusted between 0 and t is at most $K_4 \exp(-K_3 t)$ for appropriate positive constants K_3 and K_4 .

Proofs for Section 4

Consumption response with time-dependent adjustment. Consider the model with two consumption categories in Section 2.4. Suppose no aggregate shocks have taken place ($dz = 0$) for a long time period, and at t_0 a shock of size Δz_{t_0} is realized. Adjustables respond immediately: $A_{1, t_0+} = A_{1, t_0} \exp(\Delta z_{t_0} \cdot \pi/(\gamma\sigma))$, while the initial response of X_1 is zero. In the limit over time,

all households adjust. Hence, for $S > 0$

$$X_{1,t_0+S} = \exp \left\{ [\theta(S) - 1] \cdot \Delta z \cdot \frac{\pi}{\gamma\sigma} \right\} \cdot A_{1,t} \frac{\chi}{1 - \chi}$$

where $\theta(\cdot)$ is a deterministic increasing function that satisfies $\theta(0) = 0$ and $\lim_{S \rightarrow \infty} \theta(S) = 1$. Using a first order Taylor approximation of $\log C_{1,t}$ around $A/X = (1 - \chi)/\chi$ yields

$$\log C_{1,t} \approx K_8 + (1 - \chi) \log A_{1,t} + \chi \log X_{1,t}$$

where K_8 is the constant term in the approximation. Direct substitution yields (18) in the main text.

In the special case with deterministic adjustment, a log-linear approximation can be used to show that $\theta(S) \approx S/T$. Hence,

$$\Delta \log C_{1,t_0+S} \approx vS + \frac{\pi}{\gamma\sigma} \left[(1 - \chi) + \chi \frac{S}{T} \right] \cdot \Delta z_{t_0}.$$

Consumption response with state-dependent adjustment. Assume that the economy starts from the steady-state distribution that obtains in the limit when dz is zero for a long period. Here, in response to a Δz shock, a subset of households get pushed out of the (L, U) band and update their commitments. If $\Delta z_{t_0} < (L - U) \cdot \gamma\sigma/\pi$, not all households adjust. In the limit, if no subsequent aggregate shocks take place, the economy again settles in the steady-state cross-sectional distribution, in which the ratio of adjustables to commitments is a constant. It follows that there is again a deterministic function $\theta(S)$ such that total commitments for $S > 0$ are

$$X_{1,t_0+S} = \exp \left\{ [\theta(S) - 1] \cdot \Delta z \cdot \frac{\pi}{\gamma\sigma} \right\} \cdot A_{1,t} \frac{\chi}{1 - \chi}$$

and (18) is again obtained following the derivation for the time-dependent case.

Comparative Statics for (19). The comparative statics in which the endogenous parameter χ is varied can be made rigorous by changing exogenous parameters of the model. In particular, vary the relative preference for adjustables μ_1^a and μ_2^a so that $\mu^a = \left[(\mu_1^a)^{1/\gamma} + (\mu_2^a)^{1/\gamma} \right]^\gamma$ is constant. This exercise is equivalent to relabelling part of total adjustables from category 1 to category 2. Increasing μ_1^a , raises the relative preference for adjustables in consumption category 1, while reducing μ_2^a at the same time makes the consumer's problem (14) unchanged, and keeps the dynamics of a , x_1 and x_2 fixed. The function $\theta(S)$ is thus unaffected as μ_1^a and μ_2^a are varied. Hence, changing μ_1^a in this way directly generates a proportional change in χ without causing any other changes in (18), justifying the comparative statics described in the text.

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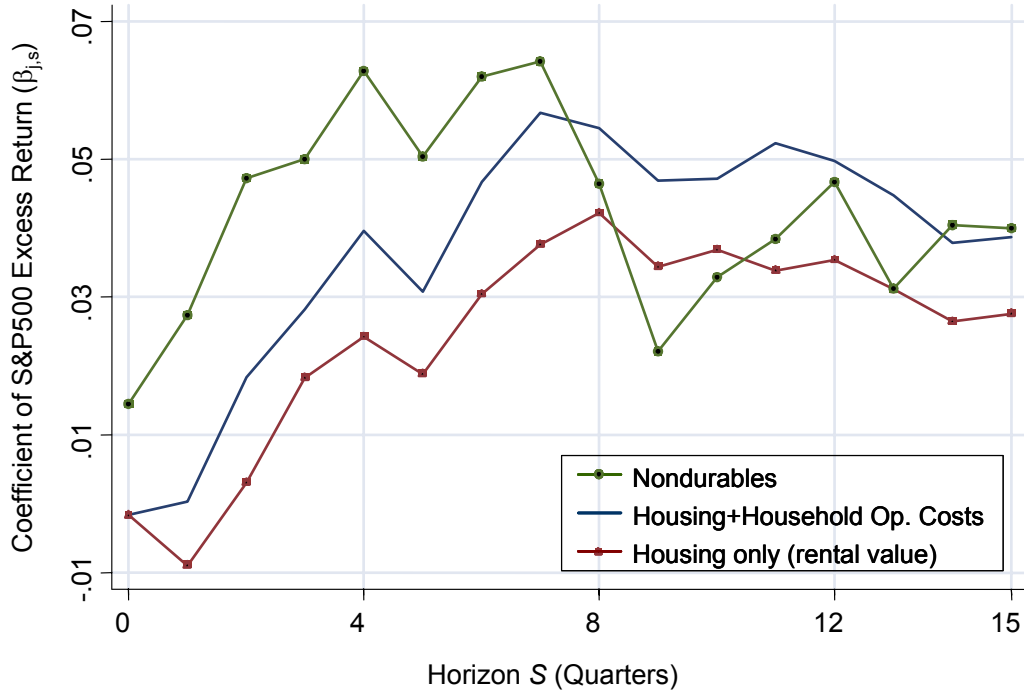
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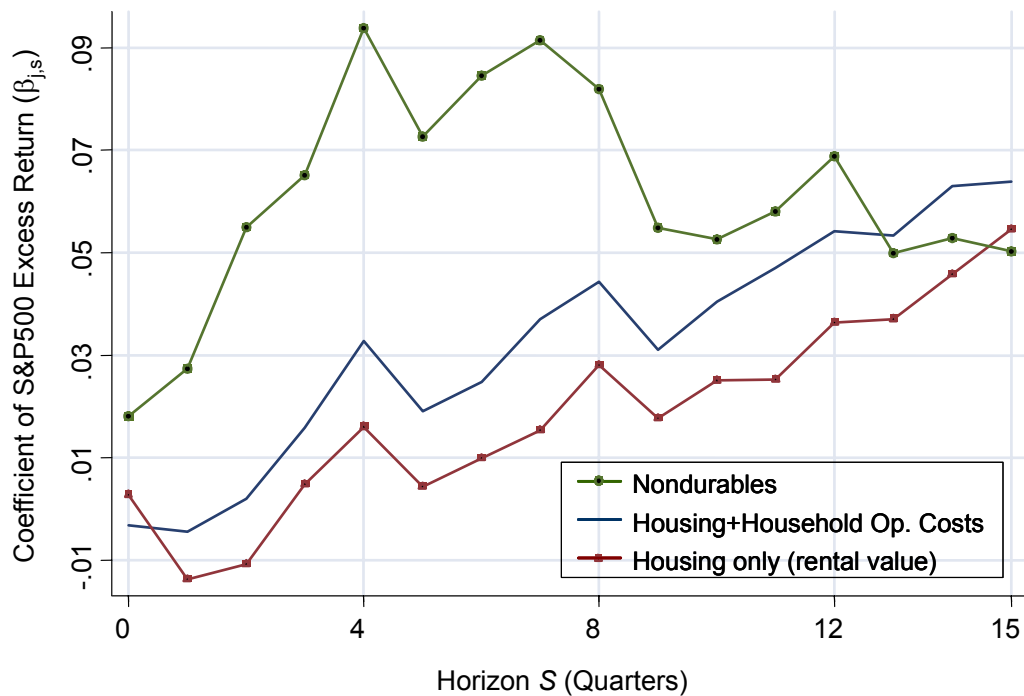
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Figure 1
Effects of Excess Returns on Consumption Growth Over Various Horizons

A. Full Sample



B. Trimmed Sample



NOTE--These figures show the effect of stock returns on consumption of different goods over time,

following the methodology of Parker and Julliard (2005) . Each point of each curve represents a coefficient from a different regression of consumption growth on S&P500 excess returns, following the end-of-period consumption convention, as explained in the text. For example, in the nondurables curve in Figure 1A, the coefficient at horizon S is $\hat{\beta}_{j,S}$ in the regression

$\ln(Nondur_{t+S}/Nondur_{t-1}) = \alpha_{j,S} + \beta_{j,S}R_t$, where R_t is the excess return on the S&P500 index in quarter t and $Nondur_t$ is per-capita real nondurable consumption in period t . The other curves are plotted analogously, replacing the dependent variable in the regressions with other consumption categories from national accounts. Figure 1B is constructed in the same manner, except that outliers are trimmed by eliminating the top and bottom 5% of the distribution of excess quarterly returns (returns that exceed 11.5% in absolute value) from the regression. Quarterly nominal consumption data is obtained from NIPA Table 2.3.5 for 1947Q1-2004Q4. These consumption measures are deflated using the chain-weighted price deflator in NIPA Table 2.3.4. Per-capita measures are obtained using annual population data from the Census Bureau.