

Intermediate Goods and Weak Links: A Theory of Economic Development

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Per capita income in the richest countries of the world exceeds that in the poorest countries by more than a factor of 50. What explains these enormous differences? This paper returns to two old ideas in development economics and proposes that complementarity and linkages are at the heart of the explanation. First, just as a chain is only as strong as its weakest link, problems at any point in a production chain can reduce output substantially if inputs enter production in a complementary fashion. Second, linkages between firms through intermediate goods deliver a multiplier similar to the one associated with capital accumulation in a neoclassical growth model. Because the intermediate goods' share of revenue is about 1/2, this multiplier is substantial. The paper builds a model with complementary inputs and links across sectors and shows that it can easily generate 50-fold aggregate income differences.

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1. INTRODUCTION

By the end of the 20th century, per capita income in the United States was more than 50 times higher than per capita income in Ethiopia and Tanzania. Dispersion across the 95th-5th percentiles of countries was more than a factor of 32. What explains these profound differences in incomes across countries?¹

This paper returns to two old ideas in the development economics literature and proposes that complementarity and linkages are at the heart of the explanation. Because of complementarity, high productivity in a firm requires a high level of performance along a large number of dimensions. Textile producers require raw materials, knitting machines, a healthy and trained labor force, knowledge of how to produce, security, business licenses, transportation networks, electricity, etc. These inputs enter in a complementary fashion, in the sense that problems with any input can substantially reduce overall output. Without electricity or production knowledge or raw materials or security or business licenses, production is likely to be severely hindered.

Intermediate goods provide links between sectors that create a productivity multiplier. Low productivity in electric power generation reduces output in banking and construction. But this reduces the ease with which the electricity industry can build new dams and therefore further reduces output in electric power generation. This multiplier effect is similar to the multiplier associated with capital accumulation in a neoclassical growth model. In fact, intermediate goods are just another form of capital, albeit one that depreciates fully in production. Because the intermediate goods' share of revenue is approximately 1/2, the intermediate goods multiplier is large.

The contribution of this paper is to build a model in which these ideas can be made precise. We show that complementarity and linkages amplify small

¹Recent work on this topic includes Romer (1994), Klenow and Rodriguez-Clare (1997), Prescott (1998), Hall and Jones (1999), Parente and Prescott (1999), Howitt (2000), Parente, Rogerson and Wright (2000), Acemoglu, Johnson and Robinson (2001), Klenow and Rodriguez-Clare (2005), Manuelli and Seshadri (2005), Caselli and Coleman (2006), Armenter and Lahiri (2006), Erosa, Koreshkova and Restuccia (2006), Marimon and Quadrini (2006), and Restuccia, Yang and Zhu (2006).

differences across economies. With plausible average differences in productivity across countries, we are able to explain 50-fold differences in per capita income.

The approach taken in this paper can be compared with the recent literature on political economy and institutions; for example, see Acemoglu and Johnson (2005) and Acemoglu and Robinson (2005). This paper is more about mechanics: can we develop a plausible mechanism for getting a big multiplier, so that relatively modest distortions lead to large income differences? The modern institutions approach builds up from political economy. This is useful in explaining why the allocations in poor countries are inferior — for example, why investment rates in physical and human capital are so low — but the institutions approach ultimately still requires a large multiplier to explain income differences. As just one example, even if a political economy model explains observed differences in investment rates across countries, the model cannot explain 50-fold income differences if it is embedded in a neoclassical framework. The political economy approach explains why resources are misallocated; the approach here explains why misallocations lead to large income differences. Clearly, both steps are needed to understand development.

2. LINKAGES AND COMPLEMENTARITY

We begin by discussing briefly the two key mechanisms at work in this paper, linkages and complementarity. These mechanisms are conceptually distinct — one can have linkages without complementarity. The linkage mechanism turns out to be quite simple and powerful, so very little time (perhaps too little) will be needed to convey its role. The complementarity mechanism is more difficult to model and hence consumes more than its share of words and effort in this paper.

2.1. Linkages through Intermediate Goods

The notion that linkages across sectors can be central to economic performance dates back at least to Leontief (1936), which launched the field of input-output economics. Hirschman (1958) emphasized the importance of complementarity

and linkages to economic development. A large subsequent empirical literature constructed input-output tables for many different countries and computed sectoral multipliers.

In what may prove to be an ill-advised omission, these insights have not generally been incorporated into modern growth theory. Linkages between sectors through intermediate goods deliver a multiplier very much like the multiplier associated with capital in the neoclassical growth model. More capital leads to more output, which in turn leads to more capital. This virtuous circle shows up mathematically as a geometric series which sums to a multiplier of $\frac{1}{1-\alpha}$, if α is capital's share of overall revenue. Because the capital share is only about 1/3, this multiplier is relatively small: differences in investment rates are too small to explain large income differences, and large total factor productivity residuals are required. This led a number of authors to broaden the definition of capital, say to include human capital or organizational capital. It is generally recognized that if one can get the capital share up to something like 2/3 — so the multiplier is 3 — large income differences are much easier to explain without appealing to a large residual.²

Intermediate goods generate this same kind of multiplier. An increase in productivity in the transportation sector raises output in the capital equipment sector which may in turn raise output in the transportation sector. In the model below, this multiplier depends on $\frac{1}{1-\sigma}$, where σ is the share of intermediate goods in total revenues. This share is approximately 1/2 in the United States and in other countries, delivering a multiplier of 2. In the model, the overall multiplier on productivity is the product of the intermediate goods and capital multipliers: $\frac{1}{1-\sigma} \times \frac{1}{1-\alpha} = 2 \times 3/2 = 3$. Combining a neoclassical story of capital accumu-

²Mankiw, Romer and Weil (1992) is an early example of this approach to human capital. Chari, Kehoe and McGrattan (1997) introduced “organizational capital” for the same reason. Howitt (2000) and Klenow and Rodriguez-Clare (2005) use the accumulation of ideas to boost the multiplier. More recently, Manuelli and Seshadri (2005) and Erosa et al. (2006) have resurrected the human capital story in a more sophisticated fashion. The controversy in each of these stories is over whether or not the additional accumulation raises the multiplier sufficiently. Typically, the problem is that the magnitude of a key parameter is difficult to pin down.

lation with a standard treatment of intermediate goods therefore delivers a very powerful engine for explaining income differences across countries. Related insights pervade the older development literature but have not had a large influence on modern growth theory. The main exception is Ciccone (2002), which appears to be underappreciated.³

2.2. The Role of Complementarity

A large multiplier in growth models is a two-edged sword. On the one hand, it is extremely useful in getting realistic differences in investment rates, productivity, and distortions to explain large income differences. However, the large multiplier has a cost. In particular, theories of economic development often suffer from a “magic bullet” critique. If the multiplier is so large, then solving the development problem may be quite easy. For example, this is a potential problem in the Manuelli and Seshadri (2005) paper: small subsidies to the production of output or small improvements in a single (exogenous) productivity level have enormous long-run effects on per capita income in their model. If there were a single magic bullet for solving the world’s development problems, one would expect that policy experimentation across countries would hit on it, at least eventually. The magic bullet would become well-known and the world’s development problems would be solved.

This is where the second insight of this paper plays its role. Because of complementarity, the development problem may be hard to solve. In any production process, there are ten things that can go wrong that will sharply reduce the value of production. In rich countries, there are enough substitution possibilities that these things do not often go wrong. In poor countries, on the other hand, any one

³Ciccone develops the multiplier formula for intermediate goods and provides some quantitative examples illustrating that the multiplier can be large. The point may be overlooked by readers of his paper because the model also features increasing returns, externalities, and multiple equilibria. Yi (2003) argues that tariffs can multiply up in much the same way when goods get traded multiple times during the stages of production. Interestingly, the intermediate goods multiplier shows up most clearly in the economic fluctuations literature; see Long and Plosser (1983), Basu (1995), Horvath (1998), Dupor (1999), Conley and Dupor (2003), and Gabaix (2005). See also Hulten (1978).

of several problems can doom a project. Obtaining the instruction manual for how to produce socks is not especially useful if the import of knitting equipment is restricted, if cotton and polyester threads are not available, if property rights are not secure, and if the market to which these socks will be sold is unknown. Complementarity is at the heart of the O-ring story put forward by Kremer (1993). The idea in this paper is similar, but the papers differ substantially in crucial ways. These differences will be discussed in detail below.

Linkages through intermediate goods provide a large multiplier, while complementarity means that there is typically not a single magic bullet that can exploit this multiplier. Occasionally, of course, there is. Fixing the last bottleneck to development can have large effects on incomes, which may help us to understand growth miracles.

2.3. An Example of Complementarity

Standard models of production often emphasize the substitutability of different inputs. While substitution will play an important role in the model that follows, so will complementarity. Since this is less familiar, we begin by focusing our attention on complementary inputs.⁴

For this purpose, it is helpful to begin with a simple example. Suppose you'd like to set up a factory in China to make socks. The overall success of this project requires success along a surprisingly large number of different dimensions. These different activities are complementary, so that inefficiencies on any one dimension can sharply reduce overall output.

First, the firm needs the basic inputs of production. These include cotton, silk, and polyester; the sock-knitting machines that spin these threads into socks; a competent, healthy, and motivated workforce; a factory building; electricity and other utilities; a means of transporting raw materials and finished goods throughout the factory, etc.

⁴Milgrom and Roberts (1990) argue that there are extensive complementarities involved in production by modern firms, related to marketing, manufacturing, engineering, design, and organization.

Apart from the physical production of socks, other activities are required to turn raw materials into revenue. The entire production process must be kept secure from theft or expropriation. The sock manufacturer must match with buyers, perhaps in foreign markets, and must find a way to deliver the socks to these buyers. Legal requirements must also be met, both domestically and in foreign markets. Firms must acquire the necessary licenses and regulatory approval for production and trade.

Finally, the managers in the firm require many different kinds of knowledge. They need to know the technical details of how to make socks. They need to know how to manage their workforce, how to run an accounting system, how to navigate a perhaps-intricate web of legal requirements, etc. Notice that even if the basic inputs are available through trade, these last two paragraphs of requirements are to a great extent nontradable. Trade may help alleviate the problems in this paper, but there are likely to be enough non-traded inputs that domestic weak links can be crucial.

The point of this somewhat tedious enumeration is that production — even of something as simple as a pair of socks — involves a large number of necessary activities. If any of these activities are performed inefficiently, overall output can be reduced considerably. Without a reliable supply of electricity, the sock-making machines cannot be utilized efficiently. If workers are not adequately trained or are unhealthy because of contaminated water supplies, productivity will suffer. If export licenses are not in order, the socks may sit in a warehouse rather than being sold. If property is not secure, the socks may be stolen before they can reach the market.

2.4. Modeling Complementary Inputs

A natural way to model the complementarity of these activities is with a CES production function:

$$Y = \left(\int_0^1 z_i^\rho di \right)^{1/\rho} . \quad (1)$$

We use z_i to denote a firm's performance along the i^{th} dimension, and we assume there are a continuum of activities indexed on the unit interval that are necessary for production. In terms of our sock example, z_a could be the quality of the instructions the firm has for making socks. z_b could be number of sock-making machines, z_c might represent the extent to which the relevant licenses have been obtained, etc.

The elasticity of substitution among these activities is $1/(1-\rho)$, but this (or its inverse) could easily be called an elasticity of complementarity instead. We will focus on the case where $\rho < 0$, so the elasticity of substitution is less than one. It is difficult to substitute electricity for transportation services or raw materials in production. Inputs are more complementary than in the usual Cobb-Douglas case ($\rho = 0$).

Complementarity puts extra weight on the activities in which the firm is least successful. This is easy to see in the limiting case where $\rho \rightarrow -\infty$; in this case, the CES function converges to the minimum function, so output is equal to the smallest of the z_i .

This intuition can be pushed further by noting that the CES combination in equation (1) is called the *power mean* of the underlying z_i in statistics. The power mean is just a generalized mean. For example, if $\rho = 1$, Y is the arithmetic mean of the z_i . If $\rho = 0$, output is the geometric mean (Cobb-Douglas). If $\rho = -1$, output is the harmonic mean, and if $\rho \rightarrow -\infty$, output is the minimum of the z_i . From a standard result in statistics, these means decline as ρ becomes more negative. Economically, a stronger degree of complementarity puts more weight on the weakest links and reduces output.⁵

2.5. Comparing to Kremer's O-Ring Approach

It is useful to compare the way we model complementarity with the O-ring theory of income differences put forward by Kremer (1993). Superficially, the

⁵Benabou (1996) studies this approach to complementarity. Interestingly, standard intertemporal preferences with a constant relative risk aversion coefficient greater than one represent a familiar example.

theories are similar, and the general story Kremer tells is helpful in understanding the current paper: the space shuttle Challenger and its seven-member crew are destroyed because of the failure of a single, inexpensive rubber seal.

This paper differs crucially, however, in terms of how the general idea gets implemented. In particular, Kremer's modeling approach appears to assume a large degree of increasing returns, which is difficult to justify.

To see this, recall that Kremer assumes there are N different tasks that must be completed for production to succeed. Suppose workers have a probability of success q at any task, and assume these probabilities are independent. Expected output in Kremer's model is then given by $Q = \prod_{i=1}^N q_i = q^N$. Suppose the richest countries are flawless in production, so $q^{rich} = 1$, while the poorest countries are successful in each task 50 percent of the time, so $q^{poor} = 1/2$. The ratio of incomes between rich and poor countries is therefore on the order of 2^N . If there are five different tasks in production, it is quite easy to explain a 32-fold difference in incomes across countries.

A problem with this approach is that the O-ring logic implies complementarity, but it does not imply the huge degree of increasing returns assumed in Kremer's $Q = q^N$ formulation. For example, an alternative production function that is also perfectly consistent with the O-ring story is $Q = q_1^{1/N} q_2^{1/N} \cdot \dots \cdot q_N^{1/N}$ — that is, a Cobb-Douglas combination of tasks with constant returns. Notice that the O-ring complementarity applies here as well: if any q_i is zero, then $Q = 0$ and the entire project fails. With symmetry so that $q_i = q$, this approach leads to $Q = q$, so that a 2-fold difference in success on each task only translates into a 2-fold difference in incomes across countries.

While the O-ring story is quite appealing, Kremer's formulation relies on an arbitrary and exceedingly strong degree of increasing returns — which is not part of the O-ring logic — to get big income differences. The approach taken

here is to drop the large increasing returns inherent in Kremer's formulation and to emphasize complementarity instead.⁶

Although it is not emphasized in his paper, Kremer's approach can be viewed as embodying a Leontief technology — the most extreme form of complementarity. Blanchard and Kremer (1997) formalize this interpretation and study a model of chains of production in order to understand the large declines in output in the former Soviet Union after 1989. They emphasize that with a chain of specialized producers, bargaining problems can lead to large losses in output.⁷

3. SETTING UP THE MODEL

We now apply this basic discussion of complementarity and linkages to construct a theory of economic development.

3.1. The Economic Environment

A single final good in this economy is produced using a continuum of activities that enter in a complementary fashion, as discussed above:

$$Y = \zeta \cdot \left(\int_0^1 Y_i^\rho di \right)^{1/\rho}, \quad \rho < 0. \quad (2)$$

In this expression, Y_i denotes the activity inputs, and ζ is a constant that we will use to simplify some expressions later.⁸

Activities are themselves produced using a relatively standard Cobb-Douglas production function:

$$Y_i = A_i \left(K_i^\alpha H_i^{1-\alpha} \right)^{1-\sigma} X_i^\sigma, \quad (3)$$

⁶Several other papers related to this one also rely heavily on increasing returns to explain income differences, including Murphy, Shleifer and Vishny (1989), Rodriguez-Clare (1996), and Rodrik (1996).

⁷Becker and Murphy (1992) also consider a production function that combines tasks in a Leontief way to produce output. They use this setup to study the division of labor and argue that it is limited by problems in coordinating the efforts of specialized workers. Grossman and Maggi (2000), motivated in part by Kremer (1993), study trade between countries when production functions across sectors involve different degrees of complementarity. They find that countries with thicker lower tails in the talent distribution will specialize in producing goods that involve less complementarity in production.

⁸In particular, we assume $\zeta = \sigma^{-\sigma}$, where σ will be defined below.

where α and σ are both between zero and one. K_i and H_i are the amounts of physical capital and human capital used to produce activity i , and A_i is an exogenously-given productivity level. The novel term in this production specification is X_i , which denotes the quantity of intermediate goods used to produce activity i .

Before discussing the role of X_i , it is convenient to specify the three resource constraints that face this economy:

$$\int_0^1 K_i di \leq K, \quad (4)$$

$$\int_0^1 H_i di \leq H, \quad (5)$$

and

$$C + \int_0^1 X_i di \leq Y. \quad (6)$$

The first two constraints are straightforward. We assume the economy is endowed with an exogenous amount of physical capital, K , and human capital, H , that can be used in production. Later on, we will endogenize K and H in standard ways, but it is convenient to take them as exogenous for now.

The last resource constraint says that final output can be used for consumption, C , or for the X_i intermediate goods. One unit of the final good can be used as one unit of the intermediate input in any activity.⁹

One can think of this as follows. Consider the production of the i^{th} activity Y_i , which we might take to be transportation services. Transportation is produced using physical capital, human capital, and some intermediate goods from other sectors (such as fuel). The share of intermediate goods in the production of the i^{th} activity is σ . To keep the model simple and tractable, we assume that

⁹An issue of timing arises here. To keep the model simple and because we are concerned with the long run, we make the seemingly strange assumption that intermediate goods are produced and used simultaneously. A better justification goes as follows. Imagine incorporating a lag so that today's final good is used as tomorrow's intermediate input. The steady state of that setup would then deliver the result we have here.

the same bundle of intermediate goods are used in each activity, and that these intermediate goods are just units of final output.

Intermediate goods are very similar to capital, except that they fully depreciate in production. The share of *produced* goods in the production of activity i is therefore $\alpha(1 - \sigma) + \sigma$. For standard parameter values like $\alpha = 1/3$ and $\sigma = 1/2$, this share is $2/3$ — the value needed for neoclassical models to explain large income differences.

The parameter σ measures the importance of linkages in our economy. If $\sigma = 0$, the productivity of physical and human capital in each activity depends only on A_i and is independent of the rest of the economy. To the extent that $\sigma > 0$, low productivity in one activity feeds back into the others. Transportation services may be unproductive in a poor country because of inadequate fuel supplies or repair services, and this low productivity will reduce output throughout the economy.

3.2. Distortions and the Exogenous Productivity Levels

In this setup, the key exogenous variables that will give rise to income differences are the productivity levels, the A_i . We take the A_i as exogenous here, and simply assume that on average, productivity is somewhat lower in poor countries than in rich countries.

In a more completely-specified model, the underlying distortions would be taxes, expropriation, and other wedges affecting the allocation of resources across firms and industries. In many (but not all) cases, these show up in ways similar to productivity distortions, which motivates the approach taken here; see Chari, Kehoe and McGrattan (forthcoming). Hsieh and Klenow (2006), for example, show how distortions to the allocation of resources across firms *within* an industry can reduce that industry's TFP. The key question then becomes: can distortions of the magnitudes we observe generate 50-fold income differences. In neoclassical models, we know the answer to this question is “no.” Hsieh and Klenow, for example, show that misallocations across firms within an industry

reduce output by a factor of 2 or 3. What is needed is a multiplier to magnify the effects of these distortions.

3.3. Substitution and Complementarity

This basic setup is not necessarily the most natural way to formulate the model. In particular, one could imagine directly replacing X_i in equation (3) with a CES combination of the different activities. Separately, the final good could be produced as a Cobb-Douglas function of the activities, as opposed to (2). Intermediate goods would then involve substantial complementarity (think of materials and energy), but when activities combine to produce the consumption good, there would be more substitutability. For example, computer services are today nearly an essential input into semiconductor design, banking, and health care, but there may be substantial substitution between computer games and other sources of entertainment in consumption. In order to produce within a firm, there are a number of complementary steps that must be taken. At the final consumption stage, however, there appears to be a reasonably high degree of substitution across goods.

Unfortunately, this more natural formulation does not lead to closed-form solutions. The simplification here replaces these two conceptually distinct production functions with the single CES combination. This makes sense at the level of the activity production function in (3), but it is a stretch when applied to the final good in (2). Nevertheless, this is the trick needed to make progress analytically. While it generally works well, we will see that this formulation does have some minor drawbacks.

4. ALLOCATING RESOURCES AND SOLVING

Taking the aggregate quantities of physical and human capital as given, we consider two alternative ways of allocating resources. The first is a symmetric allocation of resources across the activities. This allocation is not optimal, but it is quite easy to solve for and allows us to get quickly to some of the important

results in this paper. We also consider the second-best allocation of resources. This is the allocation that maximizes consumption *taking the distortions in the economy as given*; in our setting, this means taking the productivity levels A_i as given. These two allocations are defined in turn.

DEFINITION 4.1. The *symmetric allocation of resources* in this economy has $K_i = K$, $H_i = H$, $X_i = X$, and $X = \bar{s}Y$, where $0 < \bar{s} < 1$. Moreover, we assume $\bar{s} = \sigma$, which turns out to be the optimal share of output to use as intermediate goods. Y and Y_i are then determined from the production functions in (2) and (3).

DEFINITION 4.2. The *second-best allocation of resources* in this economy consists of values for the six endogenous variables $Y, C, \{Y_i, K_i, H_i, X_i\}$ that solve

$$\max_{\{X_i, K_i, H_i\}} C = Y - \int_0^1 X_i di$$

subject to

$$Y = \zeta \cdot \left(\int_0^1 Y_i^\rho di \right)^{1/\rho}$$

$$Y_i = A_i \left(K_i^\alpha H_i^{1-\alpha} \right)^{1-\sigma} X_i^\sigma$$

$$\int_0^1 K_i di = K$$

$$\int_0^1 H_i di = H$$

where the productivity levels A_i are given exogenously.

We report the solution of the model under these two allocations in a series of propositions, not because the results are especially deep, but because this helps organize the algebra in a useful way, both for presentation and for readers who wish to solve the model themselves. (Outlines of the proofs are in the Appendix.)

4.1. Solving for the Symmetric Allocation

For expositional reasons and because it is easy to solve for, we begin with the symmetric allocation. In the symmetric allocation, $Y_i = A_i m$, where $m \equiv$

$(K^\alpha H^{1-\alpha})^{1-\sigma}(\bar{s}Y)^\sigma$ is constant across activities. Therefore final output just depends on the CES combination of the A_i with curvature parameter ρ , as stated in the following proposition:

PROPOSITION 4.1. (*The Symmetric Allocation.*) *Under the symmetric allocation of resources, total production of the final good is given by*

$$Y = Q_{sym}^{\frac{1}{1-\sigma}} K^\alpha H^{1-\alpha}, \quad (7)$$

where

$$Q_{sym} \equiv \left(\int_0^1 A_i^\rho di \right)^{\frac{1}{\rho}}. \quad (8)$$

The model delivers a simple expression for final output. Y is the familiar Cobb-Douglas combination of aggregate physical and human capital with constant returns to scale. Because Y includes intermediate goods, GDP in this economy corresponds to C . However, because $C = (1 - \sigma)Y$, everything we say below about Y corresponds to GDP as well.

Two novel results also emerge, and both are related to total factor productivity. The first illustrates the role of complementarity, while the second reveals the multiplier associated with linkages through intermediate goods.

Total factor productivity is a CES combination of the productivities of the individual activities. Activities enter production in a complementary fashion, and this complementarity shows up in the aggregate production function in TFP.

To interpret this result, it is helpful to consider the special case where $\rho \rightarrow -\infty$. In this case, the CES function becomes the minimum function, so that $Q_{sym} = \min\{A_i\}$. Aggregate TFP then depends on the smallest level of TFP across the activities of the economy. That is, aggregate TFP is determined by the weakest link. Firms in the United States and Kenya may not differ that much in average efficiency, but if the distribution of Kenyan firms has a substantially worse lower tail, overall economic performance will suffer because of complementarity.

The second property of this solution worth noting is the multiplier associated with intermediate goods. Total factor productivity is equal to the CES combination of underlying productivities raised to the power $\frac{1}{1-\sigma} > 1$. A simple example should make the reason for this transparent. Suppose $Y_t = aX_t^\sigma$ and $X_t = sY_{t-1}$; output depends in part on intermediate goods, and the intermediate goods are themselves produced using output from the previous period. Solving these two equations in steady state gives $Y = a^{\frac{1}{1-\sigma}} s^{\sigma/1-\sigma}$, which is a simplified version of what is going on in our model. Notice that if we call X “capital” instead of intermediate goods, the same formulas would apply and this looks like the neoclassical growth model with full depreciation. Intermediate goods are another source of accumulable inputs in a growth model.

The economic intuition for this multiplier is also straightforward. Low productivity in electric power generation reduces output in the banking and construction industries. But problems in these industries hinder the financing and construction of new dams and electric power plants, further reducing output in electric power generation. Linkages between sectors within the economy generate an additional multiplier through which productivity problems get amplified.

At some level, the paper could end here. The main points of the model both appear in the symmetric allocation: the role of complementarity and the multiplier associated with intermediate goods. The remainder of the paper develops these points further, adds a few insights, and considers some quantitative examples.

4.2. Solving for the Second-Best Allocation

The second-best allocation is more tedious to solve for, but it is a natural one to focus on in this environment. Resources can be misallocated in many ways, and there is nothing to recommend our symmetric allocation other than its simplicity. The second-best allocation shows the best that a country can do given its endowments of inputs and exogenous productivities. The solution of the model in this case is next.

PROPOSITION 4.2. (*The Second-Best Allocation.*) *When physical capital, human capital, and intermediate goods are allocated optimally across activities, taking the A_i as given, total production of the final good is given by*

$$Y = Q_{sb}^{\frac{1}{1-\sigma}} K^\alpha H^{1-\alpha}, \quad (9)$$

where

$$Q_{sb} \equiv \left(\int_0^1 A_i^{\frac{\rho}{1-\rho}} di \right)^{\frac{1-\rho}{\rho}}. \quad (10)$$

It is useful to compare this result with the previous proposition. The aggregate production function takes the same form, and the multiplier associated with intermediate goods appears once again.

The essential difference relative to the previous result is that the curvature parameter determining the productivity aggregate is now $\frac{\rho}{1-\rho}$ rather than the original ρ . Notice that if the domain of ρ is $[0, -\infty)$, the domain of $\frac{\rho}{1-\rho}$ is $[0, -1)$, which means there is less complementarity in determining Q_{sb} than there was in the original CES combination of activities. The reason is that the second-best allocation strengthens weak links by allocating more resources to activities with low productivity. If the transportation sector has especially low productivity, the second-best allocation will put extra physical and human capital in that sector to help offset its low productivity and prevent this sector from becoming a bottleneck. Interestingly, this shows up in the math by raising the effective elasticity of substitution used to aggregate the underlying productivities.

This result can be illustrated with an example. Suppose $\rho \rightarrow -\infty$. In this case, the symmetric allocation depends on the smallest of the A_i , the pure weak link story. In contrast, the second-best allocation depends on the harmonic mean of the productivities, since $\frac{\rho}{1-\rho} \rightarrow -1$. Disasterously low productivity in a single activity is fatal in the symmetric allocation. In the second-best allocation, resources can substitute for low productivity, and weak links get strengthened.

4.3. Misallocation and TFP

Viewed together, propositions 4.1 and 4.2 illustrate a very important result found elsewhere in the macroeconomics and growth literatures: the misallocation of resources at the microeconomic level often shows up as a reduction in TFP at the macroeconomic level. This result has been emphasized by Chari et al. (forthcoming), Hsieh and Klenow (2006), Lagos (2006), and Restuccia and Rogerson (2007).

The TFP index Q is a power mean of the underlying A_i . When resources are (mis)allocated symmetrically, the curvature parameter is more negative than when resources are allocated optimally given the A_i . Q is therefore reduced: misallocating resources lowers TFP. This feature of the model will play an important role in what follows. It is one way that a lower Q in poor countries can be explained in the context of the model. This lower Q will reduce output more than one-for-one because of the intermediate goods multiplier.

5. EVALUATING TFP

The expressions for Q_{sym} and Q_{sb} above are nice, but it is not immediately obvious how to use them to quantify TFP differences across countries. At the moment, we have a continuum of exogenous productivity levels, A_i . In this section, we parameterize this continuum parsimoniously for the purpose of quantifying the predictions of the model. In this spirit, we now assume the A_i are distributed independently according to a log normal distribution, with mean μ and variance ν^2 . We allow these parameters to differ across countries.

Let η represent the absolute value of the curvature parameter in determining the productivity aggregate Q_{sb} or Q_{sym} . For the second-best allocation of resources, $\eta = -\frac{\rho}{1-\rho}$, so that $\eta \in [0, 1)$ is a positive curvature parameter. For the symmetric allocation, $\eta = -\rho \in [0, \infty)$. The parameter η measures the strength of effective complementarity. It is determined both by the underlying amount of complementarity in production and by the allocation of resources in the economy.

Finally, let Q denote the productivity aggregate, either Q_{sb} or Q_{sym} . With this notation, we have

$$Q = \left(\int_0^1 A_i^{-\eta} di \right)^{-\frac{1}{\eta}}. \quad (11)$$

Using the log-normal distribution, one can evaluate the integrals to solve for TFP:

PROPOSITION 5.1. (*The Solution for Q*). *Suppose $\log A_i \sim N(\mu, \nu^2)$. Then the aggregate productivity term Q is given by*

$$Q = e^{\mu - \frac{1}{2}\eta\nu^2}. \quad (12)$$

The first part of this result is natural: whatever mean differences we assume in the underlying A_i get translated into differences in the aggregate TFP index. The second term is where complementarity and the allocation of resources combine in an interesting way. Holding the underlying variance of A_i constant, a higher effective degree of complementarity — either because of more actual complementarity or because of the misallocation of resources — reduces Q . This is the weak-link result where the thickness of the lower tail has a stronger effect with more effective complementarity. Holding the degree of complementarity constant, a higher variance also reduces Q : it thickens the lower tail of the distribution, providing more of an opportunity for weak links to reduce TFP.

6. DEVELOPMENT ACCOUNTING

To what extent can this model with intermediate goods and complementarity explain income differences across countries? In this section, we attempt to quantify the mechanisms at work in our theory of TFP. It is possible to combine this with a neoclassical theory of physical and human capital to fully endogenize income differences. However, given that the neoclassical theory of these factors is relatively familiar, we will proceed directly to the results.

Letting $y \equiv Y/L$ denote output per worker and $h \equiv H/L$ denote human capital per worker, the Cobb-Douglas expression for output in equation (9) can be rearranged as:

$$y = Q^{\frac{1}{1-\sigma}} \left(\frac{K}{Y}\right)^{\frac{\alpha}{1-\alpha}} h. \quad (13)$$

The last two terms in this expression capture the usual neoclassical effects of physical and human capital. In Klenow and Rodriguez-Clare (1997) and Hall and Jones (1999), these neoclassical terms were measured to contribute approximately a factor of 4 to income differences between countries at the 95th and 5th percentiles. We will simply take this factor of 4 as given, noting that it can be easily explained with simple models and data on schooling and investment rates.¹⁰

In addition, we also have the TFP differences that are the focus of this paper. Moreover, these TFP differences have a multiplier that involves both the intermediate goods share and the capital share. For the usual reasons, there is a $\frac{1}{1-\alpha}$ multiplier (exponent) associated with capital accumulation: anything that increases output leads to additional capital accumulation, which further increases output, etc.

6.1. Measuring the Intermediate Goods Share, σ

For reasons that have already been explained, the crucial parameter of the model for explaining large income differences across countries is the intermediate goods share, σ . Fortunately, there is detailed empirical evidence about the magnitude of this parameter.

Basu (1995) recommends a value of 0.5 based on the numbers from Jorgenson, Gollop and Fraumeni (1987) for the U.S. economy between 1947 and 1979. Ciccone (2002), citing the extensive analysis in Chenery, Robinson and Syrquin

¹⁰Earlier versions of this paper endogenized K and H . Physical capital is easily endogenized by letting countries rent capital in a world capital market subject to country-specific distortions. For human capital, we built a Mincerian model of schooling that allowed individuals to choose the number of years they attended school so as to maximize their expected lifetime income; see Jones (2007b). This approach can rationalize the factor of 4 that is assumed for the neoclassical effects.

(1986), observes that the intermediate goods share at least sometimes rises with the level of development. However, the numbers cited for South Korea, Taiwan, and Japan in the early 1970s are all substantially higher than the U.S. number, ranging from 61% to 80%.

Fortunately, there are very rich data sets on input-output tables for many countries. For example, the OECD Input-Output database now covers 37 countries (including 9 non-OECD countries) at the level of 48 industries for a year close to 2000; see Yamano and Ahmad (2006). Historical data are available for a number of these countries as well. Using the 1-digit level summary tables in Yamano and Ahmad (2006) one can calculate intermediate good shares of gross output for different countries. For the United States, Japan, and India, these shares are all about 46%. For China, the share is 64%. Across 21 countries (mostly OECD, but including Brazil, China, and India as well), the average intermediate goods share is 52.4%, with a standard deviation of about 6%.

Given all of this evidence, we will take $\sigma = 1/2$ as a benchmark value. Notice that this choice implies a substantial multiplier that works through intermediate goods: $\frac{1}{1-\sigma} = 2$.

6.2. Explaining the 95th/5th Factor of 32

Now consider using the model to explain a 32-fold income difference between rich and poor countries. Letting r and p denote the rich and poor, our model predicts:

$$\frac{y^r}{y^p} = \underbrace{\left(\frac{Q^r}{Q^p}\right)^{\frac{1}{1-\sigma} \frac{1}{1-\alpha}}}_{\text{TFP}} \underbrace{\left(\frac{K^r/Y^r}{K^p/Y^p}\right)^{\frac{\alpha}{1-\alpha}} \frac{h^r}{h^p}}_{\text{neoclassical factor of 4}}. \quad (14)$$

If we assume the neoclassical factors contribute a factor of 4 to the difference, then we need the TFP term to contribute a factor of 8. Assuming $\sigma = 1/2$ and $\alpha = 1/3$, the multiplier on the TFP index is $\frac{1}{1-\sigma} \frac{1}{1-\alpha} = 2 \times 3/2 = 3$. So we need the ratio of the TFP indexes Q^r/Q^p to equal 2 in order to explain the 32-fold difference in incomes.

Two key remarks are relevant at this point. First, the intermediate goods and capital multipliers are crucial here. Without them, a 2-fold difference in Q would lead to an extra 2-fold difference in incomes. With just the capital multiplier, a 2-fold difference in Q leads to only a $2^{3/2} = 2.8$ -fold difference. The intermediate goods multiplier turns the factor of 2.8 into a factor of 8. Another way to say this is that with no multipliers, a two-fold difference in Q together with the factor of 4 from the neoclassical factors would generate 8-fold income differences across countries. With the capital multiplier, this rises to 11. The intermediate goods multiplier turns this 11 into 32-fold income differences.

The second key remark is that there are many possible ways to explain a fundamental 2-fold difference in the Q indexes, and the model is not tied to a single possibility. These can be seen by focusing on the model's solution for Q^r/Q^p . With the log-normal distribution, we have

$$\log\left(\frac{Q^r}{Q^p}\right) = \mu_r - \mu_p - \frac{1}{2}(\eta_r \nu_r^2 - \eta_p \nu_p^2). \quad (15)$$

In fact, there are three natural ways to get this original factor of 2. First, perhaps $\mu_p < \mu_r$. That is, there could be technological differences between the richest and poorest countries of the world that lead to a static TFP difference (i.e. absent the multipliers) of a factor of 2. Distortions to technology adoption and transfer that are not included in the model could account for this.

Second, in the presence of complementarity, a thicker lower tail in the distribution of productivities in the poorest countries could reduce Q . In equation (15), this shows up in the variance terms: a higher variance of the A_i in the poor country would reduce TFP. The weak-link story, then, is a second possible channel for explaining a 2-fold difference in the Q 's.

Finally, as we saw earlier, the misallocation of resources in this model leads directly to a lower aggregate TFP index. Even if the distribution of A_i is exactly the same in rich and poor countries, the misallocation of K_i , H_i , and X_i leads to $Q_{sym} < Q_{sb}$. As in Chari et al. (forthcoming), Restuccia and Rogerson (2007), and Hsieh and Klenow (2006), the misallocation of resources at the micro level

reduces aggregate TFP. This kind of misallocation is a third way to explain an underlying factor of 2 difference in Q .

An important opportunity to judge the relevance of this model is available, building on Hsieh and Klenow (2006) to include intermediate goods. Focusing on a model with no intermediate goods, Hsieh and Klenow find that misallocations of capital and labor in China and India across firms within an industry reduce output by a factor of two in a static sense, and slightly more taking into account the induced capital accumulation. Adding intermediate goods could either raise or lower the static effect. For example, if the allocation of intermediate goods is distorted in the same ways as the allocations of capital and labor, one would expect the static effect to remain equal to a factor of two. (Focusing on gross output instead of value-added, the capital and labor distortions will be reduced in magnitude by 1/2 — the share of value-added in gross output. This could be inflated back to a factor of two by the misallocation of intermediate goods.) However, the crucial point from the standpoint of the present paper is that the dynamic effects could be much larger because of the additional multiplier associated with intermediate goods.

6.3. Numerical Examples

To illustrate these points quantitatively, we provide some numerical examples below. First, however, it is helpful to rewrite the TFP index in a slightly different way in order to make calibrating the parameters a little easier.

Notice that since A_i is distributed log-normally, the average value of A_i , denoted \bar{A} , is given by

$$\bar{A} \equiv E[A_i] = e^{\mu + \frac{1}{2}\nu^2}.$$

Substituting this into the solution for Q in equation (12), we obtain

$$Q = \bar{A} e^{-\frac{1}{2}(1+\eta)\nu^2}.$$

Notice that if $\eta = -1$, Q is just the mean of the A_i . Otherwise, the skewness of the distribution of the A_i changes the mean of Q . With this formulation, the

ratio of Q^r/Q^p is given by

$$\frac{Q^r}{Q^p} = \frac{\bar{A}^r}{\bar{A}^p} \cdot e^{-\frac{1}{2}[(1+\eta_r)\nu_r^2 - (1+\eta_p)\nu_p^2]}. \quad (16)$$

This expression is combined with equation (14) to quantify income differences across countries. We assume neoclassical factors contribute a factor of 4 to the income difference and then examine our theory of TFP for different parameter values.

Before discussing the results, it is worth pausing to lay out the baseline parameter values we consider. As discussed earlier, we take $\sigma = 1/2$. For illustrative purposes, we will also consider results when the intermediate goods multiplier is shut off, i.e $\sigma = 0$. We pick $\alpha = 1/3$ to match the empirical evidence on capital shares; see Gollin (2002), who shows that capital shares across countries have a mean of 1/3 and are uncorrelated with GDP per worker.

For the variance of $\log A_i$, we draw on Hsieh and Klenow (2006). Hsieh and Klenow measure the standard deviation of firm-level log TFP *within* 4-digit manufacturing sectors for China, India, and the United States. Roughly speaking, they find that the standard deviation is about 1.0 for the United States and China, and about 1.3 in India. These statistics do not correspond exactly to what we want for our model. We'd like to see the deviation *across* all firms and sectors in the economy. For example, the weak link story involves electricity, transportation, replacement parts, machine tools, etc. — inputs that are taken from different sectors of the economy. We'd also like some sense of differences in things like property rights and corruption. Still, these are useful observations to get us started. Based on these numbers, we consider a standard deviation of log productivity for the “rich” country of $\nu_r = 1.0$ and for the “poor” country of $\nu_p = 1.3$ (like India). Future work on productivity differences across sectors could potentially shed better light on these parameter values.

The last parameter that needs to be calibrated is the degree of complementarity within a firm's production function. Estimating production functions is notoriously difficult, and we know of no good estimates of this parameter. As discussed earlier in the paper, a number of authors have recognized the im-

TABLE 1.
Output per Worker Ratios: Numerical Examples

Scenario	— Differing Complementarity —			No Interme- diate Goods
	$\rho = -1$	$\rho = -1/2$	$\rho = -2$	$\sigma = 0$
Baseline: No TFP differences	4.0	4.0	4.0	4.0
1. Average TFP differences: $\bar{A}^r / \bar{A}^p = 2$	32.0	32.0	32.0	11.3
2. Higher variance in poor: $\nu_p = 1.3, \nu_r = 1$	18.9	15.9	22.5	8.7
3. Misallocation: Rich=2nd best, Poor=sym.	8.5	5.1	29.6	5.8
4. 2+3: Misallocation with $\nu_p > \nu_r$	67.1	24.3	659.4	16.4
5. 1+2+3: All three together	536.9	194.1	5275.0	46.3

Note: The table reports income ratios between rich and poor countries, assuming $\alpha = 1/3$ and $\sigma = 1/2$. The baseline case has no TFP differences, so only the neoclassical factor appears — assumed to be equal to 4.0. The parameter values in the baseline case are $\bar{A}^r / \bar{A}^p = 1$ and $\nu_r = \nu_p = 1$, and resource allocation is optimal (given the A_i). The last column shows the results when the intermediate goods share is zero (and where $\rho = -1$).

portance of complementarity in production. We will take our benchmark case as $\rho = -1$, corresponding to an elasticity of substitution equal to $1/2$ — half way between Leontief and Cobb-Douglas. For robustness checks, we consider $\rho = -1/2$ and $\rho = -2$, corresponding to elasticities of substitution of $2/3$ and $1/3$ respectively.

Table 1 shows the quantitative results for the model. The baseline row of the table shows the 4-fold differences in incomes that result when only the neoclassical terms — physical and human capital — are present. TFP is identical across countries in this case, by assumption.

The next three scenarios consider different mechanisms in the model for getting TFP differences, one at a time. Scenario 1 shows the results when average TFP differs between rich and poor countries by a factor of 2. This scenario is familiar from our earlier discussion. Absent any multiplier, this difference would produce an 8-fold difference in incomes across countries. As the last column of the table shows, the capital multiplier turns this factor of 8 into a factor of 11.3, as the higher productivity in the rich country induces extra capital accumulation.

Finally, the other columns of the table show the role played by the intermediate goods multiplier. This converts the factor of 11.3 into a factor of 32.

Scenario 2 examines the role of complementarity and weak links. In particular, this scenario assumes that average TFP is the same in both countries ($\bar{A}^r = \bar{A}^p$), but the poor country has a higher variance. As discussed above, the rich country has a standard deviation of 1.0 for $\log A_i$, while the poor country has a standard deviation of 1.3. Notice that this is a relatively small difference: the ratio of the A_i two standard deviations below the mean is a factor of $e^{2 \times (1.3 - 1.0)} = e^{0.6} = 1.8$. Assuming resources are allocated optimally in both countries, this scenario generates a 19-fold difference in incomes between the two countries.

Scenario 3 considers the misallocation of resources. Assuming both the rich and poor country have the same mean and standard deviation of productivity, what happens if the rich country allocates resources optimally (given the A_i), while the poor country (mis)allocates resources symmetrically? That is, the poor country fails to adequately strengthen weak links. This scenario results in only an 8.5-fold income difference.

The remaining two scenarios examine combinations. Scenario 4 combines the misallocation of resources of Scenario 3 with the larger standard deviation in the poor country of Scenario 2. The result is quite dramatic. Whereas the previous two scenarios had 19-fold and 8.5-fold income differences, this one leads to a 67-fold income difference. A higher variance is especially costly when resources are misallocated.

Finally, Scenario 5 adds to Scenario 4 a 2-fold difference in mean TFP. Because of the intermediate goods multiplier, this expands income differences by a factor of 8. So the 67-fold difference becomes an astounding 537-fold difference.

The middle two columns of Table 1 consider the robustness of the results to different degrees of complementarity. The second column raises the elasticity of substitution to $2/3$ ($\rho = -1/2$), while the third column allows for even more complementarity, with a substitution elasticity of $1/3$ ($\rho = -2$). Overall,

these results show that the model can generate very large income differences for seemingly-plausible parameter values.

6.4. Summary

Some of the parameters of the model — like the intermediate goods share and the capital share — are quite precisely pinned down by empirical evidence. Others — like the degree of complementarity, the standard deviation of productivity, and the extent of resource misallocation — are known with much less precision.

Fortunately, the high value of the intermediate goods share by itself goes a long way toward helping us understand large income differences across countries. In particular, it generates a large multiplier. The aggregate productivity index Q may differ by a factor of 2 for many reasons: distortions to technology adoption, complementarity and weak links, or the misallocation of resources. Whatever the cause, the intermediate goods multiplier leads relatively small and plausible differences in Q to magnify into large income differences across countries. The economics of this magnification is quite intuitive. Because of linkages across sectors, the misallocation of resources in one sector affects output in others, which in turn feeds back into the original sector.

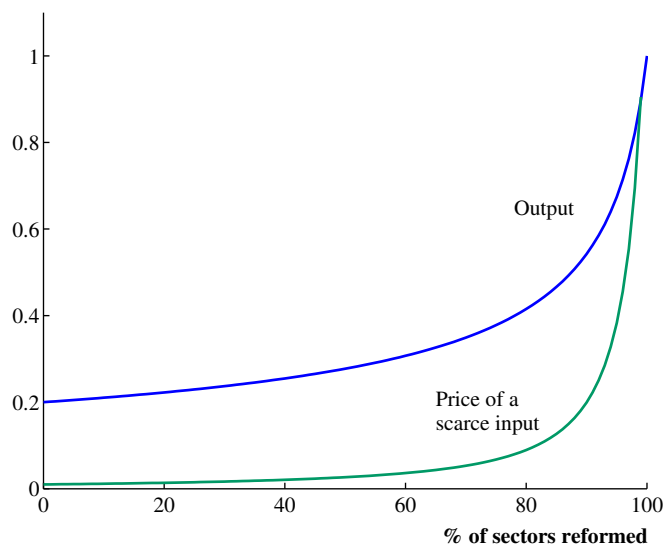
7. THE CUMULATIVE EFFECT OF REFORMS

The model possesses two key features that seem desirable in any theory designed to explain the large differences in incomes across countries. First, relatively small and plausible differences in underlying parameters can yield large differences in incomes. That is, the model generates a large multiplier.

Second, improvements in underlying productivity along any single dimension have relatively small effects on output. If a chain has a number of weak links, fixing one or two of them will not change the overall strength of the chain.

This principle is clearly true in the extreme Leontief case, but it holds more generally as well. To see this, consider a simple exercise. Suppose output is given by a symmetric CES combination of 100 inputs, for example as in equation (1).

FIGURE 1. The Cumulative Effect of Reforms



Note: Output is given by a symmetric CES combination of 100 inputs, with an elasticity of substitution equal to $1/3$. Initially, all inputs take the value 0.2. A sequence of reforms lead the inputs to increase to the value 1.0, one at a time.

Initially, all inputs take the value 0.2 and therefore output is equal to 0.2 as well. A sequence of “reforms” then leads the inputs to increase one at a time to their rich-country values of 1.0. After 100 reforms, all inputs take the value of 1.0 and output is equal to 1.0. Figure 1 shows the sequence of output levels that result from the reforms for the case $\rho = -2$, as well as the marginal product of an unreformed input.

Notice that output is relatively flat for much of the graph. The first doubling of output does not occur until nearly 80% of the sectors are reformed. In addition, the marginal product of the input in an unreformed sector remains low for a long time. When the economy suffers from many problems, reforms that address only a few may have small effects. With more complementarity, the paths would be even flatter.

Interestingly, the sharp curvature of these paths suggests that the pressure for reform can accelerate. This general setup may then help us to understand why some countries remain unreformed and poor for long periods while others — those that are close to the cusp — experience growth miracles.

Hausmann, Rodrik and Velasco (2005) advocate studying all of the distortions in an economy and quantifying the output gains from relaxing each distortion. This paper emphasizes the interactions across distortions. In particular, reforms in poor countries can “fail” because numerous other distortions keep output low. Politically, it seems important to recognize that valuable reforms can have small impacts until other complementary reforms are undertaken.

The development problem is hard because there are ten things that can go wrong in any production process. In the poorest countries of the world, productivity is low at many different stages, and complementarity means that reforms targeted at one or two problems have only modest effects.

7.1. Multinationals and Trade

Multinational firms and international trade may help to solve these problems if they are allowed to operate. For example, multinationals may bring with them knowledge of how to produce, access to transportation and foreign markets, and the appropriate capital equipment. Indeed many of the examples we know of where multinationals produce successfully in poor countries effectively give the multinational control on as many dimensions as possible: consider the maquiladoras of Mexico and the special economic zones in China and India. Countries may specialize in goods they can produce with high productivity and, to the extent possible, import the goods and services that suffer most from weak links.¹¹

And yet domestic weak links may still be a problem. A lack of contract enforcement may make intermediate inputs and other activities hard to obtain.

¹¹Nunn (2007) provides evidence along these lines, suggesting that countries that are able to enforce contracts successfully specialize in goods where contract enforcement is critical. See also Grossman and Maggi (2000).

Knowledge of which intermediate goods to buy and how to best use them in production may be missing. Weak property rights may lead to expropriation. Inadequate energy supplies and local transportation networks may reduce productivity. The right goods must be imported, and these goods must be distributed using local resources and nontradable inputs, as in Burstein, Neves and Rebelo (2003).

7.2. Should weak links get most of the resources?

The prediction of the model that appears most debatable is that in the second-best allocation, resources flow to activities according to the *inverse* of their productivity. For example, the activities with the lowest productivity will get the most resources. There are several reasons why I believe this is not a significant problem for the theory. First, it is not optimal in the first-best sense to allocate resources this way: it would be better to fix the distortions to mitigate the weak links directly.

Second, this result is partly just an artifact of the assumptions needed to solve the model in closed form; in particular, it is related to the fact that the final good combines in the complementary way while the activities are produced with a Cobb-Douglas production function. In reality, substitution and complementarity play more complicated roles. For example, in a given industry, products may be close substitutes (which is not in the model), and then you would want to devote resources to the most productive firms in a given industry. However, when a firm looks at the different activities in which it engages, it will pay a great deal of attention to the weakest links. Think about a household: we absolutely require air, water, and food. Air and water are “produced” with such a high productivity that they are cheap and we spend very little of our income on them. Everyone spends more on food than on air and water. Poor people spend a larger fraction of their income on food. But of course across types of food there is a lot of room for substitution.

8. CONCLUSION

In this theory of economic development, relatively small differences in total factor productivity at the firm or activity level translate into large differences in aggregate output per worker. There are two reasons for this. First, production at the firm level involves complementarity. For virtually any good, there is a list of activities that are essential for production. Replacement parts are an absolute requirement when machines break down. Business licenses, security, and many types of knowledge are necessary for success. Because these activities combine with an elasticity of substitution less than one, output does not depend on average productivity but rather hinges on the strength of the weakest links.

The second amplification force is both simpler and potentially more important. The presence of intermediate goods leads to a multiplier that depends on the share of intermediate goods in firm revenue. Low productivity in transportation reduces the output of many other sectors, including the truck manufacturing sector and the fuel sector. This in turn will reduce output in the transportation sector. This vicious cycle is the source of the multiplier associated with intermediate goods.

These amplification channels imply that the model makes a simple, testable prediction. In particular, if we look at total factor productivity at the micro level — using the gross output production function for a plant or firm — we should see something that at first appears puzzling: total factor productivity for firms in India or Kenya, should not be that different on average from total factor productivity in the richest countries in the world. Given the large income differences and large aggregate TFP differences, one might have expected to see large TFP differences at the firm level. Instead, firms in poor countries should look surprisingly efficient. Kenyan firms producing toasters may have 1/2 the TFP of U.S. firms producing toasters. Total output per worker of toasters in Kenya will be much lower, however, because Kenyan firms are also 1/2 as good at producing aluminum, electricity, transportation services, and property rights. These differences multiply up to explain low output per worker.

A casual reading of McKinsey studies of productivity suggests that this may be true. More directly, the analysis of large firm-level data sets in China, India, and the United States by Hsieh and Klenow (2006) suggests that this prediction can be tested in the near future.

Another important channel for future research concerns the role of intermediate goods. The present model simplifies considerably by taking the intermediate input to be units of the final output good. The input-output matrix in this model is very special. This is a good place to start. However, it is possible that the rich input-output structure in modern economies delivers a multiplier smaller than $\frac{1}{1-\sigma}$ because of “zeros” in the matrix. In work in progress, Jones (2007a) explores this issue. The preliminary results are encouraging. For example, if the share of intermediate goods in each sector is σ but the composition of this share varies arbitrarily, the aggregate multiplier is still $\frac{1}{1-\sigma}$. More generally, I plan to use actual input-output tables for both OECD and developing countries to compute the associated multipliers. I believe this will confirm the central role played by intermediate goods in amplifying distortions.

APPENDIX: PROOFS OF THE PROPOSITIONS

Proposition 4.1: The Symmetric Allocation

Proof. Follows directly from the fact that $Y_i = A_i m$, where $m = (K^\alpha H^{1-\alpha})^{1-\sigma} X^\sigma$

is constant across activities. ■

Proposition 4.2: The Second-Best Allocation

Proof. In deriving the aggregate production function, it is helpful to proceed in two steps. First, consider the optimal allocation of the intermediate goods, and then consider the optimal allocation of physical and human capital.

Define $a_i \equiv A_i (K_i^\alpha H_i^{1-\alpha})^{1-\sigma}$, so that $Y_i = a_i X_i^\sigma$. Then, the optimal allocation of X_i solves

$$\max_{\{X_i\}} C = \zeta \left(\int_0^1 a_i^\rho X_i^{\rho\sigma} di \right)^{1/\rho} - \int_0^1 X_i di.$$

Solving this problem and substituting the solution back into the production function in equation (2) gives

$$Y = \left(\int_0^1 a_i^\lambda di \right)^{\frac{1}{\lambda} \cdot \frac{1}{1-\sigma}}, \quad (\text{A.1})$$

where $\lambda \equiv \frac{\rho}{1-\rho\sigma}$. (This is where the judicious definition of ζ comes in handy.)

Using this expression, the optimal allocations of K_i and H_i solve

$$\max_{\{K_i, H_i\}} \int_0^1 a_i^\lambda di$$

subject to the resource constraints in equations (4) and (5), where $a_i \equiv A_i(K_i^\alpha H_i^{1-\alpha})^{1-\sigma}$.

The first-order conditions from this problem imply that

$$\frac{K_i}{K} = \frac{a_i^\lambda}{\int_0^1 a_i^\lambda di} = \frac{H_i}{H}.$$

These solutions for K_i and H_i can be plugged into the definition of a_i and integrated up to yield the aggregate production function. ■

Proposition 5.1: The Solution for Q

Proof. The productivity index Q is given by

$$\begin{aligned} Q &= \left(\int_0^1 A_i^{-\eta} di \right)^{-\frac{1}{\eta}} \\ &= \left(\int_0^1 e^{-\eta a_i} di \right)^{-\frac{1}{\eta}} \\ &= \left(\int_{-\infty}^{\infty} e^{-\eta a} f(a) da \right)^{-\frac{1}{\eta}} \\ &= (E[e^{-\eta a}])^{-\frac{1}{\eta}} \end{aligned} \quad (\text{A.2})$$

where $a_i \equiv \log A_i$ and $f(a)$ is the pdf of the normal distribution. This last line can be evaluated by using the moment generating function for a normal random variable. In particular,

$$E[e^{-\eta a}] = e^{-\eta\mu + \frac{1}{2}\eta^2\nu^2}.$$

Raising this expression to the $-1/\eta$ power proves the result. ■

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