Consider an economy in Romer world. Suppose that the production function for the consumption goods is $Y_t = A_t L_{yt}$. For each of period, ΔA_t amount of new ideas is invented, where $\Delta A_t = \bar{v} A_t L_{At}$. Assume that the population of the economy is $\bar{N}$. All people in this economy either produce consumption goods or invent new ideas, and the ratio is $1-\bar{\ell}$ to $\bar{\ell}$.

(a) List all unknowns, equations, and exogenous variables for this economy.

Model:

4 unknowns: Yt, At, Lyt, Lat

4 equations: $Y_t = A_t L_{yt}$ Production function of output

$\Delta A_t = \bar{v} A_t L_{At}$ Production of new ideas

$L_{yt} + L_{At} = \bar{N}$ Resource constraints

$L_{At} = \bar{\ell}\bar{N}$ Allocation of resources

Exogenous variables: $\bar{v}$, $\bar{N}$, $\bar{A}_0$, $\bar{\ell}$.

(b) Solve for this model. (Please also include per capita output.)

Firstly, $L_{At}^* = \bar{\ell}\bar{N}$, $L_{yt}^* = (1-\bar{\ell})\bar{N}$.

And then, $y_t^* = Y_t^* / \bar{N} = A_t^* L_{yt}^* / \bar{N} = A_t^*(1-\bar{\ell})\bar{N} / \bar{N} = A_t^*(1-\bar{\ell})$.

In order to get A_t^*, we need to find the growth rate of At as follows:

$$\bar{g}_A \equiv \Delta A_t / A_t = \bar{v} A_t L_{At} / A_t = \bar{v} L_{At} = \bar{v}\bar{\ell}\bar{N}.$$

So At can be written as: $A_t^* = \bar{A}_0 (1+\bar{g}_A)^t = \bar{A}_0 (1+\bar{v}\bar{\ell}\bar{N})^t$

Thus $y_t^* = \bar{A}_o (1+\bar{v}\bar{\ell}\bar{N})^t (1-\bar{\ell})$ and $Y_t^* = \bar{A}_o (1+\bar{v}\bar{\ell}\bar{N})^t (1-\bar{\ell})\bar{N}$.

So the solutions to this model are:

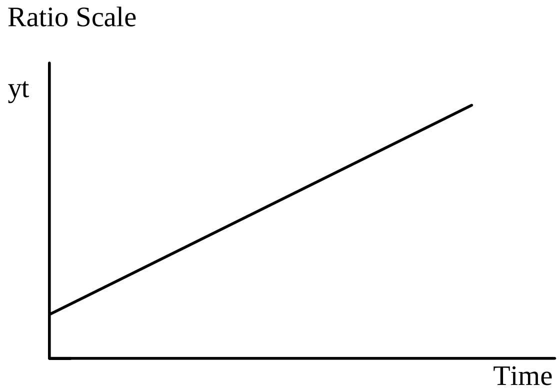
$L_{At}^* = \bar{\ell}\bar{N}$, $L_{yt}^* = (1-\bar{\ell})\bar{N}$, $A_t^* = \bar{A}_0 (1+\bar{v}\bar{\ell}\bar{N})^t$

$Y_t^* = \bar{A}_o (1+\bar{v}\bar{\ell}\bar{N})^t (1-\bar{\ell})\bar{N}$ and $y_t^* = \bar{A}_o (1+\bar{v}\bar{\ell}\bar{N})^t (1-\bar{\ell})$

(c) Will this economy grow forever? Why or why not?

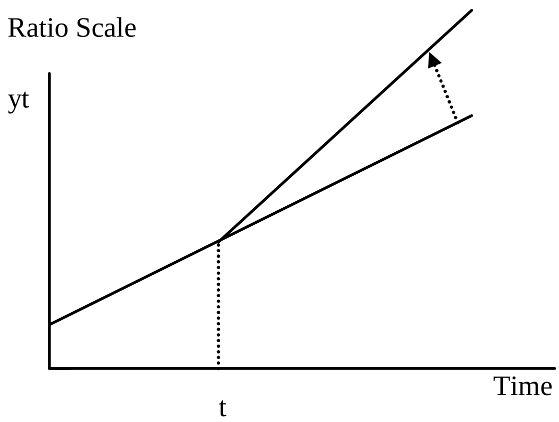
Yes, this economy will grow at the same rate $\bar{g}_A = \bar{v}\bar{\ell}\bar{N}$ forever. This is because the production function of output we assume in this model is increasing return to scale.

(d) Draw a time path diagram for per capita output.



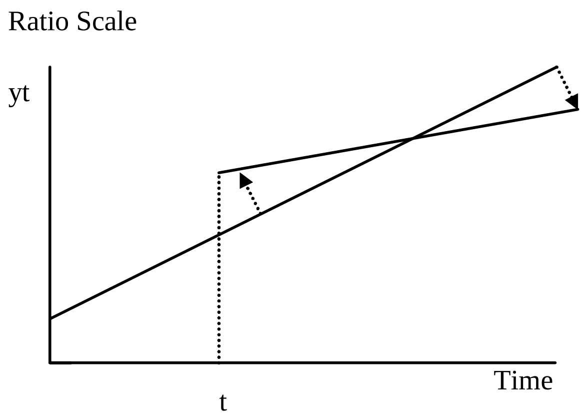
Since the economy grows at a fixed rate, the path of per capita output on the ratio scale time path diagram is a positive sloped straight line.

(e) Suppose at time t, the productivity of inventing new ideas increases from $\bar{v}$ to $\bar{v}'$. What effects will this event bring to this economy? (Explain your answer with a time path diagram.)



Since the productivity of inventing new ideas increases from $\bar{v}$ to $\bar{v}'$, the growth rate of the economy will increase after time t. Therefore the path of yt will rotate upwards after t as shown above.

(f) Suppose at time t, the ratio of population in inventing ideas to producing consumption goods drops from $\bar{\ell}$ to $\bar{\ell}'$. What effects will this event bring to this economy? (Explain your answer with a time path diagram.)



Since the ratio of population in inventing ideas to producing consumption goods drops from $\bar{\ell}$ to $\bar{\ell}'$, there are two effects due to this event. Firstly, the yt level at time t will jump up because of the increase of $\bar{\ell}$. Secondly, the growth rate of the economy will decrease after time t, thus lower the slope of yt time path line.

To sum up, in the diagram we will see a jump up of yt at time t, and then after time t the growth rate of yt will be lower than the periods before time t.