Economics 100b, Fall 2005
Sample Midterm Questions
Chapter 6

1. Define (1) nonrivalry and (2) increasing returns to scale in production. Explain how the nonrivalry of ideas relates to increasing returns to scale in production.

Answer: Something is called nonrivalrous if the use of it by one person does not reduce the ability of another person to use it. Increasing returns to scale in production refers to the property of the production function that, when inputs are increased by some factor, output increases by more than that factor. To see the relationship between the nonrivalry of ideas and increasing returns to scale, notice that in the standard constant returns to scale production function, $Y = F(K, L) = AK^{1/3}L^{2/3}$, output can be doubled by doubling the physical inputs (K and L) while keeping the stock of ideas (A) constant. When physical inputs and ideas are doubled, we see that output will more than double. Intuitively, doubling only the number of workers and machines will lead to double the output; doubling the number of workers and machines and the knowledge of how to use workers and machines will more than double output.

2. Define a balanced growth path. How does a balanced growth path differ from a steady state?

Answer: A balanced growth path is a period when all of the endogenous variables growth at a constant rate. A steady state refers to a period when the growth rate of an endogenous variable is zero. The Romer model exhibits a balanced growth path while the Solow model exhibits a steady state.

3. Explain the difference between a "level effect" and a "growth effect."

Answer: A level effect refers to the impact that a parameter or exogenous variable has on the level of per capita income. A growth effect refers to the impact that a parameter or exogenous variable has on the growth rate of per capita income. In particular, a change in some parameters or exogenous variables may lead to an increase in the level of per capita income without leading to an increase in the growth rate of per capita income.

4. Idea Driven Growth with 2 Countries.

Consider a world with two countries, the USA and Ethiopia. Both countries have researchers that produce ideas. Since ideas are nonrivalrous, all of the ideas produced can be freely used by either country.

The economies of both countries are described by the following equations

	USA	Ethiopia
Output Production Functions	$Y_t^{USA} = A_t^{World} L_{yt}^{USA}$	$Y_t^{Eth} = A_t^{World} L_{yt}^{Eth}$
Idea Production Function	$\Delta A_t^{World} = \bar{v} A_t^{World}(L_{at}^{USA} + L_{at}^{Eth})$, $\bar{A}_0^{World} > 0$	
Country Population Constraint	$L_{yt}^{USA} + L_{at}^{USA} = \bar{N}^{USA}$	$L_{yt}^{Eth} + L_{at}^{Eth} = \bar{N}^{Eth}$
Research Population	$L_{at}^{USA} = \bar{l}^{USA} \bar{N}^{USA}$	$L_{at}^{Eth} = \bar{l}^{Eth} \bar{N}^{Eth}$

(a) What is the growth of per capita output for each country?

Answer: Using the output production function and the research population equation, we can write output per capita for the USA as follows

$$y_t^{USA} = \frac{Y_t^{USA}}{\bar{N}^{USA}} = A_t^{World} \frac{L_{yt}^{USA}}{\bar{N}^{USA}} = A_t^{World} \frac{(1 - \bar{l}^{USA})\bar{N}^{USA}}{\bar{N}^{USA}} = A_t^{World}(1 - \bar{l}^{USA}).$$

Similarly, we can get the equation for output per capita in Ethiopia as $y_t^{Eth} = A_t^{World}(1 - \bar{l}^{Eth})$. Since $\bar{l}^{USA}$ and $\bar{l}^{Eth}$ are constants, these equations tell us that the growth rate of output per capita in each country will be equal to the growth rate of ideas in the world. Therefore, each country will have the same rate of growth which we can denote by g_A^{World}. To solve for g_A^{World}, we can use the idea production function and the research population equations as follows

$$g_A^{World} = \frac{\Delta A_t^{World}}{A_t^{World}} = \bar{v}(L_{at}^{USA} + L_{at}^{Eth}) = \bar{v}(\bar{l}^{USA}\bar{N}^{USA} + \bar{l}^{Eth}\bar{N}^{Eth}).$$

(b) Suppose that you are given the following parameter values

Parameter	Value
$\bar{A}_0^{World}$	2
$\bar{l}^{USA}$	.5
$\bar{l}^{Eth}$	.25
$\bar{N}^{USA}$	400
$\bar{N}^{Eth}$	200
$\bar{v}$	$\frac{1}{10000}$

Write a formula for computing per capita output for time t for each country. On a single graph, sketch a plot of per capita output against time for both countries. Put per capita output on the vertical axis, and use a ratio scale for per capita output.

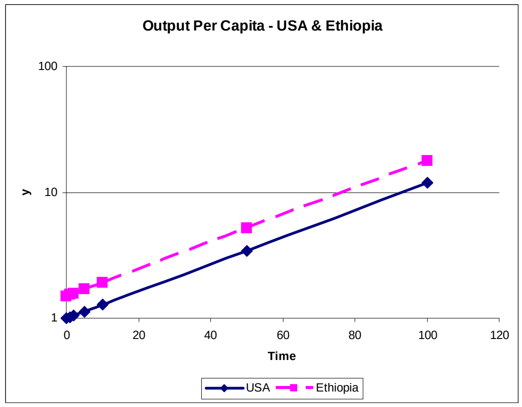
Answer: From part (a), we have $y_t^{USA} = A_t^{World}(1 - \bar{l}^{USA})$, and we know that $A_t^{World} = \bar{A}_o^{World}(1+g_A^{World})^t$, so after substituting, we get $y_t^{USA} = (1-.5)(2)(1+g_A^{World})^t = (1+g_A^{World})^t$. Similarly for Ethiopia, $y_t^{Eth} = (1-\bar{l}^{Eth})\bar{A}_o^{World}(1+g_A^{World})^t = (1.5)(1+g_A^{World})^t$. Additionally, we can compute g_A^{World} as follows

$$g_A^{World} = \bar{v}(\bar{l}^{USA}\bar{N}^{USA} + \bar{l}^{Eth}\bar{N}^{Eth}) = \frac{1}{10000}[(.5)(400) + (.25)(200)] = \frac{250}{10000} = .025.$$

Therefore, we have

$$\begin{aligned} y_t^{USA} &= (1.025)^t \\ y_t^{Eth} &= (1.5)(1.025)^t. \end{aligned}$$

Since g_A^{World} is constant, the plots of y_t^{USA} and y_t^{Eth} using a ratio scale, we will be linear. Notice that Ethiopia will have higher levels of output per capita. Intuitively, Ethiopia is able to have higher levels of output per capita since they can devote their workers to producing goods and still use the ideas produced by the USA. The following figure plots per capita output for both countries over time using a ratio scale.



(c) Suppose now that Ethiopia devoted all of it workers to producing output so that $\bar{l}^{Eth} = 0$. Write a formula for computing per capita output in Ethiopia for time t. On a new graph, sketch a plot of per capita output against time for just Ethiopia when $\bar{l}^{Eth} = .25$ and when $\bar{l}^{Eth} = 0$. How do the results compare when $\bar{l}^{Eth} = .25$ versus $\bar{l}^{Eth} = 0$? Does Ethiopia still have a positive growth rate even though no one in Ethiopia is producing ideas? Explain.

Answer: When $\bar{l}^{Eth} = 0$, we have $y_t^{Eth} = (1 - \bar{l}^{Eth})\bar{A}_o^{World}(1 + g_A^{World})^t = 2(1 + g_A^{World})^t$ and $g_A^{World} = \bar{v}(\bar{l}^{USA}\bar{N}^{USA} + \bar{l}^{Eth}\bar{N}^{Eth}) = \frac{1}{10000}[(.5)(400)] = \frac{200}{10000} = .02$. Therefore, when $\bar{l}^{Eth} = 0$, we have $y_t^{Eth} = 2(1.02)^t$. We see that Ethiopia still has positive growth even though no Ethiopians produce ideas because Ethiopia can take advantage of the ideas produced by the USA. Further, notice that $y_o^{Eth} = 2$ when $\bar{l}^{Eth} = 0$ which is higher than y_o^{Eth} when $\bar{l}^{Eth} = .25$ since more workers are producing output. Nevertheless, the growth rate for both countries is lower when $\bar{l}^{Eth} = 0$ since less ideas are being produced in the world. Even though Ethiopia starts out better when $\bar{l}^{Eth} = 0$, levels of income in the future will be lower than when $\bar{l}^{Eth} = .25$ because of the lower growth rate. The following figure plots output per capita for Ethiopia using a ratio scale when $\bar{l}^{Eth} = .25$ and when $\bar{l}^{Eth} = 0$.