Department of Economics
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Econ 100B

Solutions to Review Questions - Chapter 4

1. Suppose we have an economy with a single, profit-maximizing firm, as in the production model. Assume that the production function is given by $Y = F(K, L) = AK^{\alpha}L^{1-\alpha}$, where A is a constant and α is between 0 and 1.

(a) The firm is a profit maximizer. It therefore solves the following problem: $\max_{K, L} = F(K, L) - rK - wL$, or more precisely, $\max_{K, L} = AK^{\alpha}L^{1-\alpha} - rK - wL$.

(b) Because the production function exhibits constant returns to scale, the firm's demand for capital is given by $r = \alpha Y/K$, or $r = \alpha A(L/K)^{1-\alpha}$. Similarly, the firm's demand for labor is $w = (1 - \alpha)Y/K$, or $w = (1 - \alpha)A(K/L)^{\alpha}$.

(c) If the capital stock in the economy is 1000 machines, and there are 100 workers in the economy, then $r^* = \alpha A(1/10)^{1-\alpha}$ and $w^* = (1 - \alpha)A(10)^{\alpha}$.

(d) Y^* is given by $F(K^* = 1000, L^* = 100) = 100A(10)^{\alpha}$. Equilibrium GDP per worker is simply this amount divided by 100, or $A(10)^{\alpha}$.

(e) The share of capital income, r^*K^*/Y^*, is equal to $(\alpha A(1/10)^{1-\alpha} * 1000)/ 100A(10)^{\alpha}$, which simplifies to α. The share of labor income, w^*L^*/Y^*, is given by $((1 - \alpha)A(10)^{\alpha} * 100)/ 100A(10)^{\alpha}$, which simplifies to $1 - \alpha$. Clearly $\alpha + (1 - \alpha) = 1$.

(f) Because the production function has constant returns to scale, the share of capital income is always going to be a fixed fraction of the total income in the economy. The same can be said for labor's share of total income.