

Chapter 4 Practice Questions:

1) Suppose we know that output in the economy is given by the production function: $Y_t = A_t K_t^{1/3} L_t^{2/3}$

If technology is growing at a rate of 1% per year, the capital stock by 3%, and the labor supply by 2%, what will total growth in the economy be?

$$\begin{aligned} g(y) &= g(a) + (1/3)g(k) + (2/3)g(l) \\ &= 1 + (1/3)3 + (2/3)2 = 3.3\% \end{aligned}$$

2) What is a Cobb-Douglas Production Function?

A Cobb-Douglas Production function is of the form $Y_t = K_t^\alpha L_t^{1-\alpha}$

With the key idea being that the exponents sum to 1 so that there is CRS. ($\alpha + (1 - \alpha) = 1$)

3) Mathematically prove CRS of a Cobb-Douglas Production Function. What is one way in which you could you describe CRS non-mathematically?

So what if we double K,L?

$$\begin{aligned} F(2K, 2L) &= 2K^\alpha 2L^{1-\alpha} \\ &= 2 * K^\alpha L^{1-\alpha} \\ &= 2 * F(K, L) = 2Y \end{aligned}$$

We just proved mathematically that $Y = K^\alpha L^{1-\alpha}$ exhibits CRS. This is true for any α between 0 and 1 it turns out.

Non-Mathematically one could appeal to the *standard replication argument* – which simply states that if you were to double all the inputs (i.e. buy a second factory and hire a second set of workers) then you would double the output.

4) Write out the production model and solve it graphically to get equilibrium values.

See the text.

5) Use the solution to the production model and the fact that we assume perfect competition to calculate the payments to labor and capital. What in how we wrote down the production model determined these payments? What does this imply for the appropriateness of this equation given what we know to be true about the US economy payments to labor and capital in the data?

Payments to capital and labor are equal to the Cobb-Douglas exponents (see explanation pg85 in text). This implies that the production function we wrote down may be using appropriate exponents on capital and labor because we know from the first lecture that in the United States, historically payments to capital have been about 1/3rd of GDP and to labor have been 2/3rd and these shares have been relatively constant.

6) Suppose output is given by the production function: $Y = F(K, L) = \bar{A}K^{1/3}L^{2/3}$
What happens when you double L, leaving K fixed?

$Y = F(L) = \bar{A}K^{1/3}L^{2/3}$ should read $L(\text{bar})$

$$\begin{aligned} F(2L) &= \bar{A}K^{1/3}2^{2/3}L^{2/3} \\ F(2L) &= 2^{2/3}\bar{A}K^{1/3}L^{2/3} \end{aligned}$$

$$= 1.58Y$$

What if you were to quadruple L? Comment on the result.

$Y = F(L) = \bar{A}K^{1/3}L^{2/3}$ should read $K(\bar{K})$

$$\begin{aligned} F(4L) &= \bar{A}K^{1/3}4^{2/3}L^{2/3} \\ F(4L) &= 4^{2/3}\bar{A}K^{1/3}L^{2/3} \\ &= 2.519Y \end{aligned}$$

So while you still get more output, you no longer double the output, you get less than a proportional increase. The diminishing returns apply to Capital as well if we hold labor constant. Another point to make is that the proportionality of increase in output falls with L.

7) The production model suggests that differences in capital have some explanatory power in terms of predicting output per capita across countries. How is this different from the Solow Model? Where do differences in measured levels of K across countries come from? What does the Solow model suggest might be part of the answer?

The production model is different from the Solow model in that capital is assumed to be exogenous. When we endogenous capital in the Solow model we learn that one reason some countries have higher levels of K than others is that they have higher levels of TFP (A). Thus, the production model attributes more explanatory power to K in determining Y than the Solow model does because the Solow model recognizes that some of the variation in K across countries is actually coming from the countries differences in efficient in employing that capital.