A Policy Lesson from an Overlapping Generations Model of Habit Persistence

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Abstract

This paper analyzes the implication of habit persistence on the effects of tax policy. It shows that within the framework of an OLG economy with production, habit persistence generally increases savings (and steady state capital intensity), a result previously shown only for OLG exchange economies. Using an OLG economy with productive capital that considers habit persistence, an intensely-discussed policy issue is addressed: Does the introduction of a consumption tax foster capital accumulation? The answer the model gives is simple and striking. Contrary to the conclusion often drawn in the analysis of this issue, the model shows that because of habit persistence effects consumption taxation can result in a lessening of capital accumulation.

Keywords: *habit persistence, overlapping generations, tax policy, consumption tax*

JEL Classification: D11, D91, H31

1 Introduction

This paper addresses two questions: What is the impact of habit persistence on capital accumulation? Can conventional wisdom regarding tax policy be

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misleading if habit persistence is a factor? Both questions are investigated by means of an overlapping generations (OLG) model. By employing the answer to the first question, it is shown that the presence of habit persistence can reverse the effects of a tax program.

Habit persistence (formation) was first discussed in the context of addictive behavior relating to cigarettes, alcohol and other drugs. Before long the discussion of habit formation was extended to consumption in general. For a discussion see *Houthakker* and *Taylor* [6], *Spinnewyn* [11] or *Becker* and *Murphy* [3]. *Arrow* [2, p. 26] supposes that any consumption bundle (any lifestyle) forms habits. This view is also emphasized by psychological findings (see *Scitovsky* [10] for a treatise on the psychology of human satisfaction). Furthermore, habit formation has been used extensively in empirical work concerning the savings and growth debate (see *Alessie* and *Lusardi* [1], *Caroll* and *Weil* [4]), as well as asset pricing (see *Kocherlakota* [8]). Given the significance of habit formation, it is most important to investigate whether policy conclusions are robust with regard to habit formation.

The contribution of this paper to existing literature is twofold. First, it generalizes the analysis of *Lahiri* [9], who showed the impact of habit formation for a general OLG exchange economy. The present paper analyzes the impact of habit formation on capital accumulation within the framework of a general OLG economy with production. Second, the paper makes use of this framework to demonstrate that in the presence of habit formation, tax policy may have unexpected effects on savings and capital accumulation.

The main result of the paper is that habit formation generally promotes capital accumulation. The economic intuition behind this result is straightforward. Since habit formation lowers the marginal rate of intertemporal substitution, it is rational to reduce consumption early in life and increase savings (for consumption when old). Secondly, it is shown that the introduction of a consumption tax scheme can increase or reduce savings. However, when the tax rate is high enough (i.e., exceeds a "critical" tax rate, which can be as low as zero per cent) the tax will unambiguously result in higher savings and capital accumulation. When habit formation is considered, this critical tax rate shifts up. The conclusion of the model is that the introduction of this type of tax is likely to in fact lower savings, while a model without habit formation predicts an increase in savings under the same tax program.

Section 2 of the paper presents the OLG framework and investigates the impact of habit formation on both capital accumulation and steady state capital intensity of the OLG economy. Section 3 addresses a simple type of tax policy. A consumption tax is introduced and its impact on capital accumulation analyzed. The analysis exhibits that habit formation qualitatively alters the impact of the tax policy. Section 4 concludes the paper.

2 An Overlapping Generations Economy with Habit Formation

We consider a fully competitive economy with production that has two overlapping generations, one young and one old. Each generation is alive for two periods and has perfect foresight. Economic activity is performed over infinite discrete time. Each young generation is endowed with L_t units of labor, which are inelastically supplied to the labor market in the first period of life. Over time, the endowment of labor is evolving via (1). Parameter n denotes the exogenous rate of population growth.

$$L_{t+1} = (1+n)L_t \ , \quad n \geq -1 \tag{1}$$

The young generation receives a wage rate w_t per unit of labor. This is allocated to consumption c_t^1 and savings s_t . Superscript 1 denotes consumption in the first period of life, i.e., consumption of the young generation. Savings are equal to the purchased (depreciated) capital stock of the old generation plus investment.

$$c_t^1 + s_t = w_t \tag{2}$$

Once the generation becomes old and enters period 2 the only economic activity is consumption. Both savings and interest on savings are fully consumed.

$$c_{t+1}^2 = (1 + r_{t+1})s_t \tag{3}$$

Superscript 2 refers to the older generation and r denotes the real interest rate. Each young generation maximizes utility subject to its intertemporal budget constraint.

$$c_t^1 + \frac{c_{t+1}^2}{1 + r_{t+1}} = w_t \tag{4}$$

Utility is derived from consumption in both periods. However, in the presence of habit formation, utility of a given level of consumption when old is not independent of consumption when young. Specifically, the absolute level of consumption in the second period as well as the increase of second period relative to first period consumption are important. The more that was consumed when young, the more is required to derive the same level of utility in the following period. This phenomenon is when we refer to habit formation or habit persistence. In psychological terms this refers to the notion that continuous comfort leads to boredom, and stimulation is needed to relieve this boredom. The intertemporal utility function of a generation born at time t becomes:

$$U_t = u\left(c_t^1\right) + \beta\, u\left(\hat{c}_{t+1}^2\right) \quad \text{where} \quad \hat{c}_{t+1}^2 = c_{t+1}^2 - \delta\, c_t^1. \tag{5}$$

The parameter β is the subjective discount factor and δ denotes the strength of habit formation. This formulation of habit formation is quite standard in the literature.

ASSUMPTION 1 *(i) The felicity function $u : \mathbb{R}_+ \to \mathbb{R}$ is defined over non-negative consumption during the first and second periods of life. It is twice continually differentiable, increasing in both arguments and strictly quasiconcave. In particular, it satisfies $u'(\cdot) > 0,\ u''(\cdot) < 0$, as $\arg u \to 0 : u'(\cdot) \to \infty$, as $\arg u \to \infty : u'(\cdot) \to 0$. (ii) Furthermore it is characterized by a constant coefficient of risk aversion: $\sigma(c^1) = -c^1\, u''(c^1)/u'(c^1) = \sigma(c^2) = -c^2\, u''(c^2)/u'(c^2) = \sigma > 0$.*

The marginal rate of intertemporal substitution becomes:

$$\frac{d\, c_{t+1}^2}{d\, c_t^1} = -\frac{u'\left(c_t^1\right) - \delta\, \beta\, u'\left(\hat{c}_{t+1}^2\right)}{\beta\, u'\left(\hat{c}_{t+1}^2\right)} \tag{6}$$

As (6) shows, habit formation lowers the marginal rate of intertemporal substitution in absolute value. The intuition behind this is the following. The reduction of one unit of consumption in period t is rewarded not only by an interest in period $t+1$ but also by a “rise-of-consumption effect”, which contributes to felicity. Thus, for any given level of consumption in the first period, habit formation is stronger as less compensation is required to be equally well off in period 2. Figure 1 shows a picture of this impact on intertemporal indifference curves.

Figure 1 shows that intertemporal indifference curves become flatter in the presence of habit formation. Moreover, it shows that the domain of δ is limited. At points X and Y, the indifference curves with $\delta > 0$ are horizontal. At both points, $\delta\, \beta\, u'(\hat{c}_{t+1}^2) = u'(c_t^1)$. In order for the indifference curves to be downward sloping, the parameters of the model must satisfy (7).

$$0 \leq \delta \leq \frac{u'\left(c_t^1\right)}{\beta\, u'\left(\hat{c}_{t+1}^2\right)} \tag{7}$$

Output in period t is allocated to consumption and investment. Since we assume that one generation lives for two periods - implying that each period is about thirty years - we set the rate of depreciation of physical capital equal to one.[3] Capital intensity therefore evolves according to (9).

$$(1+n)k_{t+1} = y_t - c_t^1 - \frac{c_t^2}{1+n} \tag{9}$$

Since all agents are assumed to be price takers, the factors of production are paid their respective marginal product. The wage rate is denoted w.

$$w_t = f(k_t) - k_t f'(k_t) \tag{10}$$

$$1 + r_t = f'(k_t) \tag{11}$$

Considering the market clearing condition (9) along with (10) and (11), it follows that savings in period t constitute capital in period $t+1$.

$$k_{t+1}(1+n) = s_t(w_t,\, r_{t+1}) \tag{12}$$

As young generations maximize utility (5) subject to the intertemporal budget constraint (4) they choose savings such that:

$$s_t^*(w_t,\, r_{t+1}) = \arg\max_s u(w_t - s_t) + \beta\, u\left((1+r_{t+1})s_t - \delta(w_t - s_t)\right) \tag{13}$$

DEFINITION 1 *An intertemporal equilibrium in this OLG economy is an exogenously given endowment k_0 and a sequence $\{k_t\}_{t=0}^{\infty}$ given by the equation of motion (12) evaluated at (13) in each period t. A steady state equilibrium*[4] *is a stationary capital intensity k, such that*

$$k_t = k = \frac{s^*\left(f(k) - k\,f'(k),\, f'(k) - 1\right)}{1+n}. \tag{14}$$

The analysis that follows is mainly concerned with steady states. Therefore time indices are generally omitted.[5] Throughout this paper it will be

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$\lim_{k\to 0} \alpha(\alpha-1)k^{\alpha-2} = -\infty$. $\lim_{k\to 0} k\, f''(k) = \lim_{k\to 0} \alpha(\alpha-1)k^{\alpha-1} = -\infty$. $k\, f''(k) + f'(k) = \alpha^2 k^{\alpha-1} > 0$. Furthermore, $f'''(k) + f''(k)/k = k^{\alpha-3}\alpha(\alpha-1)(\alpha-1) > 0\,,\ k > 0$.

[3] Both assumptions, that the rate of depreciation is equal to one and that there is no technological progress in no way change the results of the analysis.

[4] Existence, stability and uniqueness of equilibrium are not analyzed in this paper. The sufficient conditions for stable equilibrium as stated in *Galor* and *Ryder* [5] are assumed to hold. None of the assumptions (a) to (e) of *Galor* and *Ryder* [5, p. 372] are violated by the present analysis.

[5] Note that all variables are already transformed such that they do not change in a steady state.

assumed that the steady state is not dynamically inefficient. That is, the paper only looks at economies, for which: $f'(k) > 1 + n$.

The final step in this section consists of analyzing the impact of habit formation on the behavior of the agents in the OLG economy. *Lahiri* [9] analyzed the impact of habit formation on savings in a similar OLG exchange economy. However, in this section the impact on savings and capital intensity will be analyzed within the framework of an OLG economy with productive capital.

Considering (13) the first order condition (FOC) for the optimization problem becomes:

$$-u'\left(c_t^1\right) + \beta\left[f'(k_t) + \delta\right] u'\left(\hat{c}_{t+1}^2\right) = 0. \tag{15}$$

LEMMA 1 *In the short run, for given k_0, habit formation increases savings of the young generation: $d\,s(k_0)/d\,\delta > 0$.*

Proof. Differentiating the FOC with respect to δ and considering that $k = k_0$, gives rise to the following differential:

$$\frac{d\,s(k_0)}{d\,\delta} = \frac{-\beta\,u'\left(\hat{c}_1^2\right) - \left[f'(k_0) + \delta\right]\beta\,u''\left(\hat{c}_1^2\right)\left(-w_0 + s_0\right)}{u''\left(c_0^1\right) + \beta\,u''\left(\hat{c}_1^2\right)\left[f'(k_0) + \delta\right]^2}. \tag{16}$$

Since $(-w_t + s_t) \leq 0$, it follows that $d\,s_0/d\,\delta > 0$. ■

PROPOSITION 1 *Habit formation affects the steady state capital intensity of an OLG economy with production. The higher is the strength of the habit formation, the higher is the steady state capital intensity: $d\,k/d\,\delta > 0,\ \forall k > 0,\ 0 < \delta < u'(c^1)/\beta\,u'(\hat{c}^2)$.*

In the proof we will first consider the following case: $-k\,f''(k) - (1+n) < 0$ (case A). This case represents all k high enough such that $\partial c^1/\partial k < 0$ (see Figure 2). The second step consists in considering two cases where k is lower: the limit as k approaches zero (case B) and the case where k becomes larger than zero and is still lower than in case A (case C). It is shown that in all three cases $\sum_{i=1-4}\gamma^i < 0$ (see below) indicating that $d\,k/d\,\delta > 0$ for all $k > 0$.

Proof. Differentiating FOC (15) with respect to δ gives rise to the following differential:

$$\frac{d\,k}{d\,\delta} = \frac{-\beta\,u'\left(\hat{c}^2\right) + \left[f'(k) + \delta\right]\beta\,u''\left(\hat{c}^2\right)c^1}{\gamma^1 + \gamma^2 + \gamma^3 + \gamma^4}.$$

$$
\begin{aligned}
\gamma^1 &\equiv u''\left(c^1\right)\left[k\,f''(k)+(1+n)\right] \\
\gamma^2 &\equiv \beta\,u''\left(\hat{c}^2\right)\left[f'(k)+\delta\right]\left[k\,f''(k)+f'(k)\right](1+n) \\
\gamma^3 &\equiv \beta\,u''\left(\hat{c}^2\right)\left[f'(k)+\delta\right]\left[k\,f''(k)+(1+n)\right]\delta \\
\gamma^4 &\equiv \beta\,u'\left(\hat{c}^2\right)f''(k)
\end{aligned}
\tag{17}
$$

Case A: The capital intensity k is high enough such that $-k\,f''(k)-(1+n)<0$. As can be easily verified, differential (17) is positive in this case: $d\,k/d\,\delta>0$. This case refers to the "efficient zone" of *Ihori*'s [7] intertemporal consumption possibility curve, i.e. $\partial c^2/\partial c^1<0$.

Case B: However, as *Galor* and *Ryder* [5] demonstrate, a necessary condition for existence of a unique, stable equilibrium is: $\lim_{k\to 0}-k\,f''(k)-(1+n)>0$. Thus, we have to consider two more cases: firstly, the case of k approaching zero and secondly the case where k is increasing and still $-k\,f''(k)-(1+n)>0$.

LEMMA 2 *As k tends to zero, $\sum_{i=1-4}\gamma^i<0$.*

The proof of Lemma 2 is given in Appendix A. Because of Lemma 2, as k tends to zero, $\sum\gamma^i<0$. Since the numerator of (17) is always less than zero, it follows that $d\,k/d\,\delta>0$ in this case.

Case C: If k is increasing, then by assumption 2(iv) $k\,f''(k)$ does not decrease. Therefore the only term that causes difficulties in evaluating (17) — $k\,f''(k)+(1+n)$ in γ^1 and γ^3 — is becoming less and less negative as k increases. Consequently $\sum_{i=1-4}\gamma^i$ remains negative as k increases. Thus, for any $k>0$, $\sum_{i=1-4}\gamma^i<0$. Accordingly, for any $k>0$, $d\,k/d\,\delta>0$. ■

Remark. Since $k=s/(1+n)$ it follows that $d\,k/d\,\delta=\partial k/\partial s\ d\,s/d\,\delta=1/(1+n)\,d\,s/d\,\delta>0$. So an increase of δ is always associated with an increase of capital formation: $d\,s/d\,\delta>0$.

The marginal rate of intertemporal substitution is lowered by habit formation. At a given interest rate, households shift consumption from period one to period two in the short run, fostering savings. At the new steady state, which has a higher δ , capital intensity, savings and the wage rate are higher, and the interest rate is lower. Figure 2 illustrates this process. Figure 2 makes use of *Ihori*'s [7] consumption possibility curve ($OXYZ$). Each point along this curve is associated with a specific capital intensity, where k is zero at the origin and steadily increases as one moves upward and along $OXYZ$. For every k, the curve depicts the steady state $\left(c^1,\,c^2\right)$ -tuple that maximizes utility (5) subject to the intertemporal budget constraint (4). As one moves upward, c^2 rises, the interest rate becomes lower and

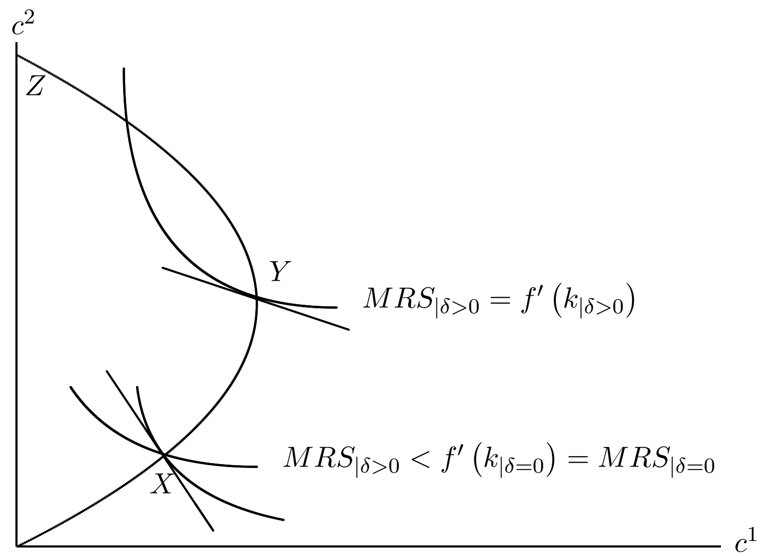

Figure 2: Optimum Consumption Plans and Habit Formation

$c^1 = f(k) - k\,f'(k) - (1+n)k$ initially rises and then falls. Point X depicts an optimum for the case of δ equal to zero. However if δ becomes larger, the marginal rate of substitution becomes lower and X is no longer an optimum. Henceforth savings become larger and the economy moves toward point Y, which represents the new steady state for $\delta > 0$. This result is important for analyzing the impact of tax policy in an economy with habit formation.

3 An OLG Tax Model with Habit Formation

The previous section demonstrates that habit formation generally has an impact on the savings behavior of households. This section addresses the question of the significance of this impact. Particularly, the impact of habit formation on the effects of a tax program is analyzed. It is shown that in the case of the introduction of a specific tax program that fosters savings and capital accumulation, the presence of habit formation reverses this effect: savings and capital accumulation actually decline.

The tax program, which is analyzed below, amounts to a simple type of consumption taxation. This tax was chosen because of the well-known positive impact of consumption taxation on savings and short-run growth.[6] The analysis below is not intended to provide another evaluation of con-

[6] Welfare and distributional effects of the tax program are not considered in this paper.

sumption taxation. Its objective is rather to demonstrate that a given tax program can result in contrary effects in the presence of habit formation. To keep the tax program as simple as possible, we assume that the tax rate is uniform across generations and does not change over time. In particular, this implies that the introduction of the tax is permanent. To prevent a public authority from running surpluses or deficits, we assume that in each period the revenue is recycled to the older generation. Since the analysis focuses on steady states, time indices are omitted as before.

The tax program changes the budget constraints of the generations in the following way.

$$c^1(1+\tau) + s = w \tag{18}$$

$$c^2(1+\tau) = (1+r)s + t \tag{19}$$

$$t = c^2\tau + (1+n)c^1\tau \tag{20}$$

$$c^1 + \frac{c^2}{1+r} = w + c^1\tau\frac{n-r}{1+r} \tag{21}$$

The parameter τ in equation (18) denotes the tax rate, variable t in (19) and (20) shows the transfers. Equation (19) shows the budget constraint for the older generations. Equation (21) shows the intertemporal budget constraint. By the assumption of dynamic efficiency, $r > n$. Therefore, for a given wage and interest rate, the tax program represents a redistribution of resources from the younger to the older generation. The implementation of this tax program is therefore associated with one effect: an income effect, which lowers lifetime income for given k.[7]

The first order condition of this tax model becomes:

$$-u'\left(c^1\right) + \beta\left(f'(k) + \delta\right) u'\left(\hat{c}^2\right) = 0 \tag{22}$$

Consumption of the younger and older generations can be written respectively as:

$$c^1 = \left[f(k) - k\,f'(k) - k(1+n)\right]/(1+\tau) \tag{23}$$

$$\hat{c}^2 = \left\{k\,f'(k)(1+n+\delta) + [f(k) - (1+n)k][(1+n)\tau - \delta]\right\}/(1+\tau). \tag{24}$$

[7] It might be more appealing to recycle the tax revenues back to each generation in proportion to each generations tax payment. However notice that there is no substitution effect caused by the tax program. If we recycle the revenues in proportion to each generations tax payment, then there is no income effect as well and the tax program has no effect at all.

Considering these together with FOC (22), simple algebra leads to the following differential.

$$\frac{d\,k}{d\,\tau} = \frac{-u''\left(c^1\right)c^1/(1+\tau) - \beta(f'(k)+\delta)\,u''\left(\hat{c}^2\right)(1+n+\delta)/(1+\tau)c^1}{-u''\left(c^1\right)\gamma^1 + \beta(f'(k)+\delta)\,u''\left(\hat{c}^2\right)\gamma^2 + \beta\,u'\left(\hat{c}^2\right)f''(k)} \tag{25}$$

$$\begin{aligned} \text{where } \gamma^1 &\equiv [-k\,f''(k) - (1+n)]/(1+\tau) \\ \gamma^2 &\equiv (1+n+\delta)/(1+\tau)[k\,f''(k) + (1+n)] + (1+n)\Delta \\ \Delta &\equiv f'(k) - (1+n) = r - n \end{aligned}$$

It can be seen that the numerator is always larger than zero. However the sign of the denominator is not obvious.

Case A. If $\gamma^1 < 0$, i.e. as consumption of young households decreases as a result of capital accumulation, then the denominator is unambiguously lower than zero. As a consequence, the implementation of the tax program unambiguously reduces capital intensity in steady state.

Case B. If $\gamma^1 > 0$, i.e. as consumption of young households increases as a result of capital accumulation (this is the case for lower k), then the denominator may be either positive or negative. Further on the paper deals with the ambiguous case B rather than with case A.

Considering the FOC as well as the constant rate of risk aversion, the denominator of (25) becomes

$$u'(c_1)/\hat{c}^2\left\{\gamma^1\sigma[c^2/c^1 + (1+n)] - \sigma(1+n)\Delta + \hat{c}^2 f''/(f'+\delta)\right\}. \tag{26}$$

The sign of this expression may be positive or negative.[8] In particular, there exists a critical tax rate τ^*, which makes (26) equal to zero. Since the tax rate enters expression (26) only in the denominator of γ^1, any larger (smaller) tax rate makes (26) larger (smaller) than zero.

PROPOSITION 2 *A tax program under which consumption is taxed and the revenues are fully rebated to the older generation within each period results in an increase in short-run growth and capital formation and in a higher steady state capital intensity if the applied tax rate exceeds a critical rate τ^*.*

$$\tau^* = \frac{f''[-f + k\,f' + (1+n)k][f'\,k(1+n) - \delta(f - k\,f' - (1+n)k)] - \gamma^3}{(1+n)f''[f - k\,f' - (1+n)k][f - k(1+n)] + \gamma^3}$$

$$\gamma^3 \equiv \sigma(\delta + f')[(-k\,f'' - (1+n))(f - k(1+n)) - \Delta(f - k\,f' - (1+n)k)](1+n)$$

[8] The derivation of this result makes use of the fact that $\gamma^2 = -(1+n+\delta)\gamma^1 + (1+n)\Delta$.

Proof. Set (26) equal to zero and solve the expression in the curly brackets for the tax rate τ. Then the critical tax rate, i.e., the tax rate that makes this expression equal to zero is τ^*. Each tax rate $\tau > \tau^*$ makes (26) positive. Then, according to (25) each tax rate $\tau > \tau^*$ fosters capital formation and raises steady state capital intensity. ■

This result indicates that the proposed tax program encourages capital formation and short run growth only if two conditions are satisfied. First, $\partial c^1/\partial k > 0$ (i.e. "small" initial capital intensity). Second, the tax rate is higher than the critical tax rate τ^*. Otherwise, the tax program results in crowding out and lower steady state capital intensity.

The economic reason for this is the following. The tax program is associated with an income effect.[9] There is an income effect because the program shifts resources from the younger generation to the older generation. Each period, the younger generation is taxed an amount per capita of $c^1\tau$. When the generation enters its second period of life, it is given a lump sum transfer of $(1+n)c^1\tau$. Since $n < r$ (by dynamic efficiency) lifetime income is reduced according to (21). To see what is going on, let's look at the first period first. Capital intensity, k_0, is given (and determined by last periods' investment) in the period of implementation of the tax program. By (10) and (11), so are the wage and interest rate. Hence, by (19) consumption of the old generation c_0^2 increases. At the same time, consumption of the young generation c_0^1 certainly decreases.

$$\frac{d\,c_0^1}{d\,\tau} = \frac{-\beta(1+r_0+\delta)u''(\hat{c}_0^2)(n-r_0)c_0^1}{-u''(c_0^1)+\beta(1+r_0+\delta)[(n-r_0)\tau-(1+r_0+\delta)]u''(\hat{c}_0^2)} < 0 \quad (27)$$

Savings s_0 are equal to $\overline{w_0} - c_0^1(1-\tau)$. So whether savings are increased or reduced as a consequence of implementation of the tax program depends upon by how much first period's consumption of young households declines. The higher the tax rate τ is, the stronger is the decline of c_0^1. Therefore we conclude that for all $\tau > \tau^*$, the decline of c_0^1 is strong enough for savings to increase. Conversely, for all $\tau < \tau^*$, the decline of c_0^1 is weak so that savings actually decline. The strength of this effect mainly depends on the curvature of the utility function and of the level of the tax rate.

If $\tau > \tau^*$, savings are increased in the first period. So the rate of interest of subsequent periods declines. From FOC (22) it follows that $u'(c^1)/u'(\hat{c}^2)$ must decrease as well. Therefore $c^1/c^2 < 0$ raises. If $\tau < \tau^*$, savings are decreased in the first period. So the rate of interest of subsequent periods raises and so c^1/c^2 declines.

[9] Notice that due to exogeneity of labor supply, the tax program has no substitution effect.

So far the impact of the tax program was described without regard to habit formation. It was shown that the implementation of the tax program can result in both, a higher and a lower steady state capital intensity. For a given utility function, if the tax rate exceeds a critical tax rate, then the tax program crowds in capital. However, habit formation has an influence on the critical tax rate τ^*.

PROPOSITION 3 *The tax program described in Proposition 2 promotes capital formation when the tax rate exceeds τ^*. The same tax program crowds out capital whenever the tax rate lies in the following interval where the first expression equals τ^* evaluated at $\delta = 0$.*

$$\hat{\tau^*} < \tau < \tau^*, \ \hat{\tau^*} \equiv \tau^*|_{\delta=0}$$

The critical tax rate is higher in the presence than in the absence of habit formation. As a consequence, an analysis of a tax program that does not consider habit formation can result in inaccurate conclusions concerning the impact of the tax program.

Proof. Consider first a "traditional" tax model without habit formation. The critical tax rate in this case is $\hat{\tau}^* = \{f''[-f + k\,f' + (1+n)k][f'\,k(1+n)] - \gamma^3|_{\delta=0}\}/\{(1+n)f''[f - k\,f' - (1+n)k][f - k(1+n)] + \gamma^3|_{\delta=0}\}$. Any tax rate above $\hat{\tau}^*$ can be assumed to encourage savings, while any tax rate below results in a decline in savings. Now consider the presence of habit formation. Any tax rate above τ^* can be assumed to encourage savings, any tax rate below results in a decline in savings. Since $\tau^* > \hat{\tau}^*$ it follows that any tax rate τ in between these rates supports two interpretations of its impact on capital formation: it lowers savings when habit formation is considered and it increases savings when habit formation is discounted. Thus, in the presence of habit formation any τ, such that $\hat{\tau}^* < \tau < \tau^*$, lowers savings, short-run growth and the steady state capital intensity when habit formation is considered and it increases savings when habit formation is discounted. Formally, $\partial\,\tau^*/\partial\,\delta > 0$. ■

The procedure adopted above can generally be improved in the following way. As shown in section 1, steady state capital intensity increases with δ. So the expression $d\,\tau^*/d\,\delta$ does not reflect by how much the critical tax rate changes for a *given* k but for *different* steady state capital intensities. In order to compare the critical τ^* *for the same* k, it is necessary to adjust preferences, such that the raise in δ is compensated for. In the present case this is possible by adjusting β according to the following compensation rule: $d\,\beta/d\,\delta = -\beta/(f' + \delta) + \beta\,u''(\hat{c}^2)c^1/u'(\hat{c}^2)$. Regardless of the specific value of δ, this adjustment rule for β ensures exactly the same initial choices and capital intensity (in the absence of the tax program).

Proposition 2 shows that for the present tax program, $d\,\tau^*/d\,\delta$ is independent of β. Therefore the application of the compensation rule does not affect τ^*. So Proposition 3 is robust with regard to the compensation rule. Though the compensation rule does not affect the level of the critical tax rate, according to (25) it has an impact on the quantitative impact of the tax program on steady state capital intensity.

It is not immediately obvious why the critical tax rate is independent of β. Consider equation (25). The sign of the denominator determines the qualitative impact of the tax program on capital accumulation and thus also determines the critical tax rate. In the following, the denominator of (25) is transformed into expression (26), which no longer contains β. This transformation makes use of FOC (22) as well as of Assumption 1 (ii), which states that one property of the utility function is a constant coefficient of risk aversion. Parameter β is eliminated by both transformations. The latter assumption allows us to express u'' in terms of u', while the FOC allows to express the expression $\beta(f' + \delta)u'(\hat{c}^2)$ in terms of $u'(c^1)$. So the reason for the independence of the critical tax rate from β is the assumption of a constant coefficient of risk aversion.

The economic intuition behind Proposition 3 is provided by propositions 1 and 2. Proposition 1 demonstrates that the optimum $\left(c^1, c^2\right)$ -tuple in the presence of habit formation is located above the optimum $\left(c^1, c^2\right)$ -tuple with $\delta = 0$. That is, savings, second-period-consumption and capital intensity are increasing in δ. Proposition 2 implies that the higher is δ the higher is steady state savings and the more incentive is needed by the young generation to further increase savings, so $\partial\,\tau^*/\partial\,\delta > 0$.

4 Conclusions

The two questions that are posed in the introduction of the paper can be answered in the following way. First, habit formation generally leads to higher savings and capital intensity. Second, habit persistence effects can reverse the impact of tax policy on capital formation, short-run growth and steady state capital intensity. So "conventional" analysis of tax policy can be misleading if habit formation is taken into account.

The intuition behind the first answer is the decline of the marginal rate of intertemporal substitution due to habit persistence. Households therefore decrease first period consumption and increase savings. This results in a higher capital intensity in the new steady state. The intuition behind the second answer lies in the impact of habit persistence on the income effect of the tax program. The tax program always lowers consumption of the young household. However, by dynamic efficiency lifetime income is also reduced. So whether the reduction of c^1 exceeds the reduction in lifetime income depends on the slope of the utility function and the level of the tax rate. Whenever c^1 declines by more than the present value of the lifetime income, savings will increase. Furthermore, c^1 declines the more, the higher the tax rate τ is. Thus, for high τ, savings increase while for low τ savings will decline. For a critical tax rate τ^*, the tax program does not influence steady state capital intensity.

Habit persistence raises the critical tax rate. In order to compensate for the negative income effect, the tax rate must be higher. This result suggests that the analysis of a tax program can be significantly misleading if it fails to consider habit persistence.

The analysis presented here is not intended to evaluate any consumption tax proposal. Its only objective is the demonstration of the importance of habit persistence for the analysis of tax programs of which the consumption tax program, as formulated in this paper, is but one example. It is also important to note that the results presented in this paper may not be significant for all tax proposals. There may be many for which habit persistence effects do not change the qualitative impact of the tax program. However, the central message remains. Since there are some programs for which habit formation matters, it has to be carefully considered in the analysis of all tax programs.

A Appendix: Proof of Lemma 2

To show Lemma 2 let's simplify $\sum_{i=1-4}\gamma^i$. Considering the constant coefficients of risk aversion along with FOC (15) we can reformulate $\sum\gamma^i$:

$$\sum_{i=1-4}\gamma^i = -\beta\, u'\left(\hat{c}^2\right)\left\{\gamma - f''(k)\right\} \tag{28}$$

$$\text{with } \gamma \equiv \bar{\sigma}\left[f'(k)+\delta\right] \left\{\left[\frac{1}{c^1}+\frac{\delta}{\hat{c}^2}\right]\left[k\,f''(k)+(1+n)\right]+\frac{1+n}{\hat{c}^2}\left[k\,f''(k)+f'(k)\right]\right\}.$$

By Assumptions 1 and 2 and by the proper choice of terms we can evaluate the limit of expression (28) as follows. All of the following limits refer to the limit as k approaches zero, $\lim_{k\to 0}$.

$$\lim \sum_{i=1-4} \gamma^i = \lim \left[-\beta\, u'\left(\hat{c}^2\right)\right] \left[\lim \gamma - \lim f''(k)\right]$$

We first start considering $\lim \gamma$.

$$\begin{aligned}
&\lim \gamma = \lim \left\{\bar{\sigma}\left[f'(k)+\delta\right]\right\} \\
&\left\{\lim \left[\frac{1}{c^1} + \frac{\delta}{\hat{c}^2}\right] \lim \left[k\, f''(k) + (1+n)\right] + \lim \left[\frac{1+n}{\hat{c}^2}\left[k\, f''(k) + f'(k)\right]\right]\right\}
\end{aligned}$$

$$\begin{aligned}
= \;& \lim \left\{\bar{\sigma}\left[f'(k)+\delta\right]\right\} \\
& \left\{\lim \left[\frac{1}{c^1} + \frac{\delta}{\hat{c}^2}\right] \lim \left[k\, f''(k)\right] + \lim \left[\frac{1+n}{\hat{c}^2}\left[k\, f''(k) + f'(k)\right]\right]\right\} \\
= \;& \lim \left\{\bar{\sigma}\left[f'(k)+\delta\right]\right\} \lim \left\{\left[\frac{1}{c^1} + \frac{1+n+\delta}{\hat{c}^2}\right] k\, f''(k) + \frac{1+n}{\hat{c}^2} f'(k)\right\} \\
= \;& \lim \left\{\bar{\sigma}\left[f'(k)+\delta\right]\right\} \\
& \lim \left\{\left[\frac{1}{c^1} + \frac{1+n+\delta}{\hat{c}^2}\right] \left[k\, f''(k) + f'(k)\right] - \left[\frac{1}{c^1} + \frac{\delta}{\hat{c}^2}\right] f'(k)\right\} \\
= \;& \lim \left\{\bar{\sigma}\left[f'(k)+\delta\right]\right\} \\
& \lim \left\{\left[\frac{1}{c^1} + \frac{1+n+\delta}{\hat{c}^2}\right] \left[\left[k f''(k) + f'(k)\right] - \left[\frac{c^2}{c^2+(1+n)c^1}\right] f'(k)\right]\right\} \\
= \;& \lim \left\{\bar{\sigma}\left[f'(k)+\delta\right]\right\} \lim \left\{\left[\frac{1}{c^1} + \frac{1+n+\delta}{\hat{c}^2}\right] \left[k\, f''(k) + f'(k)\right]\right\} \\
& - \lim \left\{\bar{\sigma}\left[f'(k)+\delta\right]\right\} \lim \left[\frac{1}{c^1} + \frac{1+n+\delta}{\hat{c}^2}\right] \lim \left[\frac{c^2}{c^2+(1+n)c^1} f'(k)\right]
\end{aligned}$$

Because

$$\begin{aligned}
& \lim \left[\frac{c^2}{c^2+(1+n)c^1} f'(k)\right] = \lim f'(k) \lim \left[\frac{c^2}{c^2+(1+n)c^1}\right] \\
= \;& \lim f'(k) \lim \left[\frac{f'(k)k}{f(k)-(1+n)k}\right] = \lim f'(k) \lim \left[\frac{1+f''(k)k/f'(k)}{1-(1+n)/f'(k)}\right] \\
= \;& \lim f'(k) \lim \left[\frac{f''(k)k}{f'(k)} + 1\right] = \lim \left[f''(k)k + f'(k)\right],
\end{aligned}$$

it follows that $\lim_{k\to 0} \gamma = 0$. So $\lim \sum \gamma^i = \lim[-\beta\, u'(\hat{c}^2)][-\lim f''(k)] \to -\infty < 0$. ■

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