Economics 204–Final Exam–August 21, 2007, 9am-12pm
Each of the four questions is worth 25% of the total
Please use three *separate* bluebooks, one for each of the three Parts

Part I

1. State and prove the Supremum Property.

2. Prove that if a finite set X has exactly n elements, then 2^X, the set of all subsets of X, has exactly 2^n elements. *Hint*: use induction.

Part II

3. Consider the function

$$f(x,y) = x^3 - y^3 + 2x^2 + 2xy + 8y^2 - 9x - 15y - 2$$

   (a) Compute the first order conditions for a local maximum or minimum of f. Show that the first order conditions are satisfied at the point $(x_0, y_0) = (1, 1)$.

   (b) Compute $D^2 f(x_0, y_0)$ and give the quadratic Taylor polynomial for f at the point (x_0, y_0).

   (c) Find the eigenvalues of $D^2 f(x_0, y_0)$ and determine whether f has a local max, a local min, or a saddle at (x_0, y_0).

   (d) Does f have a global max, a global min, or neither, at (x_0, y_0)?

   (e) Find the eigenvectors of $D^2 f(x_0, y_0)$ and provide an orthonormal basis for $\mathbf{R}^2$ consisting of eigenvectors. Rewrite the quadratic Taylor polynomial for f at the point (x_0, y_0) in terms of this basis.

   (f) Use the quadratic Taylor polynomial found in part (e) to describe the level sets of f near the point (x_0, y_0); include the shape, the directions of the principle axes, and (where appropriate) the lengths of the principal axes.

Part III

4. Let $F : C \times \mathbf{R}^p \to \mathbf{R}^n$ be a continuous function, where $C \subseteq \mathbf{R}^n$.

   (a) Given $\varepsilon > 0$, define

$$\Psi_\varepsilon(\omega) = \{x \in \mathbf{R}^n : |F(x,\omega)| < \varepsilon\}$$

   Show directly from the definition that for each $\varepsilon > 0$, Ψ_ε is a lower hemicontinuous correspondence. (We are using correspondence in the sense of the Lectures, so that we do not require that $\Psi_\varepsilon(\omega) \neq \emptyset$).

   (b) Let

$$\Psi(\omega) = \{x \in \mathbf{R}^n : F(x,\omega) = 0\}$$

   Show directly from the definition that if C is compact, then Ψ is an upper hemicontinuous correspondence. (We are using correspondence in the sense of the Lectures, so that we do not require that $\Psi(\omega) \neq \emptyset$). *Hint*: The proof is by contradiction. Suppose that Ψ is not upper hemicontinuous at some ω_0; this tells you that there is a sequence $\omega_n \to \omega_0$ with certain properties.