University of California, Berkeley
Economics 201A
Spring 2005 Final Exam–May 20, 2005

Instructions: You have three hours to do this exam. The exam is out of a total of 300 points; allocate your time accordingly. **Please write your solutions to Parts I and II in separate bluebooks.**

Part I

1. (100 points) Define or state and *briefly* discuss the importance of each of the following within or for economic theory:
   (a) Shapley-Folkman Theorem on Sums of Sets
   (b) local nonsatiation
   (c) Walrasian Quasiequilibrium
   (d) Second Welfare Theorem in an Arrow-Debreu Economy
   (e) Debreu-Gale-Kuhn-Nikaido Lemma

2. (80 points) Consider an Edgeworth Box economy, with endowments $\omega_1 = (3, 1)$, $\omega_2 = (1, 3)$, and utility functions $u_1(x_{11}, x_{21}) = x_{11}^{1/3} x_{21}^{2/3}$, $u_2(x_{12}, x_{22}) = x_{12}^{2/3} x_{22}^{1/3}$.
   (a) Find all the Walrasian Equilibria of this economy.
   (b) Compute the set of Pareto Optima of this economy.
   (c) Find prices and transfers that make the allocation $x_1 = \left(1, \frac{16}{7}\right)$, $x_2 = \left(3, \frac{12}{7}\right)$ a Walrasian equilibrium with transfers.
   (d) Compute the core of this economy. [You should obtain equations for the end points of the core, but you needn't solve these equations explicitly.]

Part II

3. (120 points) Consider an exchange economy with I consumers and $L = 2$ goods. The vector of endowments, $\omega \in \mathbf{R}^{2I}$, is fixed, and $\omega_1 > 0$. Let $\mathcal{U}$ denote the set of utility functions u on $\mathbf{R}^2_+$ satisfying

   - u is C^2 (the second partial derivatives all exist and are continuous)
   - $\bigtriangledown u|_x >> 0$ and the Hessian matrix $Hu|_x$ is negative definite for all $x \in \mathbf{R}^2_{++}$
   - $u(x) = 0$ for $x \in \mathbf{R}^2_+ \setminus \mathbf{R}^2_{++}$
   - $u(x) > 0$ for $x \in \mathbf{R}^2_{++}$

   The preferences of consumers $i = 2, ..., I$ are fixed and generated by utility functions $u_2, \ldots, u_I \in \mathcal{U}$. $i = 1$'s preference is described by a parametrized utility function $u_1 : \mathbf{R}^2_+ \times ((0, \infty) \times (0, 1)) \to \mathbf{R}$ given by $u_1(x_1, \alpha) = v(x_{11}, x_{21}) + \alpha_1 x_{11}^{\alpha_2} x_{21}^{1-\alpha_2}$, for some $v \in \mathcal{U}$.

   (a) Write down the first-order conditions defining the demand of agent 1, and show that they are necessary and sufficient to characterize the demand.

   (b) Using the Implicit Function Theorem, show that the demand of agent 1 is a C^1 function of (p, α).

   (c) Using the Transversality Theorem, show that for almost all α, the economy is regular.

   (d) Using the Implicit Function Theorem, show that for almost all α, the economy has finitely many equilibria which move in a C^1 fashion in response to changes in α.