University of California, Berkeley
Economics 201A
Fall 2001 Second Midterm Test–December 11, 2001

Instructions: You have three hours to do this test. The test is out of a total of 300 points; allocate your time accordingly. Please write your solution to each question in a **separate** bluebook.

1. (100 points) Define or state and *briefly* discuss the importance of each of the following within or for economic theory:
   (a) Kakutani's Fixed Point Theorem
   (b) Lebesgue measure zero
   (c) Core of an exchange economy
   (d) First Welfare Theorem in an Arrow-Debreu economy
   (e) Index Theorem

2. (80 points) Consider an Edgeworth Box economy, where

$$\begin{array}{cc} \omega_1 = (2,1) & \omega_2 = (1,2) \\ u_1(x_{11}, x_{21}) = \sqrt{x_{11}x_{21}} & u_2(x_{12}, x_{22}) = \sqrt{x_{12}x_{22}} \end{array}$$

   (a) Find a Walrasian equilibrium.
   (b) Show that the allocation $x_1 = (1,1)$, $x_2 = (2,2)$ is Pareto optimal. Without using the Second Welfare Theorem, show that this allocation is a Walrasian equilibrium with transfers.

3. (120 points) Consider the function $z : \Delta^0 \times \mathbf{R} \to \mathbf{R}^2$ defined by

$$z(p, \alpha) = \left(\frac{1}{p_1} + \alpha \cos(2\pi p_1), -\frac{1 + \alpha p_1 \cos(2\pi p_1)}{p_2} \right)$$

Note that $\cos(0) = 1$ and $\frac{d}{dx} \cos x = -\sin x$.

(a) For what values of α does the function $z_\alpha(p) = z(p, \alpha)$ satisfy the conditions of the Debreu-Gale-Kuhn-Nikaido Lemma?

(b) For what values of α does there exist $p \in \Delta$ such that $z(p, \alpha) = 0$?

(c) Show that for every $\alpha \in \mathbf{R}$ and every $\varepsilon > 0$, there is an exchange economy with two agents whose excess demand function agrees with z_α on $\{p \in \Delta : p_1 \in [\varepsilon, 1 - \varepsilon]\}$.

(d) Show that there is a set $A \subset \mathbf{R}$ of Lebesgue measure zero such that for every $\alpha \notin A$, the economy with excess demand z_α is regular.