UC Berkeley **Fall Semester 2004**

Economics 241A **Guido Imbens**

PROBLEM SET I, Part 4

Due: Wednesday February 18^{th}

1. (a) Suppose that $X_1, \ldots, X_N$ are independent and identically distributed random variables with normal distributions with mean μ and variance σ^2. Describe an information matrix test using the $(1,2)$ element of the information matrix. What alternatives is this test powerful against?

   (b) Suppose that $X_1, \ldots, X_N$ are independent and identically distributed random variables with Poisson distributions with parameter λ. Give the form of the information matrix test. What feature of the Poisson distribution is being tested?

2. Use the matlab data set NLS2.MAT which is posted on the website. It contains data on 935 observations on five variables: a binary indicator for finishing high school, two tests scores, iq and kww, and mother's and father's education. We will look at logistic regression models with as the outcome variable the highschool indicator. The model we consider is

$$\Pr(\text{highschool} = 1 | \text{kww}, \text{iq}, \text{med}, \text{fed})$$

$$= \frac{\exp(\beta_0 + \beta_1 \cdot \text{kww} + \beta_2 \cdot \text{iq} + \beta_3 \cdot \text{med} + \beta_4 \cdot \text{fed})}{1 + \exp(\beta_0 + \beta_1 \cdot \text{kww} + \beta_2 \cdot \text{iq} + \beta_3 \cdot \text{med} + \beta_4 \cdot \text{fed})}.$$

   (a) Show that the log likelihood function at $\beta_0 = 2$, $\beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$ is equal to -294.6777.

   (b) Calculate the first derivatives of the log likelihood function analytically at these values for the parameters. Compare them to numerical derivatives using $c = 0.001$.

   (c) Estimate the parameters by maximum likelihood, using the DFP algorithm. Stop when the sum of the absolute values of the first derivatives is less than 0.001. Report the usual statistics for each iteration.

(d) Estimate the standard errors using the outer product of the first derivatives. Compare them to standard errors based on the A_k matrix, using the fact that the A_k matrix approximate the second derivative of the objective function.

(e) Now estimate the parameters given $\beta_1 = 0$ by maximum likelihood, using the DFP algorithm. No need to report any statistics for each iteration, just the total number of iterations.

(f) Estimate the standard errors using the outer product of the first derivatives.

(g) Next we consider a Hausman-Wu test of the hypothesis $\beta_1 = 0$ by comparing the restricted and unrestricted estimates of β_3. What is the difference in the restricted and unrestricted estimates?

(h) What are the two variances for $\hat{\beta}_3$ from (d) and (f)?

(i) What is the value of the HW test statistic based on these variances?

(j) Using the information matrix estimated on the unrestricted estimates to estimate the two variances and calculate the HW statistic based on this.