

Economics 101A

(Lecture 26)

Stefano DellaVigna

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Outline

1. Walrasian Equilibrium
2. Example
3. Summary of General Equilibrium

1 Walrasian Equilibrium

- Prices p_1, p_2

- Consumer 1 faces a budget set:

$$p_1 x_1^1 + p_2 x_2^1 \leq p_1 \omega_1^1 + p_2 \omega_2^1$$

- How about consumer 2?

- Budget set of consumer 2:

$$p_1 x_1^2 + p_2 x_2^2 \leq p_1 \omega_1^2 + p_2 \omega_2^2$$

or (assuming $x_i^1 + x_i^2 = \omega_i$)

$$p_1(\omega_1 - x_1^1) + p_2(\omega_2 - x_2^1) \leq p_1(\omega_1 - \omega_1^1) + p_2(\omega_2 - \omega_2^1)$$

or

$$p_1 x_1^1 + p_2 x_2^1 \geq p_1 \omega_1^1 + p_2 \omega_2^1$$

- **Walrasian Equilibrium.** $((x_1^{1*}, x_2^{1*}), (x_1^{2*}, x_2^{2*}), p_1^*, p_2^*)$ is a Walrasian Equilibrium if:

- Each consumer maximizes utility subject to budget constraint:

$$(x_1^{i*}, x_2^{i*}) = \arg \max_{x_1^i, x_2^i} u_i((x_1^i, x_2^i))$$
$$s.t. p_1^* x_1^i + p_2^* x_2^i \leq p_1^* \omega_1^i + p_2^* \omega_2^i$$

- All markets clear:

$$x_j^{1*} + x_j^{2*} \leq \omega_j^1 + \omega_j^2 \text{ for all } j.$$

- Compare with partial (Marshallian) equilibrium:
 - each consumer maximizes utility
 - market for good i clears.
 - (no requirement that all markets clear)

- How do we find the Walrasian Equilibria?

- **Graphical method.**

1. Compute first for each consumer set of utility-maximizing points as function of prices
2. Check that market-clearing condition holds

- *Step 1.* Compute optimal points as prices p_1 and p_2 vary

- Start with Consumer 1. Find points of tangency between budget sets and indifference curves

- Figure

- **Offer curve** for consumer 1:

$$(x_1^{1*}(p_1, p_2, (\omega_1, \omega_2)), x_2^{1*}(p_1, p_2, (\omega_1, \omega_2)))$$

- Offer curve is set of points that maximize utility as function of prices p_1 and p_2 .

- Then find offer curve for consumer 2:

$$(x_1^{2*}(p_1, p_2, (\omega_1, \omega_2)), x_2^{2*}(p_1, p_2, (\omega_1, \omega_2)))$$

- Figure

- *Step 2.* Find intersection(s) of two offer curves
- Walrasian Equilibrium is intersection of the two offer curves!
 - Both individuals maximize utility given prices
 - Total quantity demanded equals total endowment

- Relate Walrasian Equilibrium to barter equilibrium.

- Walrasian Equilibrium is a subset of barter equilibrium:
 - Does WE satisfy Individual Rationality condition?

 - Does WE satisfy the Pareto Efficiency condition?

- Walrasian Equilibrium therefore picks one (or more) point(s) on contract curve.

2 Example

- Consumer 1 has Leontieff preferences:

$$u(x_1, x_2) = \min(x_1, x_2)$$

- Bundle demanded by consumer 1:

$$\begin{aligned}x_1^{1*} &= x_2^{1*} = x^{1*} = \frac{p_1\omega_1^1 + p_2\omega_2^1}{p_1 + p_2} = \\ &= \frac{\omega_1^1 + (p_2/p_1)\omega_2^1}{1 + (p_2/p_1)}\end{aligned}$$

- Graphically

- Comparative statics:

- increase in ω

- increase in p_2/p_1 :

$$\begin{aligned} \frac{dx_1^{1*}}{dp_2/p_1} &= \frac{\omega_2^1 (1 + (p_2/p_1)) - (\omega_1^1 + (p_2/p_1) \omega_2^1)}{(1 + (p_2/p_1))^2} = \\ &= \frac{\omega_2^1 - \omega_1^1}{(1 + (p_2/p_1))^2} \end{aligned}$$

- Effect depends on income effect through endowments:

- * A lot of good 2 \rightarrow increase in price of good 2 makes richer

- * Little good 2 \rightarrow increase in price of good 2 makes poorer

- Notice: Only ratio of prices matters (general feature)

- Consumer 2 has Cobb-Douglas preferences:

$$u(x_1, x_2) = (x_1^2)^{.5} (x_2^2)^{.5}$$

- Demands of consumer 2:

$$x_1^{2*} = \frac{.5 (p_1 \omega_1^1 + p_2 \omega_2^1)}{p_1} = .5 \left(\omega_1^1 + \frac{p_2}{p_1} \omega_2^1 \right)$$

and

$$x_2^{2*} = \frac{.5 (p_1 \omega_1^1 + p_2 \omega_2^1)}{p_2} = .5 \left(\frac{p_1}{p_2} \omega_1^1 + \omega_2^1 \right)$$

- Impose Walrasian equilibrium in market 1:

$$x_1^{1*} + x_1^{2*} = \omega_1^1 + \omega_1^2$$

This implies

$$\frac{\omega_1^1 + (p_2/p_1)\omega_2^1}{1 + (p_2/p_1)} + .5 \left(\omega_1^1 + \frac{p_2}{p_1}\omega_2^1 \right) = \omega_1^1 + \omega_1^2$$

or

$$\frac{.5 - .5(p_2/p_1)}{1 + (p_2/p_1)}\omega_1^1 + \frac{.5(p_2/p_1) + .5(p_2/p_1)^2 - 1}{1 + (p_2/p_1)}\omega_2^1 = 0$$

or

$$\left(\omega_1^1 - 2\omega_2^1 \right) + \left(\omega_1^1 + \omega_2^1 \right) (p_2/p_1) + \omega_2^1 (p_2/p_1)^2 = 0$$

- Solution for p_2/p_1 :

$$\frac{p_2}{p_1} = \frac{-\left(\omega_1^1 - 2\omega_2^1\right) + \sqrt{\left(\omega_1^1 + \omega_2^1\right)^2 - 4\left(\omega_1^1 - 2\omega_2^1\right)\omega_2^1}}{2\left(\omega_1^1 - 2\omega_2^1\right)}$$

- Some complicated solution!

- Problem set has solution that is much easier to compute (and interpret)

3 Summary of General Equilibrium

- Does Walrasian Equilibrium always exist? In general, yes, as long as preference convex
- Is Walrasian Equilibrium always unique? Not necessarily
- Is Walrasian Equilibrium efficient? Yes.

- **First Fundamental Welfare Theorem.** All Walrasian Equilibria are on Contract Curve (and therefore are Pareto Efficient).

- Smithian Invisible Hand. Market leads to an allocation that is Pareto Efficient.
 - BUT: problems with externalities and public good
 - BUT: what about distribution?

4 Next and last lecture

- Empirical Economics
- Some examples of Empirical Economics
 - House insurance
 - Save More Tomorrow
 - Fox News