Economics 101A
(Lecture 22)

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Outline

1. Game Theory II

2. Oligopoly: Cournot

3. Oligopoly: Bertrand

1 Game Theory II

- Definitions:
  - Players: $1, ..., I$
  - Strategy $s_i \in S_i$
  - Payoffs: $U_i(s_i, s_{-i})$
- Example: Prisoner's Dilemma
  - $I = 2$
  - $s_i = \{D, ND\}$
  - Payoffs matrix:

$1 \setminus 2$	D	ND
D	$-4, -4$	$-1, -5$
ND	$-5, -1$	$-2, -2$

- What prediction?

- Maximize sum of payoffs?

- Choose dominant strategies

- **Equilibrium in dominant stategies**

- Strategies $s^* = \left(s_i^*, s_{-i}^*\right)$ are an Equilibrium in dominant stategies if

$$U_i\left(s_i^*, s_{-i}\right) \geq U_i\left(s_i, s_{-i}\right)$$

for all $s_i \in S_i$, for all $s_{-i} \in S_{-i}$ and all $i = 1, ..., I$

- Battle of the Sexes game:

He \ She	Ballet	Football
Ballet	2, 1	0, 0
Football	0, 0	1, 2

- Choose dominant strategies? Do not exist

- **Nash Equilibrium**.

- Strategies $s^* = \left(s_i^*, s_{-i}^*\right)$ are a Nash Equilibrium if

$$U_i\left(s_i^*, s_{-i}^*\right) \geq U_i\left(s_i, s_{-i}^*\right)$$

for all $s_i \in S_i$ and $i = 1, ..., I$

- Is Nash Equilibrium unique?

- Does it always exist?

- Penalty kick in soccer (matching pennies)

Kicker \ Goalie	L	R
L	0, 1	1, 0
R	1, 0	0, 1

- Equilibrium always exists in mixed strategies σ

- Mixed strategy: allow for probability distibution.

- Back to penalty kick:
  - Kicker kicks left with probability k
  - Goalie kicks left with probability g

  - utility for kicker of playing L :

$$\begin{aligned} U_K(L,\sigma) &= gU_K(L,L) + (1-g)\,U_K(L,R) \\ &= (1-g) \end{aligned}$$

  - utility for kicker of playing R :

$$\begin{aligned} U_K(R,\sigma) &= gU_K(R,L) + (1-g)\,U_K(R,R) \\ &= g \end{aligned}$$

- Optimum?
  - $L \succ R$ if $1 - g > g$ or $g < 1/2$
  - $R \succ L$ if $1 - g < g$ or $g > 1/2$
  - $L \sim R$ if $1 - g = g$ or $g = 1/2$

- Plot best response for kicker

- Plot best response for goalie

- Nash Equilibrium is:
  - fixed point of best response correspondence
  - crossing of best response correspondences

2 Oligopoly: Cournot

- Nicholson, Ch. 14, pp. 418–419, 421–422 [OLD: p. 531, 534–535].

- Back to oligopoly maximization problem

- Assume 2 firms, cost $c_i(y_i) = cy_i$, $i = 1, 2$

- Firms choose simultaneously quantity y_i

- Firm i maximizes:

$$\max_{y_i} p(y_i + y_{-i}) y_i - cy_i.$$

- First order condition with respect to y_i:

$$p'_Y\left(y_i^* + y_{-i}^*\right) y_i^* + p - c = 0,\ i = 1, 2.$$

- Nash equilibrium:
  - y_1 optimal given y_2;
  - y_2 optimal given y_1.

- Solve equations:

$$p'_Y(y_1^* + y_2^*)\, y_1^* + p - c = 0 \text{ and}$$

$$p'_Y(y_2^* + y_1^*)\, y_2^* + p - c = 0.$$

- Cournot -> Pricing above marginal cost

3 Oligopoly: Bertrand

- Cournot oligopoly: firms choose quantities

- Bertrand oligolpoly: firms first choose prices, and then produce quantity demanded by market

- Market demand function $Y(p)$

- 2 firms

- Profits:

$$\pi_i(p_i, p_{-i}) = \begin{cases} (p_i - c)\, Y(p_i) & \text{if} \quad p_i < p_{-i} \\ (p_i - c)\, Y(p_i)/2 & \text{if} \quad p_i = p_{-i} \\ 0 & \text{if} \quad p_i > p_{-i} \end{cases}$$

- First show that $p_1 = c = p_2$ is Nash Equilibrium

- Does any firm have a (strict) incentive to deviate?

- Check profits for Firm 1

- Symmetric argument for Firm 2

- Second, show that this equilibrium is unique.

- For each of the next 5 cases at least on firm has a profitable deviation

- Case 1. $p_1 > p_2 > c$

- Case 2. $p_1 = p_2 > c$

- Case 3. $p_1 > c \geq p_2$

- Case 4. $c > p_1 \geq p_2$

- Case 5. $p_1 = c > p_2$

- Only Case 6 remains: $p_1 = c = p_2$, which is Nash Equilibrium

- It is unique!

- Surprising result of Bertrand Competition

- Marginal cost pricing

- Two firms are enough to guarantee perfect competition!

- Realistic? Price wars between PC makers

4 Next lecture

- Auctions
- Mixed Strategy Equilibria
- Dynamic Games
- Stackelberg duopoly