Economics 101A
(Lecture 18)

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Outline

1. Second Order Conditions in P-Max: Cobb-Douglas
2. Comparative Statics of Equilibrium
3. Elasticities
4. Response to Taxes

1 Second Order Conditions in P-Max: Cobb-Douglas

- How do the second order conditions relate for:
  - Cost Minimization
  - Profit Maximization?

- Check for Cobb-Douglas production function

$$y = AK^{\alpha}L^{\beta}$$

- **Cost Minimization.** S.o.c.:

$$c_y''(y^*, w, r) > 0$$

- As we showed, for CD prod. ftn.,

$$c_y''(y^*, w, r) = -\frac{1}{\alpha+\beta}\frac{1-(\alpha+\beta)}{\alpha+\beta}\frac{B}{A^2}\left(\frac{y}{A}\right)^{\frac{1-2(\alpha+\beta)}{\alpha+\beta}}$$

which is > 0 as long as $\alpha + \beta < 1$ (DRS)

- **Profit Maximization.** S.o.c.: $pf''_{L,L}(L,K) < 0$ and

$$|H| = p^2 \left[f''_{L,L} f''_{K,K} - \left(f''_{L,K} \right)^2 \right] > 0$$

- As long as $\beta < 1$,

$$pf''_{L,L} = p\beta(\beta - 1) AK^{\alpha} L^{\beta-2} < 0$$

- Then,

$$\begin{aligned} |H| &= p^2 \left[f''_{L,L} f''_{K,K} - \left(f''_{L,K} \right)^2 \right] = \\ &= p^2 \left[\begin{array}{c} \beta(\beta-1) AK^{\alpha} L^{\beta-2} * \\ \alpha(\alpha-1) AK^{\alpha-2} L^{\beta} - \\ \left(\alpha\beta AK^{\alpha-1} L^{\beta-1} \right)^2 \end{array} \right] = \\ &= p^2 A^2 K^{2\alpha-2} L^{2\beta-2} \alpha\beta [1 - \alpha - \beta] \end{aligned}$$

- Therefore, $|H| > 0$ iff $\alpha + \beta < 1$ (DRS)

- The two conditions coincide

2 Comparative statics of equilibrium

- Supply and Demand function of parameter α :
  - $Y_i^S(p_i, w, r, \alpha)$
  - $X_i^D(\mathbf{p}, \mathbf{M}, \alpha)$

- How does α affect p^* and Y^*?

- Comparative statics with respect to α

- Equilibrium:

$$Y_i^S(p_i, w, r, \alpha) = X_i^D(\mathbf{p}, \mathbf{M}, \alpha)$$

- Can write equilibrium as implicit function:

$$Y_i^S(p_i, w, r, \alpha) - X_i^D(\mathbf{p}, \mathbf{M}, \alpha) = 0$$

- What is $dp^*/d\alpha$?

- Implicit function theorem:

$$\frac{\partial p^*}{\partial \alpha} = -\frac{\frac{\partial Y^S}{\partial \alpha} - \frac{\partial X^D}{\partial \alpha}}{\frac{\partial Y^S}{\partial p} - \frac{\partial X^D}{\partial p}}$$

- What is sign of denominator?

- Sign of $\partial p^*/\partial \alpha$ is negative of sign of numerator

- Examples:

  1. *Fad.* Good becomes more fashionable: $\frac{\partial X^D}{\partial \alpha} > 0 \Longrightarrow \frac{\partial p^*}{\partial \alpha} > 0$

  2. *Recession in Europe.* Negative demand shock for US firms: $\frac{\partial X^D}{\partial \alpha} < 0 \Longrightarrow \frac{\partial p^*}{\partial \alpha} < 0$

  3. *Oil shock.* Import prices increase: $\frac{\partial Y^S}{\partial \alpha} < 0 \Longrightarrow \frac{\partial p^*}{\partial \alpha} > 0$

  4. *Computerization.* Improvement in technology. $\frac{\partial Y^S}{\partial \alpha} > 0 \Longrightarrow \frac{\partial p^*}{\partial \alpha} < 0$

3 Elasticities

- Nicholson, Ch.1, pp. 27–28 [OLD: Ch.7, pp. 176–177]

- How do we interpret magnitudes of $\partial p^*/\partial \alpha$?

- Result depends on units of measure.

- Can we write $\partial p^*/\partial \alpha$ in a unit-free way?

- Yes! Use **elasticities.**

- Elasticity of x with respect to parameter p is

$$\varepsilon_{x,p} = \frac{\partial x}{\partial p}\frac{p}{x}$$

- Interpretation: Percent response in x to percent change in p :

$$\begin{aligned} \varepsilon_{x,p} &= \frac{\partial x}{\partial p}\frac{p}{x} = \lim_{dp \to 0} \frac{x(p+dp) - x(p)}{dp}\frac{p}{x} = \\ &= \lim_{dp \to 0} \frac{dx/x}{dp/p} \end{aligned}$$

where $dx \equiv x(p+dp) - x(p)$.

- Now, show

$$\varepsilon_{x,p} = \frac{\partial \ln x}{\partial \ln p}$$

- Notice: This makes sense only for $x > 0$ and $p > 0$

- Proof. Consider function

$$x = f(p)$$

- Rewrite as

$$\ln(x) = \ln f(p) = \ln f\left(e^{\ln(p)}\right)$$

- Define $\hat{x} = \ln(x)$ and $\hat{p} = \ln(p)$

- This implies

$$\hat{x} = \ln f\left(e^{\hat{p}}\right)$$

- Get

$$\begin{aligned} \frac{\partial \hat{x}}{\partial \hat{p}} &= \frac{\partial \ln x}{\partial \ln p} = \\ &= \frac{1}{f\left(e^{\hat{p}}\right)} \frac{\partial f\left(e^{\hat{p}}\right)}{\partial \hat{p}} e^{\hat{p}} = \frac{\partial x}{\partial p} \frac{p}{x} \end{aligned}$$

- Example with Cobb-Douglas utility function

- $U(x,y) = x^{\alpha} y^{1-\alpha}$ implies solutions

$$x^* = \alpha \frac{M}{p_x}, y^* = (1-\alpha)\frac{M}{p_y}$$

- Elasticity of demand with respect to own price ε_{x,p_x}:

$$\varepsilon_{x,p_x} = \frac{\partial x^*}{\partial p_x}\frac{p_x}{x^*} = -\frac{\alpha M}{(p_x)^2}\frac{p_x}{\alpha \frac{M}{p_x}} = -1$$

- Elasticity of demand with respect to other price $\varepsilon_{x,p_y} = 0$

- Go back to problem above:

$$\frac{\partial p^*}{\partial \alpha} = -\frac{\frac{\partial Y^S}{\partial \alpha} - \frac{\partial X^D}{\partial \alpha}}{\frac{\partial Y^S}{\partial p} - \frac{\partial X^D}{\partial p}}$$

- Use elasticities to rewrite response of p to change in α :

$$\frac{\partial p^*}{\partial \alpha}\frac{\alpha}{p} = -\frac{\left(\frac{\partial Y^S}{\partial \alpha} - \frac{\partial X^D}{\partial \alpha}\right)\frac{\alpha}{Y}}{\left(\frac{\partial Y^S}{\partial p} - \frac{\partial X^D}{\partial p}\right)\frac{p}{Y}}$$

or (using fact that $X^{D*} = Y^{s*}$)

$$\varepsilon_{p,\alpha} = -\frac{\varepsilon_{S,\alpha} - \varepsilon_{D,\alpha}}{\varepsilon_{S,p} - \varepsilon_{D,p}}$$

- We are likely to know elasticities from empirical studies.

4 Response to taxes

- Nicholson, Ch. 11, pp. 322–323 [OLD: Ch. 15, pp. 407–408]

- Per-unit tax t

- Write price p_i as price including tax

- Supply: $Y_i^S(p_i - t, w, r)$

- Demand: $X_i^D(\mathbf{p}, \mathbf{M})$

$$Y_i^S(p_i - t, w, r) - X_i^D(\mathbf{p}, \mathbf{M}) = 0$$

- What is dp^*/dt?

- Comparative statics:

$$\frac{\partial p^*}{\partial t} = -\frac{\frac{\partial Y^S}{\partial t}}{\frac{\partial Y^S}{\partial p} - \frac{\partial X^D}{\partial p}} =$$

$$= -\frac{-\frac{\partial Y^S}{\partial p}\frac{p}{X}}{\left(\frac{\partial Y^S}{\partial p} - \frac{\partial X^D}{\partial p}\right)\frac{p}{X}} =$$

$$= \frac{\varepsilon_{S,p}}{\varepsilon_{S,p} - \varepsilon_{D,p}}$$

- How about price received by suppliers $p^* - t$?

$$\frac{\partial\left(p^* - t\right)}{\partial t} = \frac{\frac{\partial Y^S}{\partial p}}{\frac{\partial Y^S}{\partial p} - \frac{\partial X^D}{\partial p}} - 1 =$$

$$= \frac{\varepsilon_{D,p}}{\varepsilon_{S,p} - \varepsilon_{D,p}}$$

- *Inflexible Supply.* (Capacity is fixed) Supply curve vertical ($\varepsilon_{S,p} = 0$)

- Producers bear burden of tax

- *Flexible Supply.* (Constant Returns to Scale) Supply curve horizontal ($\varepsilon_{S,p} \to \infty$)

- Consumers bear burden of tax

- *Inflexible demand.* Demand curve vertical ($\varepsilon_{D,p} = 0$)?

- Consumers bear burden

- General lesson: Least elastic side bears larger part of burden

- What happens with a subsidy ($t < 0$)?

- What happens to quantity sold?

- Use demand curve:

$$\frac{\partial X^{D*}}{\partial t} = \frac{\partial X^{D*}}{\partial p^*} \frac{\partial p^*}{\partial t}$$

  and use expression for $\partial p^* / \partial t$ above.

5 Next Lecture

- Market Equilibrium in the Long-Run
- Consumer and Producer Surplus
- Market Power
- Monopoly
- Price Discrimination