Economics 101A
(Lecture 13)

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Outline

1. Mid-Term Feedback
2. Investment in Risky Asset II
3. Measures of Risk Aversion
4. Time Consistency
5. Time Inconsistency

1 Mid-Term Feedback

- Thanks for the feedback!

2 Investment in Risk Asset II

- Individual maximization:

$$\max_{\alpha}\,(1-p)\,u\,(w\,[(1-\alpha)+\alpha\,(1+r_{-})])+$$
$$+pu\,(w\,[(1-\alpha)+\alpha\,(1+r_{+})])$$
$$s.t. 0\leq\alpha\leq 1$$

- Case of risk aversion: $u''<0$

- Assume $0\leq\alpha^{*}\leq 1$, check later

- First order conditions:

$$\begin{aligned} 0 &= (1-p)\,(wr_{-})\,u'\,(w\,[1+\alpha r_{-}])+ \\ &\quad +p\,(wr_{+})\,u'\,(w\,[1+\alpha r_{+}]) \end{aligned}$$

- Solution is $\alpha^{*}>0$ (positive investment in stock)

- Exercise: Check s.o.c.

3 Measures of Risk Aversion

- Nicholson, Ch. 18, pp. 541–545 [OLD: Ch. 8, pp. 207–210].

- How risk averse is an individual?

- Two measures:

  - Absolute Risk Aversion r_A:

$$r_A = -\frac{u''(x)}{u'(x)}$$

  - Relative Risk Aversion r_R:

$$r_R = -\frac{u''(x)}{u'(x)}x$$

- Examples in the Problem Set

4 Time consistency

- Intertemporal choice

- Three periods, $t = 0$, $t = 1$, and $t = 2$

- At each period i, agents:

  - have income $M_i' = M_i +$savings/debts from previous period

  - choose consumption c_i;

  - can save/borrow $M_i' - c_i$

  - no borrowing in last period: at $t = 2$ $M_2' = c_2$

- Utility function at $t = 0$

$$u(c_0, c_1, c_2) = U(c_0) + \frac{1}{1+\delta} EU(c_1) + \frac{1}{(1+\delta)^2} EU(c_2)$$

- Utility function at $t = 1$

$$u(c_1, c_2) = U(c_1) + \frac{1}{1+\delta} EU(c_2)$$

- Utility function at $t = 2$

$$u(c_2) = U(c_2)$$

- $U' > 0$, $U'' < 0$

- Question: Do preferences of agent in period 0 agree with preferences of agent in period 1?

- **Period 1**.

- Budget constraint at $t = 1$:

$$c_1 + \frac{1}{1+r}c_2 \leq M_1' + \frac{1}{1+r}M_2$$

- Maximization problem:

$$\max U(c_1) + \frac{1}{1+\delta}EU(c_2)$$
$$s.t.\ c_1 + \frac{1}{1+r}c_2 \leq M_1' + \frac{1}{1+r}M_2$$

- First order conditions:

- Ratio of f.o.c.s:

$$\frac{U'(c_1)}{EU'(c_2)} = \frac{1+r}{1+\delta}$$

- Back to **period 0**.

- Agent at time 0 can commit to consumption at time 1 as function of uncertain income M_1.

- Anticipated budget constraint at $t = 1$:

$$c_1 + \frac{1}{1+r}c_2 \leq M_1' + \frac{1}{1+r}M_2$$

- Maximization problem:

$$\max U(c_0) + \frac{1}{1+\delta}U(c_1) + \frac{1}{(1+\delta)^2}EU(c_2)$$
$$s.t.\ c_1 + \frac{1}{1+r}c_2 \leq M_1' + \frac{1}{1+r}M_2$$

- First order conditions:

- Ratio of f.o.c.s:

$$\frac{U'(c_1)}{EU'(c_2)} = \frac{1+r}{1+\delta}$$

- The two conditions coincide!

- **Time consistency.** Plans for future coincide with future actions.

- To see why, rewrite utility function $u(c_0, c_1, c_2)$:

$$\begin{aligned} & U(c_0) + \frac{1}{1+\delta} U(c_1) + \frac{1}{(1+\delta)^2} EU(c_2) \\ = \; & U(c_0) + \frac{1}{1+\delta} \left[U(c_1) + \frac{1}{1+\delta} EU(c_2) \right] \end{aligned}$$

- Expression in brackets coincides with utility at $t = 1$

- Is time consistency right?
  - addictive products (alcohol, drugs);
  - good actions (exercising, helping friends);
  - immediate gratification (shopping, credit card borrowing)

5 Time Inconsistency

- Alternative specification (Akerlof, 1991; Laibson, 1997; O'Donoghue and Rabin, 1999)

- Utility at time t is $u(c_t, c_{t+1}, c_{t+2})$:

$$u(c_t) + \frac{\beta}{1+\delta}u(c_{t+1}) + \frac{\beta}{(1+\delta)^2}u(c_{t+2}) + ...$$

- Discount factor is

$$1, \frac{\beta}{1+\delta}, \frac{\beta}{(1+\delta)^2}, \frac{\beta}{(1+\delta)^3}, ...$$

  instead of

$$1, \frac{1}{1+\delta}, \frac{1}{(1+\delta)^2}, \frac{1}{(1+\delta)^3}, ...$$

- What is the difference?

- *Immediate gratification:* $\beta < 1$

- Back to our problem: **Period 1**.

- Maximization problem:

$$\max U(c_1) + \frac{\beta}{1+\delta} EU(c_2)$$
$$s.t.\ c_1 + \frac{1}{1+r} c_2 \leq M_1' + \frac{1}{1+r} M_2$$

- First order conditions:

- Ratio of f.o.c.s:

$$\frac{U'(c_1^*)}{EU'(c_2^*)} = \beta \frac{1+r}{1+\delta}$$

- Now, **period 0** with commitment.

- Maximization problem:

$$\max U(c_0) + \frac{\beta}{1+\delta}U(c_1) + \frac{\beta}{(1+\delta)^2}EU(c_2)$$
$$s.t.\ c_1 + \frac{1}{1+r}c_2 \leq M_1' + \frac{1}{1+r}M_2$$

- First order conditions:

- Ratio of f.o.c.s:

$$\frac{U'(c_1^{*,c})}{EU'(c_2^{*,c})} = \frac{1+r}{1+\delta}$$

- The two conditions differ!

- Time inconsistency: $c_1^{*,c} < c_1^*$ and $c_2^{*,c} > c_2^*$

- The agent allows him/herself too much immediate consumption and saves too little

- Ok, we agree. but should we study this as economists?

- YES!

  - One trillion dollars in credit card debt;

  - Most debt is in teaser rates;

  - Two thirds of Americans are overweight or obese;

  - $10bn health-club industry

- Is this testable?

  - In the laboratory?

  - In the field?

6 Next Lecture

- An Example: Health club Attendance

- Cost Minimization

- Solve an Example

- Cases in which s.o.c. are not satisfied

- Start Profit Maximization