

Economics 101A

(Lecture 10)

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Outline

1. Application 2: Intertemporal choice
2. Application 3: Altruism and Charitable Donations

1 Intertemporal choice

- Nicholson Ch. 17, pp. 502–506 [OLD: Ch. 23, pp. 628–632.]
- So far, we assumed people live for one period only
- Now assume that people live for two periods:
 - $t = 0$ – people are young
 - $t = 1$ – people are old
- $t = 0$: income M_0 , consumption c_0 at price $p_0 = 1$
- $t = 1$: income $M_1 > M_0$, consumption c_1 at price $p_1 = 1$
- Credit market available: can lend or borrow at interest rate r

- Budget constraint in period 1?
- Sources of income:
 - M_1
 - $(M_0 - c_0) * (1 + r)$ (this can be negative)
- Budget constraint:

$$c_1 \leq M_1 + (M_0 - c_0) * (1 + r)$$

or

$$c_0 + \frac{1}{1 + r} c_1 \leq M_0 + \frac{1}{1 + r} M_1$$

- Utility function?

- Assume

$$u(c_0, c_1) = U(c_0) + \frac{1}{1 + \delta} U(c_1)$$

- $U' > 0, U'' < 0$

- δ is the discount rate

- Higher δ means higher impatience

- Elicitation of δ through hypothetical questions

- Person is indifferent between 1 hour of TV today and $1 + \delta$ hours of TV next period

- Maximization problem:

$$\begin{aligned} \max U(c_0) + \frac{1}{1 + \delta} U(c_1) \\ \text{s.t. } c_0 + \frac{1}{1 + r} c_1 \leq M_0 + \frac{1}{1 + r} M_1 \end{aligned}$$

- Lagrangean

- First order conditions:

- Ratio of f.o.c.s:

$$\frac{U'(c_0)}{U'(c_1)} = \frac{1 + r}{1 + \delta}$$

- Case $r = \delta$

- $c_0^* = c_1^*$?

- Substitute into budget constraint using $c_0^* = c_1^* = c^*$:

$$\frac{2+r}{1+r}c^* = \left[M_0 + \frac{1}{1+r}M_1 \right]$$

or

$$c^* = \frac{1+r}{2+r}M_0 + \frac{1}{2+r}M_1$$

- We solved problem virtually without any assumption on U !

- Notice: $M_0 < c^* < M_1$

- Case $r > \delta$

- $c_0^* = c_1^*$?

- Comparative statics with respect to income M_0

- Rewrite ratio of f.o.c.s as

$$U'(c_0) - \frac{1+r}{1+\delta}U'(c_1) = 0$$

- Substitute c_1 in using $c_1 = M_1 + (M_0 - c_0)(1+r)$ to get

$$U'(c_0) - \frac{1+r}{1+\delta}U'(M_1 + (M_0 - c_0)(1+r)) = 0$$

- Apply implicit function theorem:

$$\frac{\partial c_0^*(r, \mathbf{M})}{\partial M_0} = -\frac{-\frac{1+r}{1+\delta}U''(c_1)(1+r)}{U''(c_0) - \frac{1+r}{1+\delta}U''(c_1) * (-(1+r))}$$

- Denominator is always negative
- Numerator is positive
- $\partial c_0^*(r, \mathbf{M}) / \partial M_0 > 0$ — consumption at time 0 is a normal good.
- Can also show $\partial c_0^*(r, \mathbf{M}) / \partial M_1 > 0$

- Comparative statics with respect to interest rate r
- Apply implicit function theorem:

$$\frac{\partial c_0^*(r, \mathbf{M})}{\partial r} = \frac{-\frac{1}{1+\delta}U'(c_1)}{U''(c_0) - \frac{1+r}{1+\delta}U''(c_1) * (-(1+r))} \\ \frac{-\frac{1+r}{1+\delta}U''(c_1) * (M_0 - c_0)}{U''(c_0) - \frac{1+r}{1+\delta}U''(c_1) * (-(1+r))}$$

- Denominator is always negative
- Numerator: First term negative (substitution eff.)
- Numerator: Second term (income effect:)
 - positive if $M_0 > c_0$
 - negative if $M_0 < c_0$.

2 Altruism and Charitable Donations

- Maximize utility = satisfy self-interest?
- No, not necessarily
- 2-person economy:
 - Mark has income M_M and consumes c_M
 - Wendy has income M_W and consumes c_W
- One good: c , with price $p = 1$

- Utility function: $u(c)$, with $u' > 0$, $u'' < 0$
- Wendy is altruistic: she maximizes $u(c_W) + \alpha u(c_M)$ with $\alpha > 0$
- Mark simply maximizes $u(c_M)$
- Wendy can give a donation of income D to Mark.

- Wendy computes the utility of Mark as a function of the donation D

- Mark maximizes

$$\begin{aligned} \max_{c_M} u(c_M) \\ \text{s.t. } c_M \leq M_M + D \end{aligned}$$

- Solution: $c_M^* = M_M + D$

- Wendy maximizes

$$\begin{aligned} \max_{c_M, D} u(c_W) + \alpha u(M_M + D) \\ \text{s.t. } c_W \leq M_W - D \end{aligned}$$

- Rewrite as:

$$\max_D u(M_W - D) + \alpha u(M_M + D)$$

- First order condition:

$$-u'(M_W - D^*) + \alpha u'(M_M + D^*) = 0$$

- Second order conditions:

$$u''(M_W - D^*) + \alpha u''(M_M + D^*) < 0$$

- Assume $\alpha = 1$.

- Solution?

- $u'(M_W - D) = u'(M_M + D^*)$

- $M_W - D^* = M_M + D^*$ or $D^* = (M_W - M_M) / 2$

- Transfer money so as to equate incomes!

- Careful: $D < 0$ (negative donation!) if $M_M > M_W$

- Corrected maximization:

$$\begin{aligned} \max_D & u(M_W - D) + \alpha u(M_M + D) \\ \text{s.t. } & D \geq 0 \end{aligned}$$

- Solution ($\alpha = 1$):

$$D^* = \begin{cases} (M_W - M_M) / 2 & \text{if } M_W - M_M > 0 \\ 0 & \text{otherwise} \end{cases}$$

- Assume interior solution. ($D^* > 0$)

- Comparative statics 1 (altruism):

$$\frac{\partial D^*}{\partial \alpha} = -\frac{u'(M_M + D^*)}{u''(M_W - D^*) + \alpha u''(M_M + D^*)} > 0$$

- Comparative statics 2 (income of donor):

$$\frac{\partial D^*}{\partial M_W} = -\frac{-u''(M_W + D^*)}{u''(M_W - D^*) + \alpha u''(M_M + D^*)} > 0$$

- Comparative statics 3 (income of recipient):

$$\frac{\partial D^*}{\partial M_M} = -\frac{\alpha u''(M_M + D^*)}{u''(M_W - D^*) + \alpha u''(M_M + D^*)} < 0$$

3 Next Lectures

- Introduction to Probability
- Risk Aversion
- Coefficient of risk aversion
- Applications:
 - Insurance
 - Portfolio choice