Economics 101A
(Lecture 7)

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September 19, 2006

Outline

1. Utility maximization II
2. Utility maximization – Tricky Cases
3. Indirect Utility Function
4. Comparative Statics (Introduction)
5. Income Changes
6. Price Changes

1 Utility Maximization

- Example with CES utility function.

$$\max_{x_1, x_2} \left(\alpha x_1^\rho + \beta x_2^\rho\right)^{1/\rho}$$
$$s.t.\ p_1 x_1 + p_2 x_2 - M = 0$$

- Lagrangean =

- F.o.c.:

- Special case: $\rho = 0$ (Cobb-Douglas)

2 Utility maximization – tricky cases

1. Non-convex preferences. Example:

2. Example with CES utility function.

$$\max_{x_1,x_2} \left(\alpha x_1^\rho + \beta x_2^\rho\right)^{1/\rho}$$
$$s.t.\ p_1x_1 + p_2x_2 - M = 0$$

- With $\rho > 1$ the interior solution is a minimum!

- Draw indifference curves for $\rho = 1$ (boundary case) and $\rho = 2$

- Can also check using second order conditions

2. Solution does not satisfy $x_1^* > 0$ or $x_2^* > 0$. Example:

$$\max x_1 * (x_2 + 5)$$
$$s.t.\ p_1 x_1 + p_2 x_2 = M$$

- In this case consider corner conditions: what happens for $x_1^* = 0$? And $x_2^* = 0$?

3. Multiplicity of solutions. Example:

- Convex preferences that are not strictly convex

3 Indirect utility function

- Nicholson, Ch. 4, pp. 106–108 [OLD: 103–105]

- Define the indirect utility $v(\mathbf{p}, M) \equiv u(\mathbf{x}^*(\mathbf{p}, M))$, with $\mathbf{p}$ vector of prices and $\mathbf{x}^*$ vector of optimal solutions.

- $v(\mathbf{p}, M)$ is the utility at the optimimum for prices $\mathbf{p}$ and income M

- Some comparative statics: $\partial v(\mathbf{p}, M)/\partial M =$?

- Hint: Use Envelope Theorem on Lagrangean function

- What is the sign of λ?

- $\lambda = u'_{x_i}/p > 0$

- $\partial v(\mathbf{p}, M)/\partial p_i =$?

- Properties:

  – Indirect utility is always increasing in income M

  – Indirect utility is always decreasing in the price p_i

4 Comparative Statics (introduction)

- Nicholson, Ch. 5, pp. 121–131 [OLD: 116–128]

- Utility maximization yields $x_i^* = x_i^*(p_1, p_2, M)$

- Quantity consumed as a function of income and price

- What happens to quantity consumed x_i^* as prices or income varies?

- Simple case: Equal increase in prices and income.
- $M' = tM$, $p_1' = tp_1$, $p_2' = tp_2$.
- Compare $x^*(tM, tp_1, tp_2)$ and $x^*(M, p_1, p_2)$.
- What happens?
- Write budget line: $tp_1x_1 + tp_2x_2 = tM$
- Demand is homogeneous of degree 0 in $\mathbf{p}$ and M:

  $x^*(tM, tp_1, tp_2) = t^0x^*(M, p_1, p_2) = x^*(M, p_1, p_2)$.

- Consider Cobb-Douglas Case:

$$x_1^* = \frac{\alpha}{\alpha + \beta} M/p_1, x_2^* = \frac{\beta}{\alpha + \beta} M/p_2$$

- What is $\partial x^*/\partial M$?

- What is $\partial x^*/\partial p_x$?

- What is $\partial x^*/\partial p_y$?

- General results?

5 Income changes

- Income increases from M to to $M' > M$.

- Budget line $(p_1 x_1 + p_2 x_2 = M)$ shifts out:

$$x_2 = \frac{M'}{p_2} - x_1 \frac{p_1}{p_2}$$

- New optimum?

- Engel curve: $x_i^*(M)$: demand for good i as function of income M holding fixed prices p_1, p_2

- Does x_i^* increase with M?
  - Yes. Good i is *normal*
  - No. Good i is *inferior*

6 Price changes

- Price of good i increases from p_i to to $p'_i > p_i$

- For example, decrease in price of good 2, $p'_2 < p_2$

- Budget line tilts:

$$x_2 = \frac{M}{p'_2} - x_1 \frac{p_1}{p'_2}$$

- New optimum?

- Demand curve: $x_i^*(p_i)$: demand for good i as function of own price holding fixed p_j and M

- Odd convention of economists: plot price p_i on vertical axis and quantity x_i on horizontal axis. Better get used to it!

- Does x_i^* decrease with p_i?
  - Yes. Most casesfs
  - No. Good i is *Giffen*
  - Ex.: Potatoes in Ireland
  - Do not confuse with Veblen effect for luxury goods or informational asimmetries: these effects are real, but not included in current model

7 Next Class

- More comparative statics:
  - Intuition
  - Slutzky Equation
- Then moving on to applications:
  - Labor Supply
  - Intertemporal choice
  - Economics of Altruism