Economics 280C: Answer to Problem #1

(a) As a preliminary step it is useful to calculate the means of variables. Since shocks are i.i.d. here and there are no multiperiod sources of persistence, conditional means are actually *unconditional* (and will be denoted as such), which simplifies the analysis considerably. It is straightforward to check that, since m is constant, since $i^* = p^* = 0$, and since $\mathrm{E}v = \mathrm{E}\varepsilon = \mathrm{E}g = 0$,

$$\mathrm{E}e = \mathrm{E}p = w = m, \;\; \mathrm{E}y = 0.$$

By substituting $\mathrm{E}e = m$ into the interest parity relationship we derive

$$i_t = m - e_t + \varepsilon_t,$$

which may be plugged into the money-market equilibrium condition to give

$$m - p_t = y_t - \lambda(m - e_t) + \varphi_t,$$

where $\varphi_t \equiv v_t - \lambda\varepsilon_t$. Using the aggregate supply equation, which implies $y_t = \theta(p_t - m)$, to eliminate y_t, we get

$$(1+\theta)p_t = -\lambda e_t + (1+\theta+\lambda)m - u_t. \qquad (1)$$

Now equate aggregate demand and supply to infer that

$$(\theta+\delta)p_t = \delta e_t + \theta m + g_t. \qquad (2)$$

Equations (1) and (2) are two equations in the unknowns e and p. Solving, we find that

$$e_t = m - \frac{1+\theta}{\delta(1+\theta)+\lambda(\delta+\theta)}g_t - \frac{\theta+\delta}{\delta(1+\theta)+\lambda(\delta+\theta)}\varphi_t, \qquad (3)$$

$$p_t = m + \frac{\lambda}{\delta(1+\theta)+\lambda(\delta+\theta)}g_t - \frac{\delta}{\delta(1+\theta)+\lambda(\delta+\theta)}\varphi_t. \qquad (4)$$

From this last equation and the aggregate supply schedule we derive

$$y_t = \frac{\theta(\lambda g_t - \delta\varphi_t)}{\delta(1+\theta)+\lambda(\delta+\theta)}. \qquad (5)$$

(b) Since the mutual covariances of all the shocks are zero, $\sigma_\varphi^2 = \sigma_v^2 + \lambda^2\sigma_\varepsilon^2$ and, from eq. (5),

$$\sigma^2_{y|float} = \frac{\theta^2\lambda^2\sigma_g^2 + \theta^2\delta^2\sigma_\varphi^2}{[\delta(1+\theta)+\lambda(\delta+\theta)]^2}. \qquad (6)$$

(c) Setting aggregate demand and supply equal, one can show that with the exchange rate fixed at $\bar{e}$ and the money supply endogenous instead,

$$p = \bar{e} + \frac{g_t}{\theta+\delta},$$
$$y = \frac{\theta g_t}{\theta+\delta}. \qquad (7)$$

(d) The variance under a fixed exchange rate is, using eq. (7),

$$\sigma^2_{y|fix} = \frac{\theta^2\sigma_g^2}{(\theta+\delta)^2}. \qquad (8)$$

(e) From eq. (6), when $\sigma^2_\varphi = 0$,

$$\sigma^2_{y|float} = \frac{\theta^2\lambda^2\sigma_g^2}{[\delta(1+\theta)+\lambda(\delta+\theta)]^2} < \frac{\theta^2\sigma_g^2}{(\theta+\delta)^2} = \sigma^2_{y|fix}.$$

(f) Under the interest-rate rule, E$i = i^* = 0$ and E$p = 0$. For ex post values, we have

$$e_t = \frac{\theta+\delta}{\theta+\delta(1+\psi)}\varphi_t - \frac{\psi}{\theta+\delta(1+\psi)}g_t,$$

$$p_t = \frac{\delta}{\theta+\delta(1+\psi)}\varphi_t + \frac{1}{\theta+\delta(1+\psi)}g_t,$$

$$y_t = \frac{\theta(\delta\varphi_t + g_t)}{\theta+\delta(1+\psi)},$$

$$\sigma^2_{y|float} = \frac{\theta^2\delta^2\sigma^2_\varphi + \theta^2\sigma_g^2}{[\theta+\delta(1+\psi)]^2}.$$

(g) Under a (credibly) fixed exchange rate, the domestic interest rate is fixed at $i = i^* = 0$. Price and output are determined simply by the intersection of aggregate demand and supply, given $\bar{e}$, as in part (c) above, so eqs. (7) and (8) still apply. When $\sigma^2_\varphi = 0$, meaning that there are no interest-rate shocks,

$$\sigma^2_{y|float} = \frac{\theta^2\sigma_g^2}{[\theta+\delta(1+\psi)]^2} < \frac{\theta^2\sigma_g^2}{(\theta+\delta)^2} = \sigma^2_{y|fix}.$$

(In general, this could be the case too if the policy shocks u to the interest rate perfectly offset the portfolio preference shocks ε in the interest-parity relation. Of course, that would require an unrealistically high degree of knowledge of market sentiment by the central bank.) If financial shocks dominate (large σ^2_φ relative to σ^2_g), the variance inequality would be reversed, and more easily for small values of ψ. The v shocks in the LM curve are irrelevant now because the money supply is endogenous and moves to offset automatically shocks to the quantity of money demanded.