

Econ 240B – Fall 2001
Final Examination Solutions

1. Define $x=[1 \ p \ w \ a \ s]'$, $\beta=[\beta_1 \ \beta_2 \ \beta_3 \ \beta_4 \ \beta_5]'$, $\theta=[\beta' \ \sigma]'$

(a) $L(\theta)=d*\log\left[\Phi\left(\frac{x'\beta-f}{\sigma}\right)\right]+(1-d)*\log\left[1-\Phi\left(\frac{x'\beta-f}{\sigma}\right)\right]$

(b) Define $y_i = \text{Ind}\{i \in \text{sample of actual purchases}\}$. The LR statistic below is asymptotically distributed as an χ^2_6 under the null:

$$LR = 2 \left[\begin{array}{l} \max_{\beta_A, \sigma_A, \beta_H, \sigma_H} \left\{ \sum_{i=1}^{N+M} \left\{ d_i * \log \left[\Phi \left[\frac{y_i (x_i' \beta_A - f) + (1-y_i) (x_i' \beta_H - f)}{y_i \sigma_A + (1-y_i) \sigma_H} \right] \right\} + [1-d_i] * \log \left[1 - \Phi \left(\frac{y_i (x_i' \beta_A - f) + (1-y_i) (x_i' \beta_H - f)}{y_i \sigma_A + (1-y_i) \sigma_H} \right) \right] \right\} \right\} \\ - \max_{\beta, \sigma} \left\{ \sum_{i=1}^{N+M} d_i * \log \left[\Phi \left(\frac{x_i' \beta - f}{\sigma} \right) \right] + (1-d_i) * \log \left[1 - \Phi \left(\frac{x_i' \beta - f}{\sigma} \right) \right] \right\} \end{array} \right]$$

(c)

$$LR = 2 \left[\begin{array}{l} \max_{\beta, \sigma_A, \sigma_H} \left\{ \sum_{i=1}^{N+M} \left\{ d_i * \log \left[\Phi \left(\frac{x_i' \beta - f}{y_i \sigma_A + (1-y_i) \sigma_H} \right) \right] + [1-d_i] * \log \left[1 - \Phi \left(\frac{x_i' \beta - f}{y_i \sigma_A + (1-y_i) \sigma_H} \right) \right] \right\} \right\} \\ - \max_{\beta, \sigma} \left\{ \sum_{i=1}^{N+M} d_i * \log \left[\Phi \left(\frac{x_i' \beta - f}{\sigma} \right) \right] + (1-d_i) * \log \left[1 - \Phi \left(\frac{x_i' \beta - f}{\sigma} \right) \right] \right\} \end{array} \right] \xrightarrow{D} \chi^2_1$$

2. Using WESML approach:

$$\hat{\mu} = \frac{\sum_{y_i < \log(20K)} \frac{y_i}{9} + \sum_{y_i > \log(20K)} \frac{y_i}{6}}{\sum_{y_i < \log(20K)} \frac{1}{9} + \sum_{y_i > \log(20K)} \frac{1}{6}}$$

3.

(a) It will not be consistent as long as z is not clean. This condition does not depend on the true value of γ .

(b) Do the omitted variable version of the Hausman Test: Regress y on 1, x, z and $z^\#$ and test whether the coefficient of $z^\#$ is zero by means of a Wald test.

4.

(a) If $X_1=X_2=X$, then: $\hat{\epsilon}_1=Q\epsilon_1$, $\hat{\epsilon}_2=Q\epsilon_2$, $Q=I-X(X'X)^{-1}X'$ and $\text{tr}(E[\hat{\epsilon}_1\hat{\epsilon}_2']) = \text{tr}(Q)\sigma_{12} = (T-K)\sigma_{12} \neq 0$

(b) $\hat{\beta}_j = \beta_j - (X_j'X_j)^{-1}X_j'\epsilon_j$, $j = 1,2$. If $X_1'X_2 = 0$, then $E[(\hat{\beta}_1 - \beta_1)(\hat{\beta}_2 - \beta_2)'] = (X_1'X_1)^{-1}X_1'X_2(X_2'X_2)^{-1}\sigma_{12} = 0$

5.

(a) Let z_t be any random variable. Then the following must hold:

$\text{cov}(z_t, \hat{\epsilon}_{1t}) = \text{cov}(z_t, \epsilon_{1t}) - (\hat{\beta}_{12} - \beta_{12})\text{cov}(z_t, w_t)$. And, asymptotically, $\text{cov}(z_t, \hat{\epsilon}_{1t}) = \text{cov}(z_t, \epsilon_{1t})$. Thus, for $z_t = \epsilon_{2t}$: $\text{cov}(\epsilon_{2t}, \hat{\epsilon}_{1t}) = \text{cov}(\epsilon_{2t}, \epsilon_{1t}) = 0$; and for $z_t = w_t$: $\text{cov}(w_t, \hat{\epsilon}_{1t}) = \text{cov}(w_t, \epsilon_{1t}) \neq 0$.

(b) From (a) we can see that $\hat{\epsilon}_{1t}$ is a proper instrument for w_t , then supply equation can be estimated by 2SLS using 1, m_t , and $\hat{\epsilon}_{1t}$ as instruments.