

Microeconomic Analysis  
PROBLEM SET 7

1. For the following production functions calculate the marginal products, the technical rates of substitution and indicate whether they have increasing, decreasing or constant returns to scale:
  - a.  $x_1^{1/4} x_2^{3/4}$
  - b.  $x_1 + (x_2)^{1/2}$
  - c.  $(x_1^{1/3} + x_2^{1/3})^3$
2. Suppose a particular company has the production function  $Y = \min\{L, 2K\}$ .
  - a. Does this production function exhibit decreasing, increasing or constant returns to scale? Explain
  - b. Draw a few isoquants for this production function.  $L$  on the  $x$ -axis.
  - c. If the firm wants to produce ten units, calculate input demands? How do these demands depend on the relative prices of capital and labor in this case? Explain.
3. A firm has two variable factors and a production function  $f(x_1, x_2) = (2x_1 + 4x_2)^{1/2}$ .
  - a. On a graph with factor 1 on the  $x$ -axis, draw the production isoquants corresponding to an output of 3 and to an output of 4.
  - b. If the price of the output good is 4, the price of factor 1 is 2, and the price of factor 2 is 3, find the profit-maximizing amount of factor 1, of factor 2 and the profit-maximizing output.  
Hint: Pay close attention to the shape of the isoquants.
4. A firm produces a soft drink using two ingredients, sugar ( $S$ ) and bubbly water ( $B$ ) in fixed proportions: 6 tablespoons of sugar per 12 oz. of bubbly water.
  - a. What is the production function (Hint: Be careful, maybe a graph can be helpful)
  - b. Does this production function exhibit constant, increasing or decreasing returns to scale? Explain.
  - c. Write down the firm's cost minimization problem and solve for the conditional factor demands,  $S(w_S, w_B, y)$  and  $B(w_S, w_B, y)$ .
  - d. Solve for the long run cost function.