

Economics 100A
Fall 2001
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PROBLEM SET #2
(Due in section the week of September 10, 2001)

1 Univariate Calculus Review

The following two problems are intended to give students a brief review of univariate calculus. The first problem simply covers the Power Rule, Product Rule, Quotient Rule, and Chain Rule. The second problem is a common economic maximization problem applying differentiation.

1.1 Differentiation

Problem 1 Find the first derivative of the following functions:

- (a) $3x^2 - 9x + 7x^{2/5} - 3x^{1/2}$
- (b) $(x^{1/2} + x^{-1/2})(4x^5 - 3\sqrt{x})$
- (c) $\frac{e^{2x}}{(x^2 + 1)}$
- (d) $\ln(x + 3x + 1) + xe^{3-x}$

1.2 Maximization

Problem 2 Consider a firm's profit function, $\Pi(x) = R(x) - C(x)$, where $R(x)$ is total revenue as a function of output (x), and $C(x)$ is total cost as a function of output (x).

- (a) Under perfect competition, each firm is a price taker. Assuming a competitive market price, $p^* = 10$ and a cost function, $C(x) = (x - 5)^2$, express the firm's profit as a function of x .
- (b) Find the competitive firm's profit maximizing level of output, x^* (Hint: maximize the firm's profit by taking the first derivative of the profit function, setting it equal to zero, and solving for the level of output, x^*).
- (c) If the firm were only interested in minimizing costs, what level of output would it choose?

- (d) Under monopoly conditions, the firm is no longer a price taker. Rather, the firm now faces the entire market demand curve and sets the price to the level that maximizes its profit given the market demand curve. Suppose the market demand curve as a function of price is, $x = D(p) = 40 - 2p$. Write an equation for the profit of the firm as a function of p .
- (e) Solve for the monopoly firm's profit maximizing price, p^M . How much output, x^M , is supplied at the monopoly price? Assuming the same market demand curve as in (d), how much output, x^* , would be supplied to the market at the competitive price, $p^* = 10$?
- (f) If this example is representative of monopolistic versus competitive markets, what can you say, in general, about the relative prices and relative quantities of output available to consumers?

2 Consumer Choice

Problem 3 and Problem 4 cover budget constraints (Varian, Ch. 2) and Problem 5 addresses preferences (Varian, Ch. 3). This material serves as the underpinnings of utility theory (Varian, Ch. 4) which characterizes optimal consumer choice (Varian, Ch. 5) in a constrained setting.

2.1 Budget Constraints

Problem 3 *Billy Madison consumes 100 units of X and 50 units of Y. The price of X rises from 2 to 3. The price of Y remains at 4.*

- (a) How much must Billy's income rise so that he can exactly afford 100 units of X and 50 units of Y?
- (b) Draw Billy's original budget set when the price of X is 2.
- (c) Draw Billy's budget set when the price of X is 3, but his income is unchanged.
- (d) Draw Billy's budget set when the price of X is 3 and his income increases by the amount in (a) so that he can afford his original consumption bundle.

Problem 4 *If Veronica Vaughn spends all of her daily income on cigarettes and Yoo-Hoo, she can afford 10 packs of cigarettes and 10 bottles of Yoo-Hoo. She can also afford 6 packs of cigarettes and 22 bottles of Yoo-Hoo.*

- (a) What is the relative price of cigarettes in terms of bottles of Yoo-Hoo (i.e. what is the ratio of prices, $\frac{p_c}{p_y}$).

- (b) Exactly how much income does Veronica earn in one week if the price of cigarettes is \$6? Write a budget equation for Veronica that is a function of the packs of cigarettes, C , and the number of bottles of Yoo-Hoo, Y .
- (c) Draw Veronica's daily budget set with packs of cigarettes on the x -axis and bottles of Yoo-Hoo on the y -axis.
- (d) Now suppose the price of cigarettes falls by \$1 and the price of Yoo-Hoo rises by \$1. If Veronica was consuming 5 packs of cigarettes and 30 bottles of Yoo-Hoo prior to the price changes, how much must her income rise under the new prices in order for her to just afford the old bundle, $(5, 30)$? Draw this new budget line.

2.2 Preferences

Problem 5 *Assume, aside from yourself, that there are only two types of people in the world, Stanford students and UCLA students. Also assume that you, as a Berkeley student, dislike both. This suggests that from your perspective Stanford students and UCLA students are "bads."*

- (a) If both Stanford (S) students and UCLA (U) students are bads, will the indifference curves have a positive or negative slope? Draw some "smooth" indifference curves and indicate the direction of increasing preference. Label the number of Stanford students (S) on the x -axis and UCLA students (U) on the y -axis.
- (b) Now suppose your dislike for people can be characterized by $D = \max(S, 3U)$, where D is the level of dislike associated with a bundle of Stanford and UCLA students. In effect, your dislike is determined by the maximum of a particular type, with 3 UCLA students as tolerable as 1 Stanford student. Draw some indifference curves characterized by these preferences.
- (c) Assume now that you do not care one way or the other about being around UCLA students (i.e. you are neutral or indifferent). Draw some indifference curves associated with these preferences.