1 Simultaneous Equations

- Need to test if the instrumental variables have been successful. Test suggested by Hausman, J.A. (1983) "Specification and Estimation of Simultaneous Equation Models", in Griliches, Z. and Intriligator, M.D. (eds) *Handbook of Econometrics*, *Vol I*, North-Holland, Amsterdam.

- If we have an equation: $Y_i = a_1 + b_1 X_i + u_i$, where both Y and X are endogenous. Also that there is an instrument set Z_{Mi}, $M = 1...m$, such that $X_i = g_0 + g_1 Z_{1i} + ... + g_m Z_{mi} + v_i$. Once you have estimated:

$$Y_i = a_1 + b_1 \widehat{X}_i + u_i$$

take the residuals ($\widehat{u_i}$) and regress (by OLS):

$$\widehat{u_i} = h_0 + h_1 Z_{1i} + ... + h_m Z_{mi} + \eta_i$$

- Obtain the R^2 of the u on Z equation, multiply by the degrees of freedom $(n - k)$. Compare with the χ^2_{M-1} tabled value, where $M - 1$ are the number of over-identifying restrictions:

$$[R^2 * (n - k)] \sim \chi^2_{M-1}$$

- Angrist, J. and Evans, W. "Children and their Parents Labor Supply: Evidence from Exogenous Variation in Family Size", *American Economic Review*, June 1998.

- Note studies that show that parents have strong preference for mixed sex-sibling composition. Parents of same sex siblings are significantly and substantially more likely to have an additional child.

- As sex is virtually randomly assigned - a third child given the sex of the first two provides a plausible instrument.

- IF the instrument is truly random, you can use a simple instrumental variable estimator (called the Wald estimator):

$$\widehat{b} = \frac{(\overline{Y_1} - \overline{Y_0})}{(\overline{X_1} - \overline{X_0})}$$

where Y = variable of interest, X = the other endogenous variable, and Z is a binary indicator as instrument that shows if an individual was part of the 'treatment' ($Z = 1$) or 'control' ($Z = 0$) group.

- Note that it is assumed that no other co-variates play a role. If it is suspected that they do, have to use instrumental variables as set out above.