1 Heteroskedasticity

- Concerned with non-constant variance in the error term: $E(u_i) \neq \sigma^2$.
- Usually occurs when the data are cross-section (e.g. CPS on individuals, or Census of Manufactures on firms). Data has a scale effect in which you examine low to high income, or small to large employers.
- Example for today: employer size-wage effect (Brown and Medoff, Journal of Political Economy, 1989). Known empirical relationship that large employers pay more to equivalent workers.
- Consequences of heteroskedasticity:
  1. OLS estimators are still linear
  2. OLS estimators are still unbiased
  3. However, no longer have the minimum variance property
  4. Usual formulas to estimate variances are biased. Cannot tell if variances will be positive or negative.
  5. Formula: $\widehat{\sigma}^2_{YX} = \frac{\sum e^2}{n-k}$ is not an unbiased estimator of σ^2. Note that this entails that the standard errors are biased.
  6. Result: confidence intervals and hypothesis tests based on the t and F distributions are unreliable. Possibility exists of drawing wrong conclusion on hypothesis tests.
- Intuitively: under OLS each e^2 receives the same weight, whether it comes from a population with a large or small variance.
- Ideally we would like to give more weight to observations from populations with smaller variances than those from populations with large variances. This is the essence of **weighted least squares**.
- How to detect heteroskedasticity: **Glejser test** (Journal of the American Statistical Association, 1969):
  1. Run original model: $Y_i = a + bX_i + e_i$ and obtain the residuals: e_i.
  2. Use the absolute values: $|e_i|$.
  3. Regress the absolute error values onto transformations of the independent variable(s). For example: $|e_i| = g_0 + g_1 X_i + v_i$, or $|e_i| = g_0 + g_1\sqrt{X_i} + v_i$, or $|e_i| = g_0 + g_1\frac{1}{X_i} + v_i$.
  4. Test $H_0 : g_1 = 0$. If rejected, then a high probability of heterskedasticity.
- If heteroskedasticity detected, then re-estimate model using **weighted least squares**.

- Lets assume that the error variance is proportional to $X_i^2 : E(e_i)^2 = \sigma^2 X_i^2$.
- Transform the original model: $\frac{Y_i}{X_i} = a\frac{1}{X_i} + b + \frac{e_i}{X_i}$ so that: $\frac{Y_i}{X_i} = a\frac{1}{X_i} + b + \frac{e_i}{X_i}$ and $\upsilon_i = \frac{e_i}{X_i}$.
- Constant variance because: $E(\upsilon_i^2) = \frac{E(e_i^2)}{X_i^2} = \sigma^2 \frac{X_i^2}{X_i^2} = \sigma^2$.