EFFICIENT DIVISION OF PROFITS FROM COMPLEMENTARY INNOVATIONS

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Abstract

Many products—including microprocessors, telecommunications devices, and on-line auction services—make use of multiple technologies, each of which is essential to make or sell the product. This paper examines how alternative intellectual property regimes and legal institutions affect investment in the R&D necessary to develop these complex technologies. We posit several reasonable properties for an innovation reward scheme and show that any scheme possessing all of these properties cannot support the first-best R&D levels. We show that it is feasible to support efficient investment as a private equilibrium if one or more of these properties are relaxed. We also analyze the effects of a simple regime in which payoffs are proportional to patent holdings, and the regime that arises when any one intellectual property owner can obtain an injunction to block sale of the product.

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I. INTRODUCTION

Many technologies are valuable only when used together, and a firm may be unable lawfully to offer the product enabled by these technologies unless it obtains rights to utilize all of them. In 2003, for example, eBay was ordered to pay $29.5 million in damages to MercExchange after a jury had found that eBay infringed two MercExchange's patents, including one for the technology underlying eBay's popular "Buy It Now" feature.[2] In 2002, NTP, a company whose primary assets were patents and an equity stake in a mobile startup company, sued Research In Motion (RIM), the manufacturer of the Blackberry mobile email device, for infringing several of NTP patents covering wireless electronic mail systems. RIM eventually settled the NTP lawsuit with a payment to NTP of $612.5 million. In another high profile patent case, Intel reached a $525 million settlement of a suit alleging that Intel's Pentium family of microprocessors infringed Intergraph's patents.[3] Intergraph obtained other multi-million dollar patent infringement settlements and reported a total of approximately $865 million of pre-tax income before expenses for 2002 through January 2005 from its IP licensing and enforcement efforts.[4]

eBay's auction service, RIM's Blackberry Device, and Intel's microprocessors are examples of products that require many complementary technologies. Given the prevalence of

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[2] Mercexchange, L.L.C. v. eBAY, INC. and Half.com, Inc., Civil Action No. 2:01cv736, U.S. Dist. Ct. for the Eastern Dist. of VA, 275 F. Supp. 2d 695 (August 6, 2003).

[3] Intel and Intergraph reached a partial settlement worth $300 million in April 2002 and a final settlement worth $225 million in March 2004. See Intel News Release at http://www.intel.com/pressroom/archive/releases/20020416corp.htm and http://www.intel.com/pressroom/archive/releases/20040413corp_a.htm, visited September 26, 2006.

[4] Intergraph Corporation Annual Report for the fiscal year ended December 31, 2004.

complementary innovations in information technology, biotech, and other industries, an important question is whether the relevant public policies encourage efficient investment in the research and development (R&D) that generates complementary technologies. Below, we derive a function that divides product-market profits among innovators in a way that motivates efficient R&D investment when the R&D generates technologies that are valuable only when used with other technologies. The form of this R&D reward function can provide guidance to Congress and the courts in developing rules for awarding damages in patent and copyright infringement litigation, and to private organizations such as patent pools that must develop procedures to allocate licensing royalties to members of the pool.

Intuitively, one might expect that damage awards and settlements in infringement litigation and royalty shares in patent pools should correspond to the relative importance of the intellectual property that has been infringed or contributed to a pool. It can be extremely difficult, however, to assess the relative importance of different pieces of intellectual property that are components of a complex product. Consider the infringement litigation brought by NTP against RIM that led to the $600+ million settlement. Some have accused NTP of acting as a "patent troll" which hid in the thicket of intellectual property rights, waiting to pounce on the unsuspecting manufacturer of the Blackberry device to demand a huge (unfair) payment or face an injunction against the use of its technology.[5] A different perspective is that NTP owns technologies that are essential to use the Blackberry service, and consequently NTP has as great a claim as RIM to the profits from the Blackberry device. A striking application of this logic is the

[5] See *e.g.,* Alan Murray, "War on 'Patent Trolls' May Be Wrong Battle," *Wall Street Journal*, March 22, 2006.

recent jury verdict that awarded Lucent-Alcatel $1.5 billion for infringement of two patents held by Lucent-Alcatel relating to MP3 digital audio technology.[6]

Below, we develop a formal analysis of a market in which two firms conduct R&D to develop several technologies, all of which are necessary to offer a new product. To sharpen our focus, we examine the polar case of perfect complements, in which each technology is as important as any other. As we will show, even in this case it does not follow that all innovations should be rewarded equally.

Section II sets out our analytical framework and, as a baseline for further analysis, characterizes the socially optimal and profit-maximizing monopoly R&D levels. As expected, a profit-maximizing monopoly internalizes the complementarities that exist among the technologies required to produce a complex product, but the firm underinvests in R&D because the private value of the innovation is less than its social value by the amount of consumer surplus generated by the new product.

The remainder of the paper examines the case of primary interest: a duopoly, in which each firm invests to build a portfolio of patents. Section III identifies several intuitively desirable properties for an innovation reward scheme (*i.e.,* the mapping from the results of R&D to the payoffs awarded to the firms that conducted the R&D). For example, the reward scheme must be self-financing in that the firms' payoffs must sum to the resulting level of product-market profits; there are no government subsidies. We show that there is no scheme that can satisfy these properties and support first-best R&D levels. We also show, however, that it is feasible to

[6] Blackburn and Lopez, 2007. Many patents necessary to implement MP3 technology can be licensed as a package from the mp3 licensing group, but not the patents at issue owned by Lucent-Alcatel.

support efficient investment as a private equilibrium if one or more of these properties are relaxed.

The economic forces at work are fairly intuitive even if their specific interactions are not. Each duopoly innovator fails to account for the positive benefit a discovery has on the value of technologies controlled by the other innovator or on consumer surplus, which suggests too little incentive to invest in R&D for a technology that complements other technologies. In the other direction, it is well known that pecuniary externalities in the form of business-stealing effects (*e.g.,* in patent races) may generate private R&D incentives that exceed the social incentives.[7] Consequently, an optimal innovation reward scheme can be structured to use the overinvestment from racing to offset the underinvestment due to uninternalized complementarities.

One issue with respect to the innovation reward schemes characterized in Section III is whether they would be readily implementable. Section IV examines two regimes that would be. First, under a regime of equal profit shares per innovation, each firm's profit share is proportional to the number of technologies it controls that are necessary to produce the final product. Given that, ex post, each technology is as important as any other, this rule has an intuitive appeal, and is used in some patent pools to allocate licensing royalties.[8] Analysis shows, however, that this reward scheme generally leads to excessive R&D investment to obtain the last technology because firms engage in racing. Numerical simulations indicate that this result also holds for earlier innovations.

The other reward scheme arguably corresponds to the existing intellectual property rights regime in which an intellectual property owner can obtain an injunction to bar the use of its

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[7] See, for example, the analyses of patent races surveyed by Reinganum (1989).

intellectual property without a license. When each technology is essential, any intellectual property owner can block the sale of the final product. The litigation between NTP and RIM, for example, threatened to shut down RIM's widely used Blackberry service[9] Under Nash bargaining, the firms will split the value of the product equally as long as each firm has at least one patent.

The resulting equal-profit-shares-per-innovator regime scheme generally does not generate efficient incentives for R&D. If each firm holds at least one patent, the incentive to invest to discover another technology is low because of a free-riding problem—all existing rights holders benefit equally from discovery of the remaining essential technologies. On the other hand, if a firm currently has no patents, its incentive to discover another technology can be inefficiently large because successful discovery yields a reward of one half of the final product's total value even when the technology is only one of a very large number needed to produce the final product. Similarly, a firm that has to date made all of the discoveries has a strong incentive to preempt a second firm from successfully obtaining a patent because, if the second firm won a patent, then the incumbent's share of the total product profits would fall from 100 percent to 50 percent. Due to these effects running in different directions, a reward regime of equal profit shares for each innovator can lead to too little or too much R&D investment. As expected, computer simulations suggest that this reward scheme leads to too little incentive for R&D when the social value of an innovation is large relative to its private value, holding other parameters constant.

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[8] Layne-Farrar and Lerner (2006).

[9] Similarly, recent litigation between Verizon and Vonnage threatened to shut down Vonnage's voice over IP telephone service.

Before presenting our analysis, it is useful to put it in context. Several authors have examined industry settings in which firms engage in a sequence of races to obtain the intellectual property rights to technologies that are non-infringing substitutes for one another.[10] In contrast, we are interested in complementary intellectual property in situations where any one intellectual property owner can block the sale of the final good embodying the complementary technologies. The analysis of the present paper is more closely related to studies of cumulative innovation, such as Green and Scotchmer (1995), Matutes et al. (1996), Scotchmer (1996), O'Donoghue (1998), and Denicolò (2000), which allow for infringing, complementary technologies. However, research on cumulative innovation has focused on the interdependence of the rewards for a basic invention, which has standalone value, and the rewards for complementary inventions that follow and build on the basic invention. In the present analysis, there is no sense in which one of the inventions is basic and the others follow-on; all are required to generate any value.

Like us, Lemley and Shapiro (2006) examine settings in which a single product may infringe several different patents. Their focus is on identifying the effects of injunctions and the judicial determination of "reasonable royalties" on the total licensing fees paid by product suppliers to rights holders for use of their intellectual property. Our focus, in contrast, is on characterizing the optimal division of profits among innovators in order to promote efficient R&D investment.

Lastly, Dequiedt and Versaevel (2006) analyze the effects of reward policies for a patent pool on incentives for firms to invest in R&D. Their paper is related to ours in that it studies the effects of patent rewards on R&D investment for complementary innovations. There are,

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[10] Examples include Reinganum (1985), Vickers (1986), Choi (1991), and Doraszelski (2003).

however, important differences. Rather than characterize the optimal reward scheme, Dequiedt and Versaevel (2006) assume a particular reward structure and examine its effects. They make this simplification because it allows them to study dynamic issues that can arise in the presence of patent pools but do not arise in our model. Specifically, in their model, firms race to become one of the founders of the pool and the threshold size of the pool affects the incentives to invest in R&D.

II. COMPLEMENTARY INNOVATIONS

In this section, we describe the model and establish the socially optimal outcome as a benchmark. We are interested in situations where various technologies are worth more when used together than when used separately. Formally, we examine the polar case of perfect complements: there are L technologies that must be used together in order create either private or social value.

Assumption A.1: *If all L technologies have been invented, they generate social value w and private value to the innovators,* π*, with* $w > \pi$*. If fewer than L technologies have been invented, the social and private benefits are zero.*

In what follows, we will as a shorthand refer to π as the profits derived from the final product that utilizes the L technologies.[11]

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[11] The gap between π and w could be surplus enjoyed by both consumers and producers of the final product if the latter are not themselves the innovators.

As in Loury (1979) and Dasgupta and Stiglitz (1980), a firm chooses how many R&D projects to undertake and pays a lump sum of c for each project chosen. We assume a standard functional form for innovation:

Assumption A.2: *Innovation for each technology follows an independent Poisson process. If there are n active R&D projects directed to a technology, the probability that the technology will be discovered before time t is* $1-e^{-nht}$*, with h > 0.*

Our next assumption greatly simplifies the analysis. Together with A.2, the next assumption allows us to focus on strategies whereby a firm chooses its R&D expenditure conditional solely on the number of innovations completed to date and the distribution of the intellectual property rights for those innovations:

Assumption A.3: *The L technologies must be invented sequentially, and R&D projects are specific to each technology.*[12]

Next, consider socially optimal R&D investment. Let $W(K)$ denote the expected continuation social value of R&D assuming that K technologies have been discovered. Then $W(L)=w$, and for $K \in \{0,1,2,\ldots,L-1\}$

$$W(K)=\max_n \int_0^\infty \left[nhW(K+1)\right]e^{-(r+nh)t}\,dt - nc = \max_n \left[\frac{nhW(K+1)}{nh+r} - nc\right]. \qquad (1)$$

Let $n^w(K)$ denote the socially optimal number of R&D projects when K technologies

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[12] Assumption 3 can be replaced with the assumption that R&D projects are not targeted to a particular technology. Under this alternative assumption, the optimization program in equation (1) must be modified by the addition of a constraint that the number of R&D projects undertaken must be non-decreasing in the number of technologies that have been discovered to date. As shown below, this constraint is not binding and the solution derived in the text under Assumption 3 is also valid under the alternative assumption.

have been discovered. We treat n^w as a continuous variable. From (1) it follows that

$$n^w(K) = \max\left\{\frac{r}{h}\left[\left(\frac{hW(K+1)}{rc}\right)^{1/2} - 1\right], 0\right\} . \quad (2)$$

Suppose $n^w(K) > 0$ for all *K*. Then iteratively substituting equation (2) into (1) for $K = L-1, L-2, \ldots, 0$ with $W(L) = w$ yields

$$W(K) = w[1-(L-K)/\alpha]^2 \quad \text{for } K = 0,1,\ldots,L \ , \quad (3)$$

where

$$\alpha \equiv \left(\frac{hw}{rc}\right)^{1/2} .$$

Intuitively, $\frac{w}{r}$ is a measure of the benefit of more rapid innovation and $\frac{c}{h}$ is a measure of the cost. Hence, α can be interpreted as the benefit of faster innovation relative to the cost. Substituting equation (3) into equation (2) with $K = 0$ shows that $\alpha \geq L$ is necessary and sufficient for $n^w(K) > 0$ for all technologies.

Assumption A.4: $\alpha \geq L$.

Given Assumption A.4,

$$n^w(K) = \frac{r}{h}\{\alpha - (L-K)\} \ . \quad (4)$$

We next consider the behavior of a profit-maximizing monopolist. Define Π(*K*) and $n^\pi(K) > 0$ analogously to *W*(*K*) and $n^w(K)$. Then, paralleling the derivation for the social optimum,

$$\Pi(K) = \pi[1-(L-K)/\beta]^2$$

and $$n^m(K) = \max\left\{\frac{r}{h}\{\beta - (L-K)\},\ 0\right\} \quad ,$$

where $\beta \equiv \left(\frac{h\pi}{rc}\right)^{1/2}$. Furthermore, $n^\pi(K) > 0$ if $\beta > L$.[13]

From $\pi < w$, it follows immediately that $n^\pi(K) < n^w(K)$ and $\Pi(K) < W(K)$. When $\pi < w$, a profit-maximizing monopolist does less than the socially optimal amount of R&D because it ignores the benefits that innovation confers on consumers.

We use the welfare and profit-maximizing results as benchmarks to explore R&D investment incentives by a duopoly for complex innovations. Without loss of generality, we henceforth scale R&D projects so that $h = 1$.

III. THE OPTIMAL DUOPOLY REGIME

We now consider R&D investment in a market with two potential innovators, firms 1 and 2. Both firms are assumed to have the innovation technology characterized by assumptions A.2 and A.3, and assumption A.1 continues to apply to the firms' aggregate payoffs. The state of the market at time *t* is determined by the firms' discoveries (k_1, k_2), where we omit a time subscript to economize on notation. Firm-specific payoffs are $\pi_i(k_1, k_2)$ for $i = 1,2$. The firm-specific payoffs could arise in different contexts. For example, they could represent the rules that a court would follow to allocate product value in a patent dispute, or they could represent the rules that a patent pool would use to divide licensing royalties. We assume that firms have rational expectations about these firm-specific payoffs when they make their R&D investment decisions.

We consider six reasonable properties of firm-specific payoff functions.

1. **No intermediate payoffs:** firms get no reward until all of *L* discoveries have been made. Specifically, $\pi_i(k_1, k_2) = 0$ unless $k_1 + k_2 = L$.

2. **Path Independence:** only the final ownership of intellectual property rights matters, not the temporal order in which the rights were obtained. With path independence and no intermediate payoffs, we can express the payoffs solely in terms of the pattern of ownership when all *L* innovations have been obtained.

   Current intellectual property policies satisfy path independence—what matters when adjudicating disputes among owners of complementary innovations is who has which intellectual property rights, not the order in which they were obtained.[14]

3. **Anonymity:** payoffs don't depend on the identity of the firm. Under this condition, $\pi_1(k_1, k_2) = \pi_2(k_2, k_1)$.

4. **Non-negative Rewards:** $\pi_i(k_1, k_2) \geq 0$ for all k_1 and k_2. This condition is a reasonable one when innovation confers property rights but those rights do not include the ability to tax other firms.

5. **Budget Balance:** the sum of the rewards equals the aggregate value of the final product: $\pi_1(k_1, k_2) + \pi_2(k_1, k_2) = \pi$ for all $k_1 + k_2 = L$. Budget balance is a sensible property for two reasons. The obvious one is that the government typically neither

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[13] Observe that the welfare- and profit-maximizing R&D investment rates increase with the number of discoveries made. Grossman and Shapiro (1986) derive a similar result in their analysis of a monopolist undertaking a multistage R&D project.

[14] The case of follow-on innovations is somewhat different as these innovations are not pure complements.

subsidizes nor taxes innovations, beyond general measures in the tax code and the rewards from the protection of intellectual property. The more important reason is that, even if there were a political will for doing so, policy makers almost certainly lack the information that would be necessary to fine tune the amount contributed or taxed. Observe that budget balance implies that rewards can be expressed in terms of profit shares.

6. **Zero Reward for Zero Success:** a firm that has obtained no innovations should receive no reward from innovation: $\pi_1(0, L) = \pi_2(L, 0) = 0$. As we will see, absent a restriction to the contrary, a welfare-maximizing reward scheme in the duopoly setting would grant a positive reward for zero success. In a more general setting, in which a large number of firms could claim to be potential innovators, a policy of positive rewards for zero success would attract firms claiming a payment despite having conducted no R&D.

We next examine whether there is an innovation reward scheme satisfying these properties that supports socially efficient R&D investment as a market equilibrium. We begin by considering the continuation game when there is only one technology remaining to be discovered.

A. One Discovery Remaining

Assuming Properties 1 and 2, we restrict our attention to the firm payoffs when all discoveries required to produce the product have been made: $\pi_1(k, L-k)$ and $\pi_2(k, L-k)$ for $k = 0,\ldots,L$. A necessary condition for efficiency is that these payoffs induce the socially optimal R&D levels for the continuation game in which only one technology remains to be discovered, corresponding to $k_1 + k_2 = L - 1$. Specifically, let $n_i(k_1, k_2)$ be the equilibrium level of R&D activity chosen by firm i when firm 1 has k_1 patents and firm 2 has k_2 patents. We search for payoffs $\pi_1(k, L-k)$ and $\pi_2(k, L-k)$ such that

$$n_1(k_1, k_2) + n_2(k_1, k_2) = n^w(L-1) = r(\alpha - 1) \tag{5}$$

for all $k_1 + k_2 = L - 1$. Our first observation is a restatement of the monopoly analysis above:

Lemma 1: *If $\pi < w$, then any property rights regime that satisfies budget balance and non-negative rewards and that induces only one firm to conduct R&D in the final stage cannot support the first-best outcome.*

Lemma 1 implies that, if there is an intellectual property regime that induces the first-best outcome and satisfies budget balance and non-negative rewards, then that regime must induce an interior solution in which both firms invest. Intuitively, there is a tendency for a profit-maximizing firm to invest less than the socially optimal amount because it ignores the increase in consumer surplus that its innovation creates. With a monopoly innovator, this is the only effect. With competing innovators, however, there is an offsetting distortion—a firm also ignores the negative pecuniary externality that its innovation confers on the rival firm. Another way of stating this effect is that a benefit of having both firms conduct R&D is that it creates preemption incentives, which increase R&D and offset the consumer surplus wedge between private and social incentives.

We proceed as follows. Assuming payoff functions $\pi_1(k, L-k)$ and $\pi_2(k, L-k)$ for $k = 0,\ldots,L$, we solve for an interior Nash equilibrium of the duopoly continuation game corresponding to $k_1 + k_2 = L - 1$, which at this step we assume is the unique equilibrium of the game (later we show that is the case). Call this equilibrium $[\hat{n}_1(k, L-k-1), \hat{n}_2(k, L-k-1)]$. We then ask whether payoff functions exist such that $\hat{n}_1(k, L-k-1) + \hat{n}_2(k, L-k-1) = n^w(L-1)$ and, if so, whether they satisfy the six reasonable properties.

In the continuation game with $k_1 + k_2 = L-1$, Firm 1 chooses its investment rate $n_1(t)$ to maximize its expected profit conditional on the investment rate chosen by Firm 2. The exponential discovery probability and fixed cost of R&D projects imply that the optimal investment rate for both firms is a constant until the discovery is made. If Firm 1 makes the next discovery, the state of the market transitions to $(k+1, L-k-1)$. Firm 1 receives the payoff $\pi_1(k+1, L-k-1)$ and Firm 2 receives the payoff $\pi_2(k+1, L-k-1)$. If Firm 2 makes the next discovery, the state transitions to $(k, L-k)$ and Firm 1 receives the payoff $\pi_1(k, L-k)$ while Firm 2 receives $\pi_2(k, L-k)$.

Firm 1 thus chooses a constant n_1 to maximize

$$\Pi_1(k, L-k-1) = \int_0^\infty \left[n_1\pi_1(k+1, L-k-1) + n_2\pi_1(k, L-k)\right]e^{-(r+(n_1+n_2))t}\,dt - n_1 c$$

$$= \frac{n_1\pi_1(k+1, L-k-1) + n_2\pi_1(k, L-k)}{n_1 + n_2 + r} - n_1 c \ .$$

Assuming positive investments by both firms, Firm 1's best response to investment by Firm 2 must satisfy

$$N_1(k, L-k-1) = \left\{\frac{1}{c}\left[r\pi_1(k+1, L-k-1) + n_2\left(\pi_1(k+1, L-k-1) - \pi_1(k, L-k)\right)\right]\right\}^{1/2} - r - n_2 \,. \quad (6)$$

Similarly, assuming positive investments by both firms, Firm 2's best response must satisfy

$$N_2(k,L-k-1)=\left\{\frac{1}{c}\left[r\pi_2(k,L-k)+n_1\left(\pi_2(k,L-k)-\pi_2(k+1,L-k-1)\right)\right]\right\}^{1/2}-r-n_1. \qquad (7)$$

By (5), if the payoffs support efficient investment in R&D, then it must be the case that the values of $\hat{n}_1$ and $\hat{n}_2$ that satisfy (6) and (7) also satisfy

$$\hat{n}_1(k,L-k-1)+\hat{n}_2(k,L-k-1)=r(\sqrt{\frac{w}{rc}}-1) \quad . \qquad (8)$$

Equations (6)-(8) imply that, if the Nash equilibrium investments sum to the socially optimal level of investment in R&D, then we can write these investments as

$$\hat{n}_1(k,L-k-1)=r\left[\frac{w-\pi_2(k,L-k)}{\pi_2(k,L-k)-\pi_2(k+1,L-k-1)}\right] \qquad (9)$$

and

$$\hat{n}_2(k,L-k-1)=r\left[\frac{w-\pi_1(k+1,L-k-1)}{\pi_1(k+1,L-k-1)-\pi_1(k,L-k)}\right] \quad . \qquad (10)$$

Define the payoff shares $s_i(k,L-k)=\pi_i(k,L-k)/\pi$ and let

$$\theta=\frac{2w}{\pi}-1.$$

The parameter, $\theta > 1$, is related to the ratio of the social to private value of the good or service made possible when all L technologies are discovered. Using budget balance ($s_1(k_1,k_2)+s_2(k_1,k_2)=1$) and anonymity ($s_1(k_1,k_2)=s_2(k_2,k_1)$), we can write equations (9) and (10) as

$$\hat{n}_1(k,L-k-1)=r\left[\frac{\frac{1}{2}(\theta-1)+s_1(k,L-k)}{s_1(k+1,L-k-1)-s_1(k,L-k)}\right] \qquad (11)$$

and

$$\hat{n}_2(k, L-k-1) = r\left[\frac{\frac{1}{2}(\theta+1) - s_1(k+1, L-k-1)}{s_1(k+1, L-k-1) - s_1(k, L-k)}\right]. \tag{12}$$

A central question is whether the payoff shares that satisfy equations (11) and (12) have reasonable properties. In deriving equations (11) and (12) we have imposed Properties 1-5 from the beginning of this section. We now ask whether Property 6 (zero reward for zero success) also can be satisfied.

As a preliminary step, we establish that a non-negative incremental reward for invention is necessary to support efficient investment as an outcome of the duopoly game.[15]

Lemma 2: *Suppose that the rewards satisfy path independence, budget balance, non-negativity, and anonymity. If the reward shares support efficient R&D investment, then for all* $k \in \{0,1,\ldots,L-1\}$*, the incremental return from an additional discovery is positive:* $s_1(k+1, L-k-1) > s_1(k, L-k)$.

Proof. Efficiency requires $n_1(k, L-k-1) + n_2(k, L-k-1) = n^w(L-1)$. By Lemma 1, an efficient reward function must induce an interior solution. Equations (11) and (12) and $n^w(L-1) = r(\alpha - 1)$ yield

$$s_1(k+1, L-k-1) - s_1(k, L-k) = \frac{\theta}{\alpha} > 0\,. \tag{13}$$

QED

The following proposition derives the unique payoffs shares that support efficient

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[15] Observe that, even if there were a zero or slightly negative incremental return from an additional discovery, firms would still have positive R&D investment incentives because additional discoveries would advance the date at which firms received their rewards.

investment in R&D with one discovery to go.

Proposition 1: *If* $\frac{\alpha}{L} \geq \theta > 1$, *then for* $k_1 + k_2 = L - 1$, *the unique reward scheme that supports efficient R&D investment and satisfies no intermediate payoffs, path independence, budget balance, non-negativity, and anonymity is characterized by shares:*

$$s_1(k, L-k) = \frac{1}{2} + (k - \frac{L}{2})\frac{\theta}{\alpha} \tag{14}$$

and

$$s_2(k, L-k) = \frac{1}{2} - (k - \frac{L}{2})\frac{\theta}{\alpha}. \tag{15}$$

Proof: By Lemma 1, an efficient reward function must induce an interior solution. Iterating equation (13) forward from $k = 0$, we obtain

$$s_1(k, L-k) = s_1(0, L) + k\frac{\theta}{\alpha} \quad . \tag{16}$$

By budget balance and symmetry,

$$s_1(L, 0) = 1 - s_1(0, L) \ . \tag{17}$$

By equations (16) and (17), $s_1(0, L) = \frac{1}{2}\{1 - L\frac{\theta}{\alpha}\}$. Therefore, $s_1(k, L-k) = \frac{1}{2} + (k - \frac{1}{2}L)\frac{\theta}{\alpha}$, and budget balance implies $s_2(k, L-k) = \frac{1}{2} - (k - \frac{1}{2}L)\frac{\theta}{\alpha}$. There are no other payoff shares that could support efficient R&D investment levels as an equilibrium with one discovery to go. Furthermore $s_1(0, L) \geq 0$ requires $\alpha \geq \theta L$.

Substituting (14) and (15) into (6) and (7) and then solving the simultaneous equations yields investment values that satisfy equation (8), the efficiency condition. By construction, these values satisfy the first-order necessary conditions for equilibrium. Given

$\pi_1(k+1, L-k-1) > \pi_1(k, L-k)$, Firm 1's expected profit is a concave function of its investment, and similarly for Firm 2. Hence, the simultaneous equations are also sufficient conditions for a Nash equilibrium. Therefore, the shares defined by (14) and (15) support efficient investment as a Nash equilibrium. **QED**

We henceforth assume:

Assumption A.4: $\alpha \geq \theta L$.

Equations (14) and (15) define the unique payoff shares that support efficient investment and satisfy no intermediate payoffs, path independence, budget balance, anonymity, and non-negativity. Observe, however, that

$$s_1(0, L) = \frac{1}{2}(1 - \frac{\theta L}{\alpha}),$$

which is positive except in the special case for which $\alpha = \theta L$. We have established:

Corollary: *If $\alpha > \theta L$, then there is no payoff function that satisfies no intermediate payoffs, path independence, budget balance, non-negativity, and zero reward for zero innovation, and which supports the first-best level of R&D.*

Under the optimal scheme, a firm that is in the patent race, but makes no discovery, nevertheless earns a positive reward when another firm discovers all of the technologies necessary to produce the product. Intuitively, the private gains to preemptive innovation are larger than the social value of a slightly earlier innovation, so that a winner-take-all regime would lead to excessive investment. In order to reduce the difference between wining and losing the R&D race, it is necessary either to reduce the winner's payoff by violating budget balance or to raise the loser's payoff above zero. As in the theory of teams (*e.g.,* Holmstrom, 1982), efficient payoffs generally do not satisfy budget balance when there is joint production.

Payments to unsuccessful innovators raise important concerns about adverse selection because a firm that did no R&D could attempt to claim a reward. This could be a significant problem because the payment to a firm with no patents under the efficient reward function in Proposition 1 can be large. Holding θ constant, as the social value of the innovation grows (which implies that the private value also grows), in the limit a firm that patents all technologies gets the same share of the profit from innovation as a firm with no patents (*i.e.,* $\frac{\theta}{\alpha} \to 0$ and in the limit each firm's share is ½).

We have demonstrated that efficient investment can be supported as an equilibrium of the duopoly game. The following result, proved in the Appendix, establishes that there are no other equilibria when the optimal payoff shares are in place:

Lemma 3: *Given the payoff shares in* (14) *and* (15), *the unique equilibrium investment levels are the efficient levels identified in Proposition 1.*

The payoffs required to support efficient investment do not satisfy all of the six properties that we have identified as reasonable for an innovation reward scheme. In particular, they fail the requirement of zero reward for zero success. If we relax other of the six properties, then other payoffs would support efficient investment. For example, if we relax the budget balance requirement, a reward for innovation equal to $\pi_i(k, L-k) = w$ for one firm and $\pi_j(k, L-k) = 0$ for the other supports the first-best level of investment in R&D for any number of invention stages. This is so because the incentive for innovation is identical to the socially optimal incentive when only one firm has a payoff from R&D and the payoff is equal to the social value of the R&D. Of course, this policy violates anonymity and relies on the government's having information about the relationship between π and w that it is unlikely to possess in practice.

Straightforward substitution into equations (6) and (7) demonstrates that $\pi_i(k, L-k) = w$ for i = 1,2 will also induce socially optimal investment in R&D in the final stage. Under this payoff structure, each firm receives a prize equal to the social value of all of the innovations together regardless of which firm makes the final discovery. Hence, these payoffs violate the condition of no reward for no innovation as well as budget balance. There is a continuum of Nash equilibria when both firms receive a payoff from discovery equal to the social value of the invention, all of which have the property that $n_1(k, L-k-1) + n_2(k, L-k-1) = n^w(L-1)$.

B. More than One Discovery Remaining

Let $\Pi_i(k_1, k_2)$ denote firm i's expected continuation payoffs at state (k_1,k_2). By definition $\Pi_i(k_1, k_2) = \pi_i(k_1, k_2)$ when $k_1 + k_2 = L$. For $k_1 + k_2 < L$, we have the following recursion relationship

$$\Pi_i(k_1, k_2) = \int_0^\infty [n_1\Pi_i(k_1+1, k_2) + n_2\Pi_i(k_1, k_2+1)]e^{-(r+(n_1+n_2))t}dt - n_i c$$

$$= \frac{n_1\Pi_i(k_1+1, k_2) + n_2\Pi_i(k_1, k_2+1)}{n_1 + n_2 + r} - n_i c \ ,$$

where n_1 and n_2 are the equilibrium investment levels conditional on k_1 and k_2. Here $\Pi_i(k_1+1, k_2)$ and $\Pi_i(k_1, k_2+1)$ are the continuation values of the game after one more discovery. Figure 1 illustrates the possible states of the game and the continuation values when L = 3.

The following proposition, proved in the Appendix, establishes that the payoff shares given by equations (14) and (15) support efficient investment in R&D for any number of innovations provided that $\alpha \geq \theta L$, which is a necessary condition for non-negative payoffs.

Proposition 2: *If* $\frac{\alpha}{L} \geq \theta > 1$, *then for all* $k_1 + k_2 = 0, 1, ..., L-1$ *the unique reward scheme that supports efficient R&D investment and satisfies no intermediate payoffs, path independence, budget balance, non-negativity, and anonymity is characterized by the profit shares in Proposition 1.*

The payoffs that support efficient R&D investment as the outcome of the duopoly game generally have the undesirable property that a firm that makes no discoveries still earns a positive reward. Such a policy is an invitation to inefficient entry, as firms could sit on the sidelines, do no R&D, and still claim a reward when other firms make the discoveries that are necessary to produce the product. In the next section, we consider two reward schemes that have zero reward for a firm that makes zero discoveries. Neither policy, however, is completely immune from inefficient strategic conduct.

IV. ALTERNATIVE INTELLECTUAL PROPERTY REWARD REGIMES

In this section, we consider two alternative regimes, one of which has a very simple reward structure and the other of which reflects current legal institutions:

- *Equal profit shares per innovation:* Under a regime of equal profit shares per innovation, firm *i* receives $\frac{k_i \pi}{L}$ when it holds k_i patents and $k_1 + k_2 = L$. Such a regime is supported by the intuition that, when the patented technologies are perfect complements, they are all equally valuable, or at least a court would have no rational basis for determining that one was more valuable than the other.[16]

- *Equal profit shares per innovator:* Under a regime of equal profit shares per innovator, each intellectual property owner holder receives $\frac{\pi}{n}$ when there are n holders of L patents. This regime can be viewed as a reduced form for a setting in which a patent holder can obtain injunctive relief to block any activity that infringes its patent, each patent has no value except when used with the $L-1$ other patents, and the bargaining by the rights holders satisfies the Nash bargaining axioms. When the threat of an injunction is strong, a firm holding one patent has as much bargaining power as a firm holding ten. In a duopoly under a regime of equal profit shares per innovation owner, $\pi_i(k, L-k) = \frac{\pi}{2}$ for all $k \in \{1,2,...,L-1\}$, $\pi_1(L,0) = \pi_2(0,L) = \pi$, and $\pi_1(0,L) = \pi_2(L,0) = 0$.

A. Equal Profit Shares per Innovation

We begin by observing that a regime of equal profit shares per innovation provides efficient R&D investment incentives when $\alpha = \theta L$, which can be seen by substituting $\alpha = \theta L$ into equations (14) and (15) defining the optimal reward scheme.

Proposition 3: If *$\alpha = \theta L$, then equal profit shares per innovation provide efficient R&D investment incentives.*

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[16] In a more general model in which innovations differ in their difficulty (*i.e.,* the values of h and c), these cost differences might serve as the basis for determining that some innovations were more deserving of reward than others.

When $\alpha > \theta L$, the incremental private return to another innovation exceeds the incremental return with optimal payoff shares and firms have excessive R&D incentives. In the Appendix, we prove

Proposition 4: *Suppose $k_1 + k_2 = L - 1$. If $\alpha > \theta L$, then the firms' aggregate equilibrium investment rate under the equal-profit-shares-per-innovation regime exceeds the socially optimal level.*

Numerical simulations suggest that the equal-profit-shares-per-innovation regime also provides excessive R&D incentives when $k_1 + k_2 < L - 1$ and $\alpha > \theta L$. Figure 2 compares equilibrium investment rates under the equal-profit-shares-per-innovation regime to efficient R&D investment rates at different states of the game when $L = 3$. The solid lines in Figure 2 show efficient levels of investment in R&D when there are three, two, or one technology remaining to be discovered. The socially efficient investment rates are independent of the profit from the product, holding *w, r, c,* and *h* fixed. The profit levels in Figure 2 range from the lowest value for which a duopoly equilibrium can support efficient investment (corresponding to $\theta = \frac{\alpha}{L}$, where $\theta = \frac{2w}{\pi} - 1$) to the social value, *w,* corresponding to $\theta = 1$. At the lowest value of π, equal profit shares per innovation equal the rewards that support efficient investment in R&D. For higher levels of π, equal profit shares per innovation generate excessive incentives for investment in R&D.

B. Equal Profit Shares per Innovator

We next consider a regime that awards an equal profit share to each firm holding a patent. We first observe that, for $L = 2$, the per-innovation and per-innovator regimes are identical: when one firm holds both patents, it gets a 100-percent share, and when two firms each hold one patent, each gets a 50 percent share. Hence, when $L = 2$, the equal-profit-shares-per-innovator regime gives rise to excessive innovation incentives if $\alpha > \theta L$.

The consequences of the equal-profit-shares-per-innovator regime change dramatically, however, when $L \geq 3$ and each firm has at least one patent. This is so because, when each firm already has at least one patent, an additional discovery generates no additional reward for the firm that makes the discovery. Each firm's share of profits is 50 percent regardless of who makes the next discovery. The discovery hastens the expected date at which the product can be produced and the firm earns profits, but there is a free-rider problem in that each firm would prefer that the other firm make the discovery, provided that it has at least one patent itself. This suggests under-investment in R&D when firms have at least one patent and $L \geq 3$. The following lemma confirms this intuition.

Lemma 4: *Starting from any state in which both firms have at least one innovation, the firms' aggregate equilibrium R&D investment rate under the equal-profit-shares-per-innovator regime is less than the socially optimal level.*

Proof: For $k = 1,2,...,L-2$ and with equal profit shares per innovating firm (denoted by "f"), each firm's best response to investment by the other firm when each firms has at least one innovation is

$$N_1^f(n_2; k_1, L-1-k_1) = \sqrt{\frac{r\pi}{2c}} - r - n_2$$

and

$$N_2^f(n_1; k_1, L-1-k_1) = \sqrt{\frac{r\pi}{2c}} - r - n_1^f,$$

from which it follows that

$$n_1^f(k_1, L-1-k_1) + n_2^f(k_1, L-1-k_1) = r\left[\sqrt{\frac{\pi}{2rc}} - 1\right] =$$

$$r\left[\frac{\alpha}{\sqrt{\theta+1}} - 1\right] < r(\alpha - 1) = n^w(L-1). \text{ **QED**}$$

Figure 3 illustrates the R&D under-investment with equal profit shares per innovator when each firm has a patent and $L = 3$. The dotted line in the figure labeled $k_1 = 1$, $k_2 = 1$ is always below the efficient level of investment, which corresponds to the solid line labeled $n^w(2)$.

The situation is very different when only one firm has innovated to date. In this case, the marginal return from obtaining a patent to the firm that has not innovated is equal to one half of the discounted profits from the final product. Similarly, a firm that has been the sole innovator to date stands to lose half of the expected product profits if the rival firm is able to obtain a patent covering at least one of the remaining technologies. Thus, the marginal return to patenting can be high for a firm that has no patents or for a firm that has all of the patents to date. These considerations suggest that there can be private incentives to engage in socially excessive R&D investment. The following result, proved in the Appendix, confirms this intuition when there is one discovery remaining:

Lemma 5: *For π sufficiently close to w, a regime of equal profit shares per innovator leads to overinvestment when $k_1 = L-1$ and $k_2 = 0$ or when $k_1 = 0$ and $k_2 = L-1$:* $n_1^f(L-1,0) + n_2^f(L-1,0) > n^w(L-1)$.

Lastly, the relationship between equilibrium and socially optimal investment levels is more complicated when at least one firm has no patents and the other firm has fewer than $L-1$

patents. In this case, there are counteracting incentives for under-investment (if both firms obtain patents for some $k_1 + k_2 < L$) and over-investment (when one firm has patents but the other does not). The lines in Figure 3 labeled $k_1 = 1$, $k_2 = 0$ and $k_1 = 0$, $k_2 = 0$ prove the following result by example when $L = 3$:

Proposition 5: *Under a regime of equal profit shares per innovator, equilibrium investment can higher or lower than the efficient level, depending on parameter values.*

Specifically, in the examples illustrated in Figure 3, equilibrium R&D investment is inefficiently low for low values of π and inefficiently high for high values of π when neither firm has a patent or when only one firm has a patent.

V. CONCLUSION

When technologies are valuable only when used together, firms undertaking R&D to create these technologies are in a complementary relationship. This complementary relationship can give rise to a free-riding problem. However, the firms are also competitors to the extent that rewards are positively related to the amount of intellectual property that they create. This relationship can lead to business-stealing effects. We searched for an innovation reward scheme that balances these forces to support efficient R&D investment by a duopoly while satisfying the reasonable properties that the payoffs that allocate the value of the product to the patentees are independent of the sequence of discoveries, do not depend on the identities of the firms that make the discoveries, are non-negative, offer zero reward for zero success, and are not paid prior to discovery of the full set of essential technologies. We showed that, in general, efficient R&D investment cannot be supported by a reward policy that satisfies all of these assumptions.

We also showed that there exists an efficient reward policy satisfying no-intermediate payoffs, path independence, anonymity, non-negative rewards, and budget balance. In general this policy must provide a positive reward for a firm that is engaged in R&D but has no success. Furthermore, the efficient reward policy has the property that the incremental return to the discovery of another patent is less than the patent's share of the total count of patents. These results, as well as our analysis of the equal-profit-shares-per-innovation and equal-profit-shares-per-innovator regimes, suggest that there is no simple rule that courts can use to allocate the value of a product to owners of the underlying intellectual property that would provide efficient R&D incentives for complementary innovations.

We have derived these results under the assumption that discovery follows an independent exponential distribution. Although this assumption is restrictive, the fundamental results should apply to wide range of discovery technologies. The important characteristic of the discovery technology assumed in our analysis is that a monopoly would have too little incentive to invest in R&D but a firm engaged in a race to obtain an innovation may have excessive investment incentives. These opposing incentives should exist for a wide range of discovery technologies and lead to similar conclusions.

We have also derived our results under the assumption that the set of technologies and the resulting set of potential patents are exogenously given. Under some reward schemes (*e.g.,* equal profit shares per innovation) firms could have incentives to engage in strategic behavior to increase the number of patents covering a given technology. Similarly, under a regime of equal profit shares per innovator, a firm holding multiple patents would have incentives to sell all but one of those patents to other firms in order to increase its share of total profits. Lastly, firms can have incentives to obtain intellectual property rights to what they claim are technologies necessary to offer a product, even if they are not. The regime of equal profit shares per innovator can be particularly vulnerable to this type of behavior because a single intellectual property right can be sufficient to claim a large share of the value of the final product. An interesting issue for future research is to determine how considerations of private strategic behavior shape the socially optimal sharing rules.

We close by observing that it would be interesting to extend the analysis to consider application to patent pools in greater depth. The members of a patent pool have to determine how to share the pool's licensing revenues. Some patent pools allocate licensing revenues through an internal negotiation over patent values, and others have lower royalties for patentees that join the pool after its initial formation.[17] The efficient rules developed in our analysis above could provide a basis for a patent pool to allocate licensing revenues among the members of the pool. As in the case of efficient damage awards, the objective would be to design revenue sharing rules that lead to efficient investment in R&D for complementary innovations, assuming that firms correctly anticipate future licensing revenues allocated to them by a patent pool. Voluntary patent pools, however, raise interesting issues of commitment and membership that do not arise in the analysis of legally-imposed sharing regimes.

[17] See Layne-Farrar and Lerner (2007) and Lerner, Strojwas and Tirole (forthcoming).

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APPENDIX

Proof of Lemma 3: First, suppose that there is at least one interior Nash equilibrium of the continuation game when $k_1 + k_2 = L - 1$. When one technology remains to be discovered, the best-response functions (6) and (7) can be written as

$$N_1(n_2; k, L-1-k) = \left\{ \frac{\pi}{c} \left[rs_1(k+1, L-1-k) + n_2 \left(s_1(k+1, L-1-k) - s_1(k, L-k) \right) \right] \right\}^{1/2} - r - n_2 \tag{A.1}$$

and

$$N_2(n_1; k, L-1-k) = \left\{ \frac{\pi}{c} \left[r(1 - s_1(k, L-k)) + n_1 \left(s_1(k+1, L-1-k) - s_1(k, L-k) \right] \right\}^{1/2} - r - n_1 \tag{A.2}$$

where

$$s_1(k, L-k) = \frac{1}{2} + (k - \frac{L}{2}) \frac{\theta}{\alpha}$$

and

$$s_2(k, L-k) = 1 - s_1(k, L-k) .$$

The proof follows from the four following facts:

1. The best-response functions (A.1) and (A.2) and the optimal payoff shares imply that any interior equilibrium must satisfy

$$n_2 = n_1 + r(L - 1 - 2k_1) \equiv \varsigma(n_1) . \tag{A.3}$$

2. The optimal payoff shares and (A.2) imply that any interior equilibrium must satisfy

$$n_2 = \left\{ \left(\frac{r\alpha\theta}{\theta+1} \right) \left[r(L + \frac{\alpha}{\theta} - 2k_1) + 2n_1 \right] \right\}^{1/2} - r - n_1 \equiv \varphi(n_1) \tag{A.4}$$

3. If $\frac{\alpha}{L} \geq \theta > 1$, then $\varphi(0) > r(L - 1 - 2k_1) = \varsigma(0)$. This can be seen as follows. First, observe that

$$\varphi(0) = \left\{\left(\frac{\alpha\theta r}{\theta+1}\right)\left[r(L+\frac{\alpha}{\theta}-2k_1)\right]\right\}^{1/2} - r \ .$$

$\alpha \geq \theta L$ implies that

$$\left(\frac{\alpha\theta}{\theta+1}\right)\left[L-2k_1+\frac{\alpha}{\theta}\right] > L-2k_1$$

and, hence, that $\varphi(0) > \varsigma(0)$.

4. $\varphi''(n_1) < 0 = \varsigma''(n_1)$ for all n_1, as can be shown by direct calculation.

By facts (1) and (2), any interior equilibrium must satisfy $\varphi(n_1) = \varsigma(n_1)$. If the two functions intersect at least once, then by fact (3), we know that $\varphi(n_1)$ first intersects $\varsigma(n_1)$ from above. By fact (4), there can be no other intersection. Hence, there is at most one interior equilibrium.

We next show that an equilibrium in which only one firm invests does not exist. Suppose $n_1 > 0$. Firm 2 will invest a strictly positive level if

$$\frac{\partial}{\partial n_2}\left[\frac{n_1\pi_2(k+1, L-k-1) + n_2\pi_2(k, L-k)}{n_1+n_2+r}\right] - n_2 c > 0 \ \text{ at } n_2 = 0 \ .$$

This condition requires that

$$\pi_2(k, L-k) - \frac{n_1}{n_1+r}\pi_2(k+1, L-k-1) > c(n_1+r) \qquad \text{(A.5)}$$

Given optimal pay-off shares from Proposition 1, the left-hand side of (A.5) is strictly greater than $\pi\frac{\theta}{\alpha}$, while the right-hand side is less than $rc\alpha$, because a monopoly invests at less than the efficient level. Thus, if Firm 1 invests, Firm 2 also will invest at a strictly positive level if

$$\pi\frac{\theta}{\alpha} > rc\alpha \ ,$$

which is always satisfied if $\theta > 1$. The same holds for Firm 1 if Firm 2 invests. Hence, the unique equilibrium investment levels are the efficient levels identified in Proposition 1. **Q.E.D.**

Proof of Proposition 2: The proof proceeds by induction. We first observe that the socially optimal investment level depends solely on the sum of k_1 and k_2, and not the individual values. Conditional on $k_1 + k_2 = L - 1$ and socially optimal R&D investment, equilibrium expected welfare is $w_{L-1} \equiv w(1 - \frac{1}{\alpha})^2$.

Given that payoff shares sum to one and our assumptions about the R&D technology, equilibrium expected industry profits depend only on the total amount of R&D conducted. Proposition 1 and Lemma 3 established that the payoff shares given by equations (14) and (15) support efficient investment in R&D for all k_1 and k_2 such that $k_1 + k_2 = L - 1$. Because the efficient aggregate investment level is the same for all $k_1 + k_2 = L - 1$, equilibrium expected industry profits are also the same for all $k_1 + k_2 = L - 1$ when the payoff shares are given by equations (14) and (15). Henceforth, we will use $\Pi(L-1)$ to denote these profits.

Now, consider the continuation game beginning for any $k_1 + k_2 = L - 2$. Observe that the continuation game at $k_1 + k_2 = L - 2$ is simply a transformed version of the continuation game at $k_1 + k_2 = L - 1$, where $\Pi(L-1)$ replaces π, and $W(L-1)$ replaces w. Applying the logic of Proposition 1 to this game, the optimal payoff shares are

$$s_1'(k, L-k-1) = \frac{1}{2} + (k - \frac{L-1}{2})\frac{\theta'}{\alpha'} \qquad \text{(A.6)}$$

$$s_2'(k, L-k) = \frac{1}{2} - (k - \frac{L-1}{2})\frac{\theta'}{\alpha'}, \qquad \text{(A.7)}$$

where

$$\alpha' = \left(\frac{hW(L-1)}{rc}\right)^{\frac{1}{2}}, \tag{A.8}$$

and

$$\theta' = 2\frac{W(L-1)}{\Pi(L-1)} - 1. \tag{A.9}$$

We next show that the payoff shares given by equations (14) and (15) induce these payoff shares in this earlier continuation game: First, note that, given $k_1 = k$ and $k_2 = L-k-1$, the expected continuation profits are

$$\Pi_1(k,L-k-1) = \pi\left[\frac{n_1(k,L-k-1)s_1(k+1,L-k-1)+n_2(k,L-k-1)s_1(k,L-k)}{n_1(k,L-k-1)+n_2(k,L-k-1)+r}\right] - n_1(k,L-k-1)c \tag{A.10}$$

$$\Pi_2(k,L-k-1) = \pi\left[\frac{n_1(k,L-k-1)s_2(k+1,L-k-1)+n_2(k,L-k-1)s_2(k,L-k)}{n_1(k,L-k-1)+n_2(k,L-k-1)+r}\right] - n_2(k,L-k-1)c \tag{A.11}$$

The payoff shares in Proposition 1 imply that $n_1(k,L-k-1)+n_2(k,L-k-1) = n^w(L-1)$.

Using this result and the expressions for the payoff shares, the sum of the firms' payoffs with one discovery remaining is

$$\Pi(L-1) = \pi\left[\frac{n^w(L-1)}{n^w(L-1)+r}\right] - n^w(L-1)c = \frac{\alpha-1}{\alpha}(\pi - \alpha rc).$$

Furthermore, direct calculation of equation (A.10), noting that $n_1(k,L-k-1) = r(\frac{\alpha}{2}+k-\frac{L}{2})$ and $n_2(k,L-k-1) = r(\frac{\alpha}{2}-(k+1-\frac{L}{2}))$ yields

$$\Pi_1(k,L-k-1) = \pi\left[\frac{1}{2}\left(\frac{\alpha-1}{\alpha}\right)+\left(k-\frac{L-1}{2}\right)\frac{\theta}{\alpha}\right] - \alpha rc\left[\frac{1}{2}\left(\frac{\alpha-1}{\alpha}\right)+\left(k-\frac{L-1}{2}\right)\frac{1}{\alpha}\right] \tag{A.12}$$

A laborious calculation shows that equation (A.12) is equivalent to $\Pi_1(k,L-k-1) = \Pi(L-1)s_1'(k,L-1-k)$ with $s_1'(k,L-1-k)$ given by equation (A.6). A similar calculation shows that $\Pi_2(k, L-k-1) = \Pi(L-1)s_2'(k, L-1-k)$.

The proof assumes that $s_1'(k, L-1-k) \geq 0$, which requires that $\alpha' \geq \theta'(L-1)$. This condition must hold at each stage for the payoff shares in equations (A.6) and (A.7) to support efficient investment in R&D. Direct calculation using the expressions for α', θ', $\Pi(L-1)$ and $W(L\text{-}1)$ shows that $\alpha' \geq \theta'(L-1)$ if $\alpha \geq \theta L$. By induction, this result implies that $\alpha \geq \theta L$ is a sufficient condition for the payoff shares to be non-negative for any values of k_1 and k_2. **QED**

Proof of Proposition 4. Compare firm 1's best response function under the equal profit shares per innovation regime with the superscript "p." From equations (6) and (7) with

$\Pi_1(k, L-k) = \pi \dfrac{k}{L}$ and $\Pi_2(k, L-k) = \pi \dfrac{L-k}{L}$, we have

$$N_1^p(n_2; k, L-1-k) = \left\{ \frac{\pi}{cL}\left[r(k+1) + n_2 \right] \right\}^{1/2} - r - n_2, \tag{A.13}$$

and

$$N_2^p(n_1; k, L-1-k) = \left\{ \frac{\pi}{cL}\left[r(L-k) + n_1 \right] \right\}^{1/2} - r - n_1 \ . \tag{A.14}$$

These two equations imply that, in equilibrium, $r(k+1) + n_2 = r(L-k) + n_1$, or

$$n_2 = r(L-1-2k) + n_1 \ . \tag{A.15}$$

One can simultaneously solve (A.14) and (A.15) to find the equilibrium investment levels:

$$2n_1 + r(L-2k) - \left\{ \frac{\pi}{cL}\left[r(L-k) + n_1 \right] \right\}^{1/2} = 0 \ . \tag{A.16}$$

Equation (A.16) is equivalent to

$$4n_1^{\ 2} + \left[4z - y \right] n_1 + \left[z^2 - y(z + rk) \right] = 0$$

where $z \equiv r(L-2k)$ and $y \equiv \dfrac{\pi}{cL}$.

The roots of this quadratic equation are

$$n_1 = \frac{-B \pm (B^2 - 4AC)^{1/2}}{2A} \ ,$$

where $A = 4$, $B = 4z - y$, and $C = z^2 - y(z + rk)$. By assumption, $\beta > L$, which implies that $y > rL$. It is evident that $z \le rL$. Hence, C is negative. Because $C < 0 < A$, only the larger root is admissible, and

$$n_1^{\,p} = -\tfrac{1}{2} z + \tfrac{1}{8}\left(y + \sqrt{y^2 + 8yrL}\right) \ . \tag{A.17}$$

Under the efficient-investment regime (denoted by *), we have

$$N_2^*(n_1; k, L-1-k) = \left\{ \left(\frac{r\alpha\theta}{\theta+1} \right) \left[r(L + \frac{\alpha}{\theta} - 2k) + 2n_1 \right] \right\}^{1/2} - r - n_1 \ . \tag{A.18}$$

It is readily shown that (A.15) must be satisfied in this regime as well. Hence, the equilibrium investment levels under the efficient-investment regime can be found by solving

$$2n_1 + r(L - 2k) - \left\{ \left(\frac{r\alpha\theta}{\theta+1} \right) \left[r(L + \frac{\alpha}{\theta} - 2k) + 2n_1 \right] \right\}^{1/2} = 0 \ , \tag{A.19}$$

which is equivalent to

$$4n_1^{\,2} + [4z - 2x] n_1 + \left[z^2 - x(z + r\frac{\alpha}{\theta}) \right] = 0 \ ,$$

where $z \equiv r(L - 2k)$ and $x \equiv r\dfrac{\alpha\theta}{\theta+1}$.

The roots of this quadratic equation are

$$n_1 = \frac{-\hat{B} \pm (\hat{B}^2 - 4\hat{A}\hat{C})^{1/2}}{2\hat{A}} \ ,$$

where $\hat{A} = 4$, $\hat{B} = 4z - 2x$, and $\hat{C} = z^2 - x(z + r\dfrac{\alpha}{\theta})$. Observe that $\hat{C} < 0$ if $x > z$. For $x \le z$, $\hat{C}$

is increasing in z. Thus, taking $k = 0$, $\hat{C}$ is bounded from above by $\hat{C} \le (rL)^2 - xr(L + \frac{\alpha}{\theta})$.

Recall $\alpha > \theta L$. Hence,

$$\hat{C} \le (rL)^2 - 2xrL = r^2 L\left(L - 2\frac{\alpha\theta}{\theta+1}\right) < r^2 L^2\left(1 - 2\frac{\theta^2}{\theta+1}\right) < 0 .$$

Because $\hat{C} < 0 < \hat{A}$, only the larger root is admissible, and

$$n_1{}^* = -\tfrac{1}{2}z + \tfrac{1}{8}\left(2x + \sqrt{(2x)^2 + \frac{16xr\alpha}{\theta}}\right) . \qquad \text{(A.20)}$$

By equations (A.17) and (A.20),

$$n_1^p - n_1^* = y + \sqrt{y^2 + 8yrL} - 2x - \sqrt{(2x)^2 + \frac{16xr\alpha}{\theta}} .$$

Observe that

$$2x = r\frac{2\alpha\theta}{\theta+1} ,$$

$$y \equiv \frac{\pi}{cL} = r\frac{2}{\theta+1}\frac{\alpha^2}{L} = 2x(\frac{\alpha}{\theta L}) \equiv 2x\lambda ,$$

where $\lambda \ge 1$, and $\frac{16xr\alpha}{\theta} = 8yrL$. Therefore, $n_1^p - n_1^* > 0$. The fact that (A.15) holds for both regimes implies that $n_2^p - n_2^* > 0$ as well. **QED**

Proof of Lemma 5: For $\pi = w$,

$$N_1^f(n_2; L-1,0) = \left\{\frac{w}{c}\left[r + \frac{n_2}{2}\right]\right\}^{1/2} - r - n_2 ,$$

which implies

$$N_1^f(n_2; L-1,0) + n_2 = \left\{\frac{w}{c}\left[r + \frac{n_2}{2}\right]\right\}^{1/2} - r \ge \left(\frac{wr}{c}\right)^{1/2} - r = n^w(L-1) ,$$

with strict inequality if $n_2 > 0$.

We next prove that $n_2 > 0$ in equilibrium. Suppose, counterfactually, that $n_2 = 0$. Then

$N_1^f(n_2; L-1,0) = \left(\frac{wr}{c}\right)^{1/2} - r$ and

$$N_2^f\left(\left(\frac{wr}{c}\right)^{1/2} - r; L-1,0\right) = \left\{\frac{w}{2c}\left[r + n_1\right]\right\}^{1/2} - r - n_1$$

$$= \left\{\frac{w}{2c}\left(\frac{wr}{c}\right)^{1/2}\right\}^{1/2} - \left(\frac{wr}{c}\right)^{1/2}$$

$$= \left(\frac{wr}{c}\right)^{1/2}\left\{\left(\frac{1}{2}\right)^{1/2}\left(\frac{w}{rc}\right)^{1/4} - 1\right\}.$$

The first term is clearly positive. The sign of the term in curly brackets is equal to the sign of $w - 4rc$, which is positive for all $L > 2$ by Assumption A.4. By continuity, the result also holds for π sufficiently close to w. **QED**

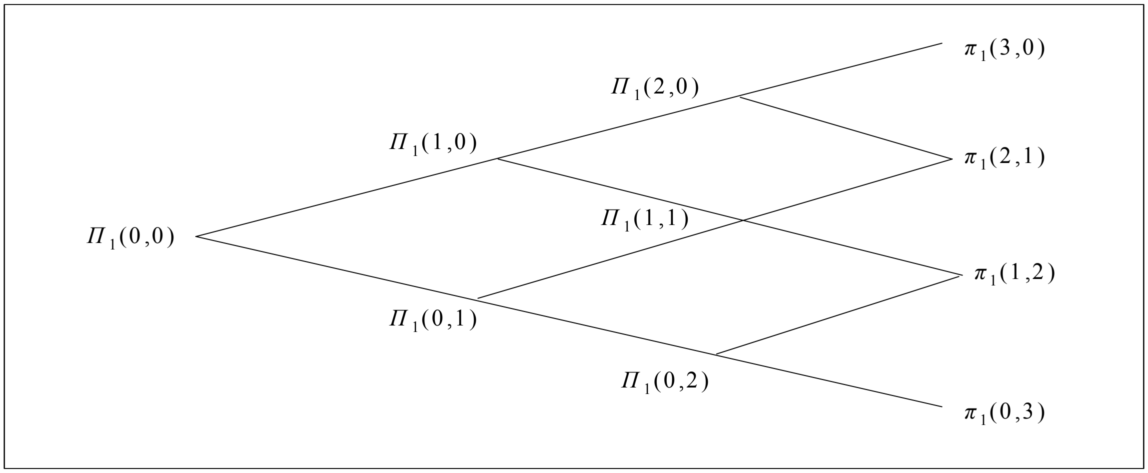

Figure 1. The states corresponding to *L* =3.

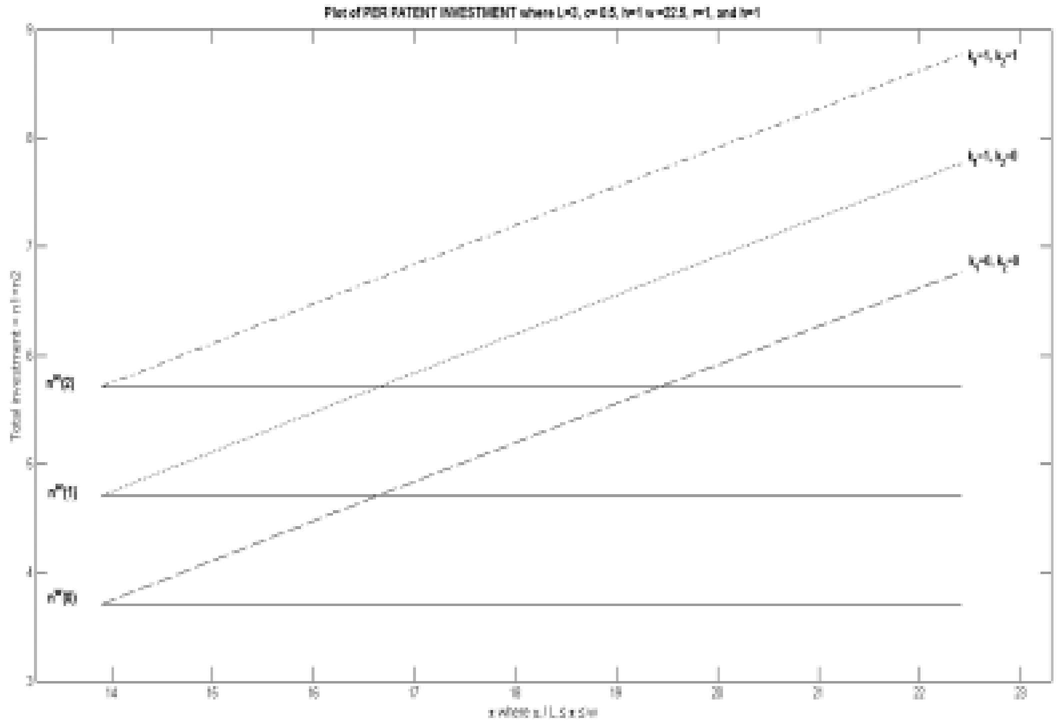

Figure 2. Investment rates with per-patent payoffs corresponding to different discovery states when *L* = 3.

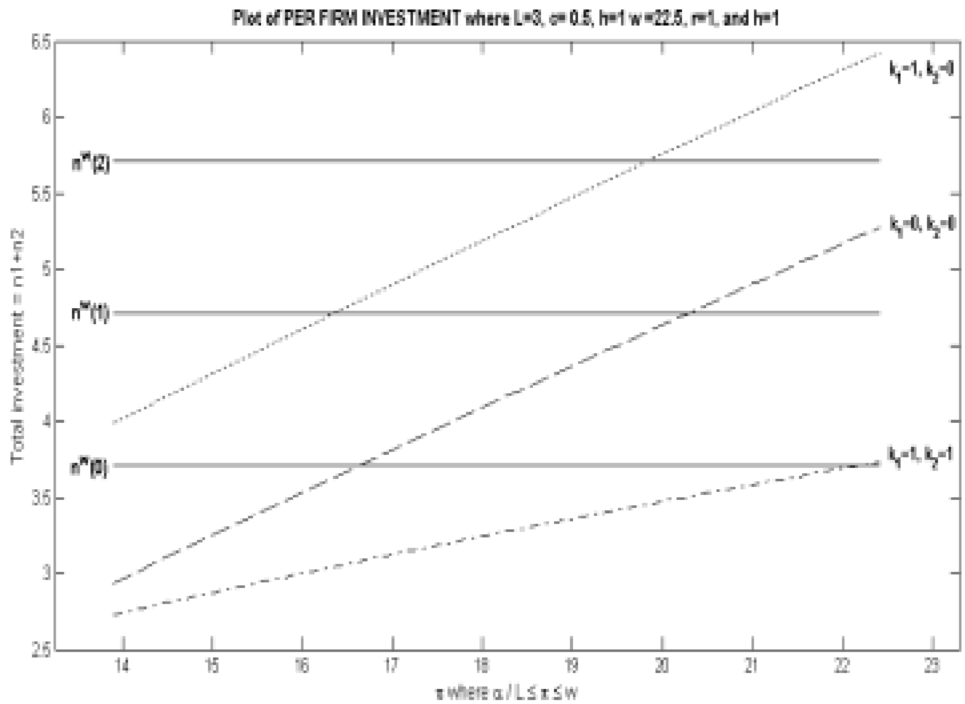

Figure 3. Investment rates with per-firm payoffs corresponding to different discovery states when *L* = 3.