A game of "matching pennies"

		column L	column R
row	T	2,0	0,1
	B	0,1	1,0

People last names A-M play ROW (choose T, B)

People last names N-Z play COLUMN (choose L, R)

A game of "matching pennies": Mixed-strategy equilibrium

		column L	column R	mixed-strategy equilibrium
row	T	2,0	0,1	.5
	B	0,1	1,0	.5
mixed-strategy equilibrium		.33	.67	

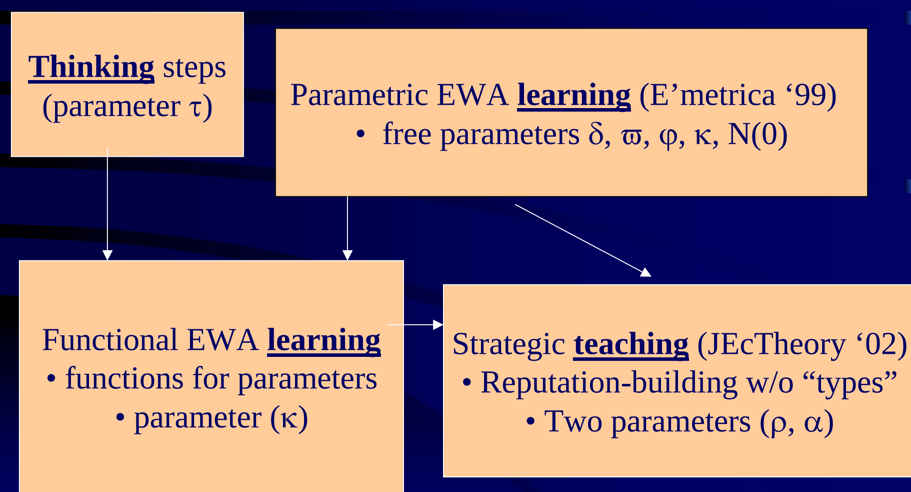
Behavioral game theory: Thinking, learning & teaching

Colin F. Camerer, Caltech
Teck Ho, Wharton
Kuan Chong, National Univ Singapore

- How to model bounded rationality?
  - Thinking steps (one-shot games)
- How to model equilibration?
  - Learning model (fEWA)
- How to model repeated game behavior?
  - Teaching model

Behavioral models use some game theory principles, relax others

Principle	Nash	Thinking	Learning	Teaching
concept of a game	↙	↙	↙	↙
strategic thinking	↙	↙	↙	↙
best response	↙			
mutual consistency	↙			
learning			↙	↙
strategic foresight	↙			↙



Potential economic applications

- Thinking
  - price bubbles, speculation, competition neglect
- Learning
  - evolution of institutions, new industries
  - Neo-Keynesian macroeconomic coordination
  - bidding, consumer choice
- Teaching
  - contracting, collusion, inflation policy

Modelling philosophy

- General (game theory)
- Precise (game theory)
- Progressive (behavioral econ)
- Empirically disciplined (experimental econ)

"...the empirical background of economic science is definitely inadequate...it would have been absurd in physics to expect Kepler and Newton without Tycho Brahe" (von Neumann & Morgenstern '44)

Modelling philosophy

- General (game theory)
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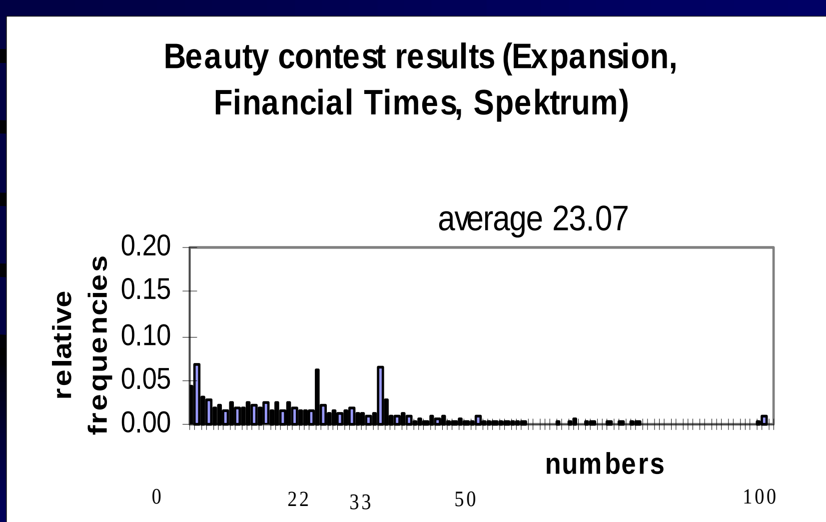
"...the empirical background of economic science is definitely inadequate...it would have been absurd in physics to expect Kepler and Newton without Tycho Brahe" (von Neumann & Morgenstern '44)

"Without having a broad set of facts on which to theorize, there is a certain danger of spending too much time on models that are mathematically elegant, yet have little connection to actual behavior. At present our empirical knowledge is inadequate..." (Eric Van Damme '95)

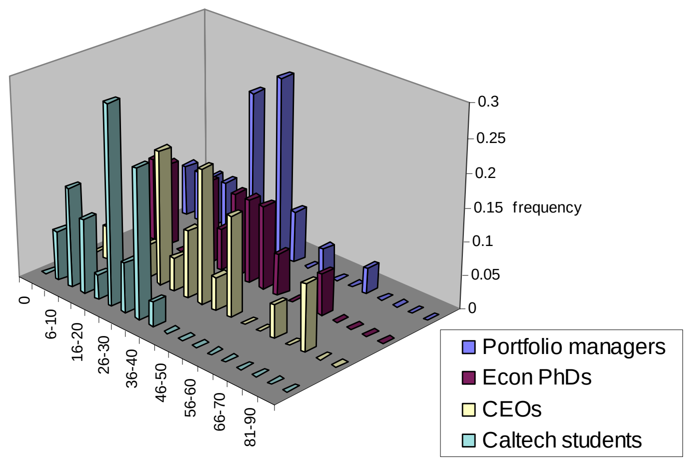
Beauty contest game

- N players choose numbers x_i in [0,100]
- Compute target $(2/3)*(\Sigma x_i /N)$
- Closest to target wins $20

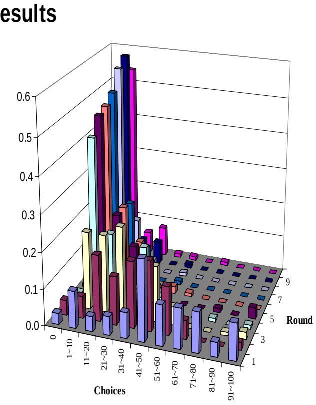
Beauty contest game: Pick numbers [0,100] closest to (2/3)*(average number) wins



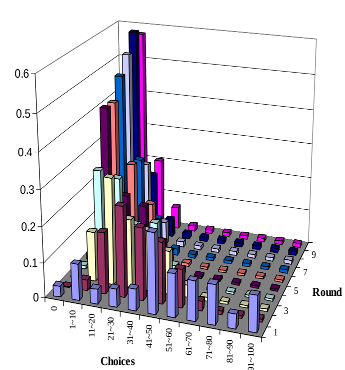
Beauty contest results



Results



Predictions



The thinking steps model

- Discrete steps of thinking

  Step 0's choose randomly

  K-step thinkers know proportions $f(0),\ldots f(K-1)^*$

  Normalize $f'(h)=f(h)/\Sigma_{h=0}^{K-1} f(h)$ and best-respond

  $A^j(K)=\Sigma_m o(s^j,s^m)\ (P^m(0) f'(0) + P^m(1) f'(1)+\ldots P^m(K-1) f'(K-1))$

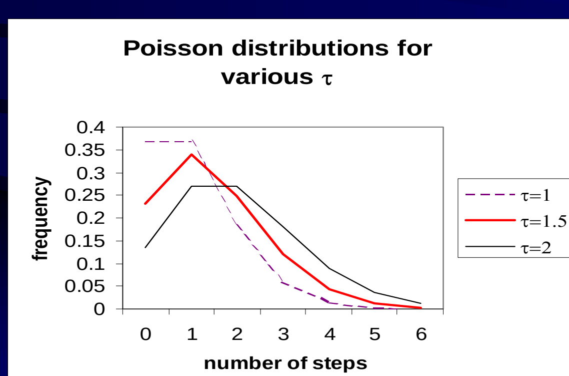
  logit probability $P^j(K)=\exp(\kappa A^j(K))/ \Sigma_h \exp(\kappa A^h(K))$

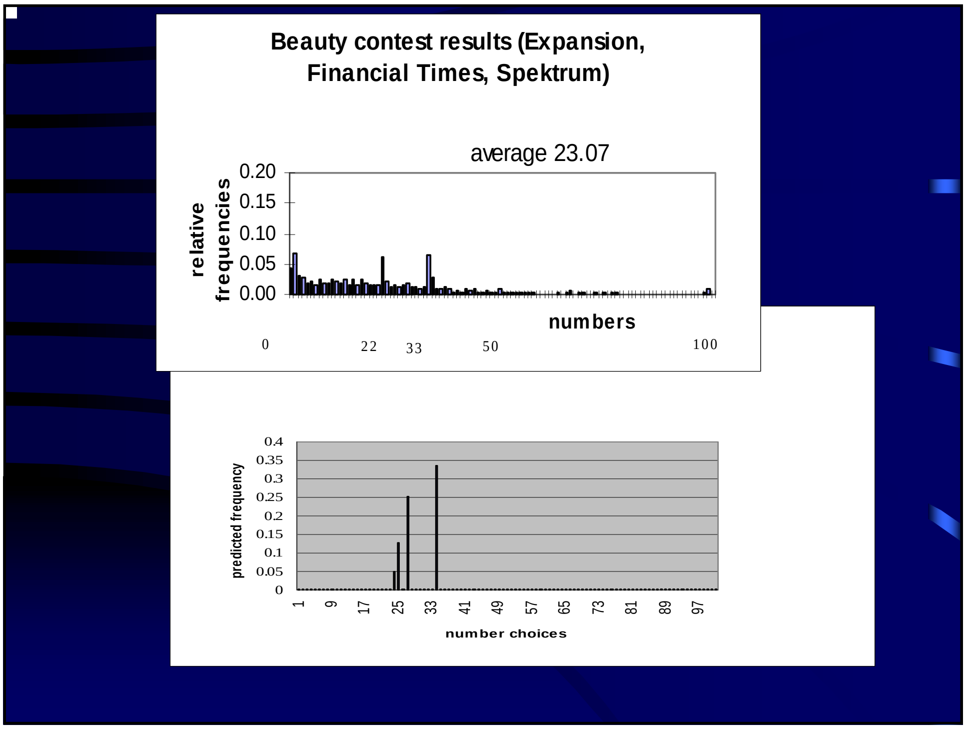
- What is the distribution of thinking steps f(K)?

*alternative: K-steps think others are one step lower (K-1)

Poisson distribution of thinking steps

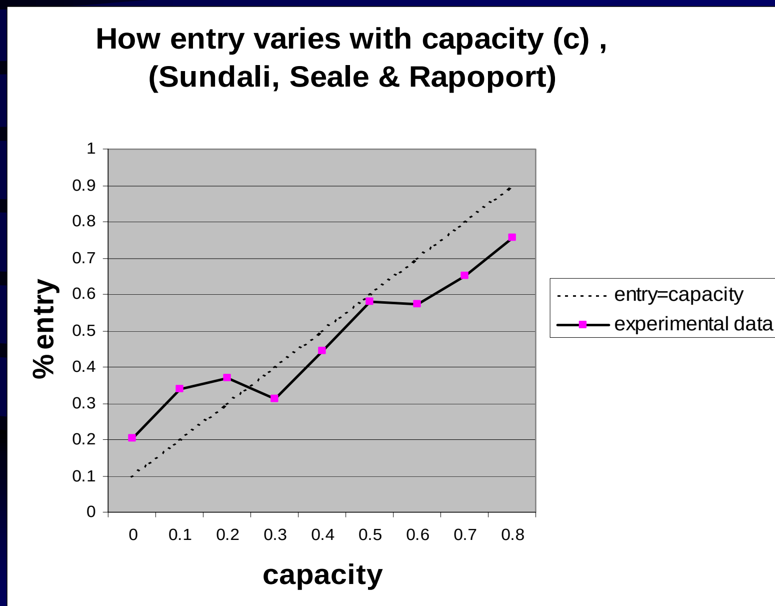
- $f(K)=\tau^K/e^\tau K!$ 56 games: median $\tau=1.78$
- Heterogeneous (♦ "spikes" in data)
- Steps > 3 are rare (working memory bound)
- Steps can be linked to cognitive measures



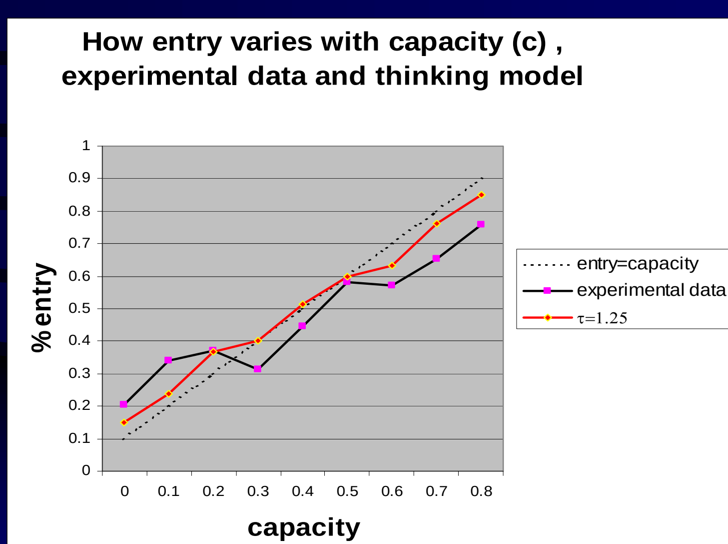


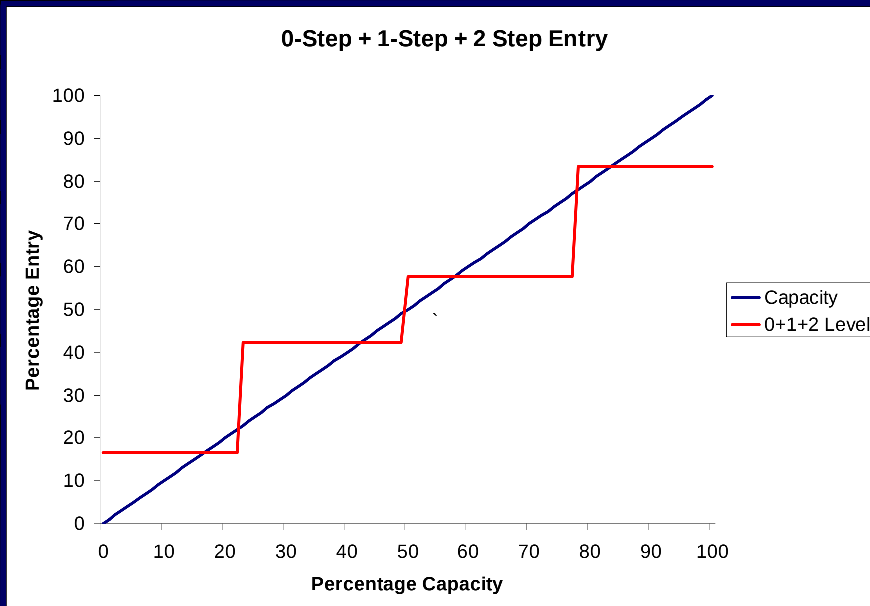
Thinking steps in entry games

- Entry games:
  - Prefer to enter if n(entrants)<c;
  - stay out if n(entrants)>c
  - All choose simultaneously
- Experimental regularity in the 1st period:
  - Close to equilibrium prediction n(entrants) ●c
  - “To a psychologist, it looks like magic”-- D. Kahneman ‘88



Thinking steps in entry games





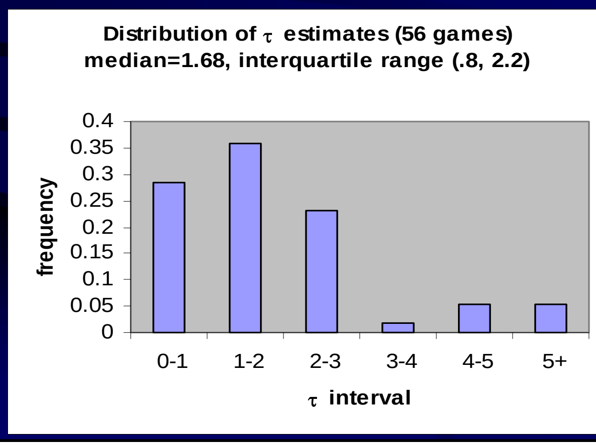
Thinking steps estimates of τ

	range of τ	common τ
• Matrix games		
Stahl, Wilson	(1.7, 18.3)	8.4
Cooper, Van Huyck	(.5, 1.3)	.8
Costa-Gomes, Crawford, Broseta	(1.3, 2.4)	2.2
• Mixed-equilibrium games	(.3, 2.7)	1.5
• First period of learning	(0, 3.9)	
• Entry games		2.0
• Signaling games	(.3,1.2)	

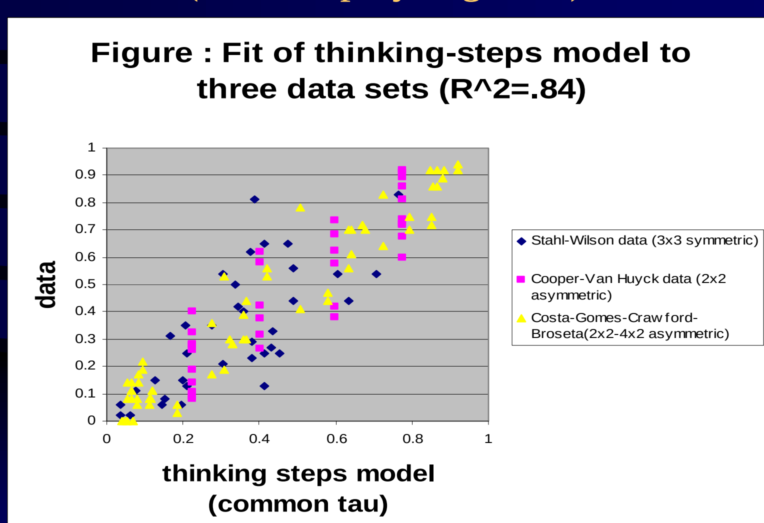
(Fits significantly better than Nash, QRE)

Estimates of mean thinking step τ

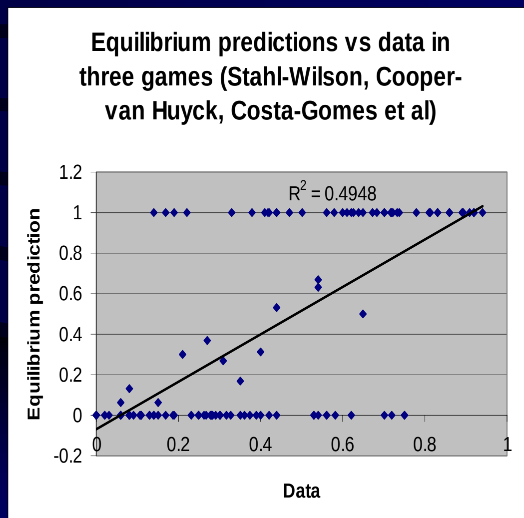
- 33 one-shot matrix games
- 15 mixed-equilibrium games
- 1 entry game
- 7 thinking-learning games



Fitting the model to normal-form games (n=1672 player-games)



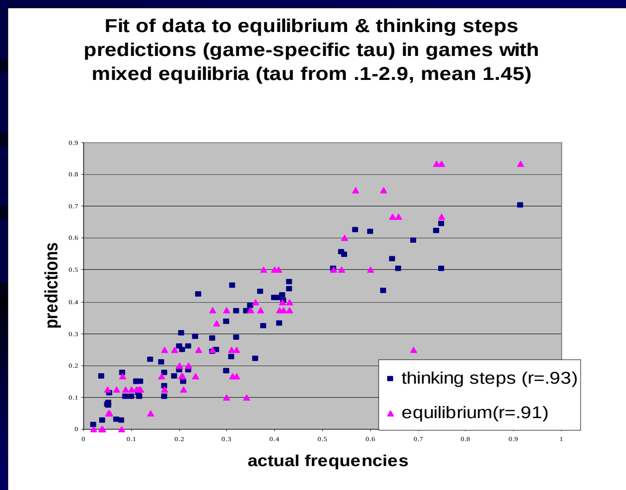
Nash equilibrium vs data in normal-form games



Thinking steps analysis (τ=1.5)

				row step thinker choices				steps	mixed	
			0	1	2	3	4...	overall	equilm	data
	2,0	0,1	.5	1	1	0	0	.72	.5	.?
	0,1	1,0	.5	0	0	1	1	.28	.5	.?
0	.5	.5								
1	.5	.5								
2	0	1								
3	0	1								
4	1	0								
5	1	0								
overall	.34	.66								
mixed	.33	.67								
data	.?	.?								

Equilibrium vs thinking-steps (overconfidence version) in mixed-equilibrium games (n=15 games)



Comparing QRE and thinking-steps

- Fit (thinking-steps slightly better)
- Heterogeneity

  ``spikes” in p-beauty contests

  noisy cutoff rules in entry games

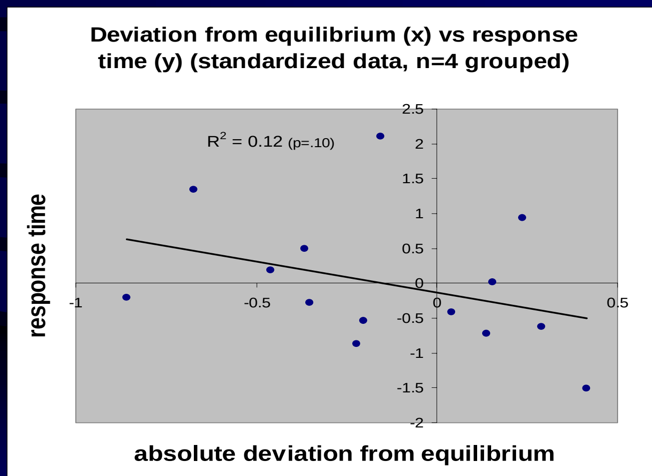
  endogeneous “purification” in mixed-equil’m games
- Cognitive measures

  Effects of “prompting” beliefs-- pushes steps up by 1?

  Response times (modest correlation with pBC choice)

  Attention measures in shrinking-pie bargaining

Response times vs deviation from equilibrium in p-beauty contest games



Conclusions

- Discrete thinking steps, frequency Poisson distributed (mean number of steps $\tau \approx 1.5$)
- One-shot games & initial conditions
- Advantages:
  - Can “solve” multiplicity problem
  - Explains “magic” of entry games
  - Sensible interpretation of mixed strategies
- Theory:
  group size effects (2 vs 3 beauty contest)
  approximates Nash equilm in some games (dominance solvable)
  refinements in signaling games (intuitive criterion)

Conclusions

- **<u>Thinking</u>** (τ, κ) steps model

  τ fairly regular ($\approx$1.5) in entry, mixed, matrix, dominance-solvable games

  Easy to use
- **<u>Learning</u>** (κ)

  Hybrid fits & predicts well (20+ games)

  One-parameter fEWA fits well, easy to estimate
- Next?

  Field applications

  Theoretical properties of thinking model

Parametric EWA learning

- Attraction $A_i^j(t)$ for strategy j updated by

  $A_i^j(t) = (\varpi A_i^j(t-1) + o(\text{actual}))/(\varpi(1-\varphi)+1)$ (chosen j)

  $A_i^j(t) = (\varpi A_i^j(t-1) + \delta\, o(\text{foregone}))/(\varpi(1-\varphi)+1)$ (unchosen j)
- key parameters: δ imagination, ϖ decay

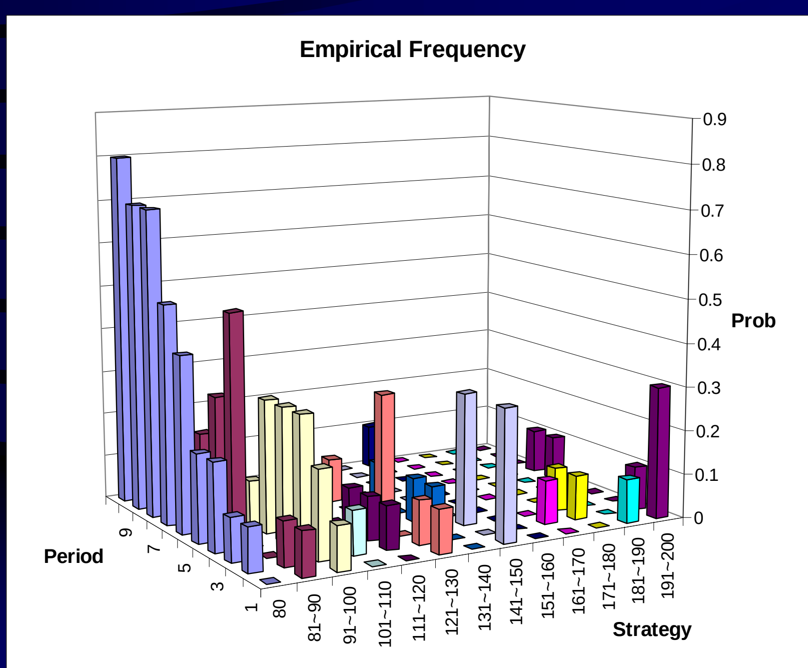
- "In nature a hybrid [species] is usually sterile, but in science the opposite is often true"-- sFrancis Crick '88

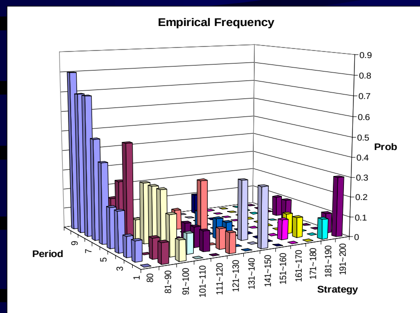
  Weighted fictitious play (δ=1, φ=0)

  Choice reinforcement (δ=0)

Example: Price matching with loyalty rewards
(Capra, Goeree, Gomez, Holt AER '99)

- Players 1, 2 pick prices [80,200] ¢
  Price is $P=\min(P_{1},P_{2})$
  Low price firm earns P+R
  High price firm earns P-R
- What happens? (e.g., R=50)

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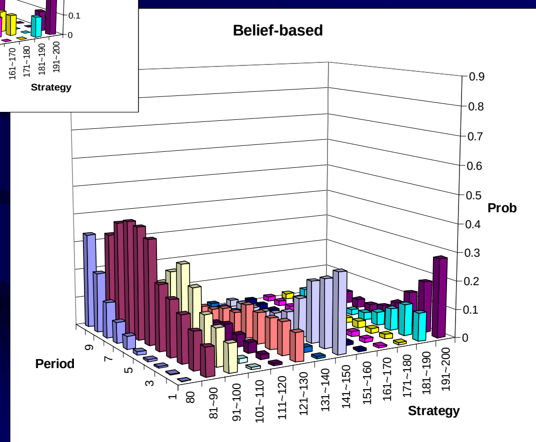
181~190
191~200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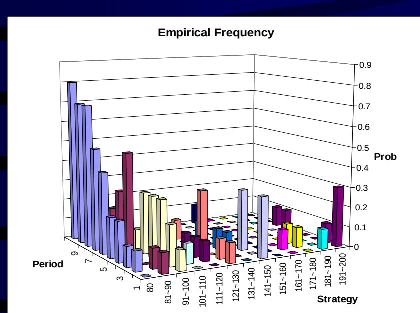
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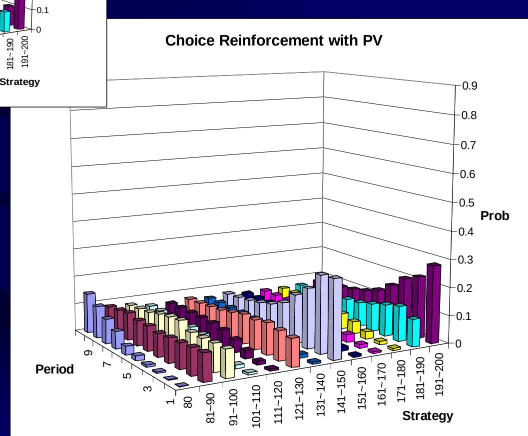
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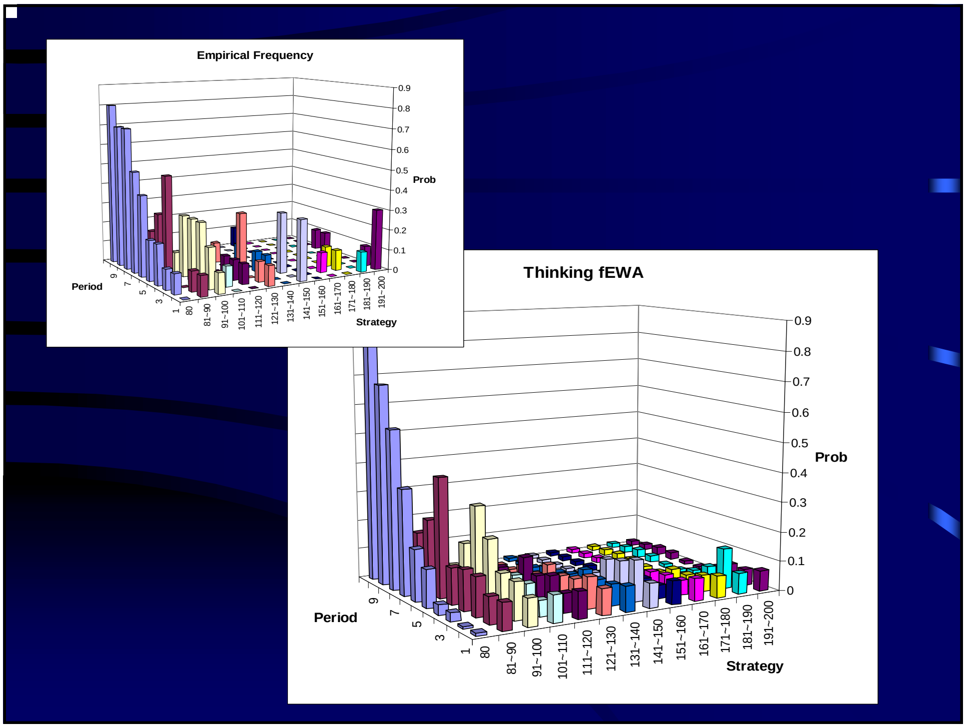
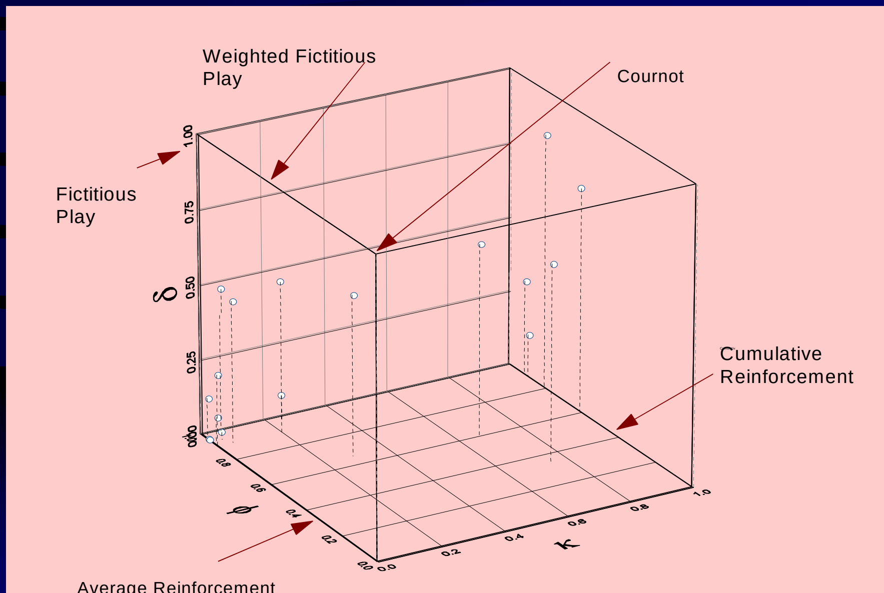


Studies comparing EWA and other learning models

Reference	Type of game
Amaldoss and Jain (Mgt Sci, in press)	cooperate-to-compete games
Cabrales, Nagel and Ermenter ('01)	stag hunt "global games"
Camerer and Anderson ('99, Ec Theory)	sender-receiver signaling
Camerer and Ho ('99, Econometrica)	median-action coordination 4x4 mixed-equilibrium games p-beauty contest
Camerer, Ho and Wang ('99)	normal form centipede
Camerer, Hsia and Ho (in press)	sealed bid mechanism
Chen ('99)	cost allocation
Haruvy and Erev ('00)	binary risky choice decisions
Ho, Camerer and Chong ('01)	"continental divide" coordination price-matching patent races two-market entry games
Hsia ('99)	N-person call markets
Morgan & Sefton (Games Ec Beh, '01)	"unprofitable" games
Rapoport and Amaldoss ('00 OBHDP, '01)	alliances patent races
Stahl ('99)	5x5 matrix games
Sutter et al ('01)	p-beauty contest (groups, individuals)

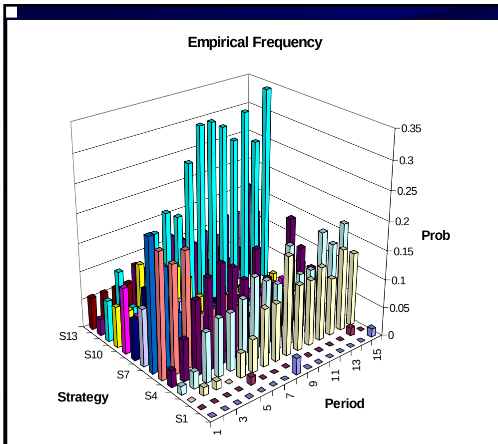
20 estimates of learning model parameters

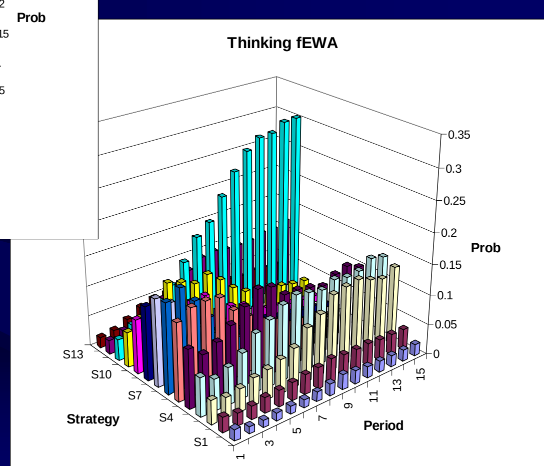


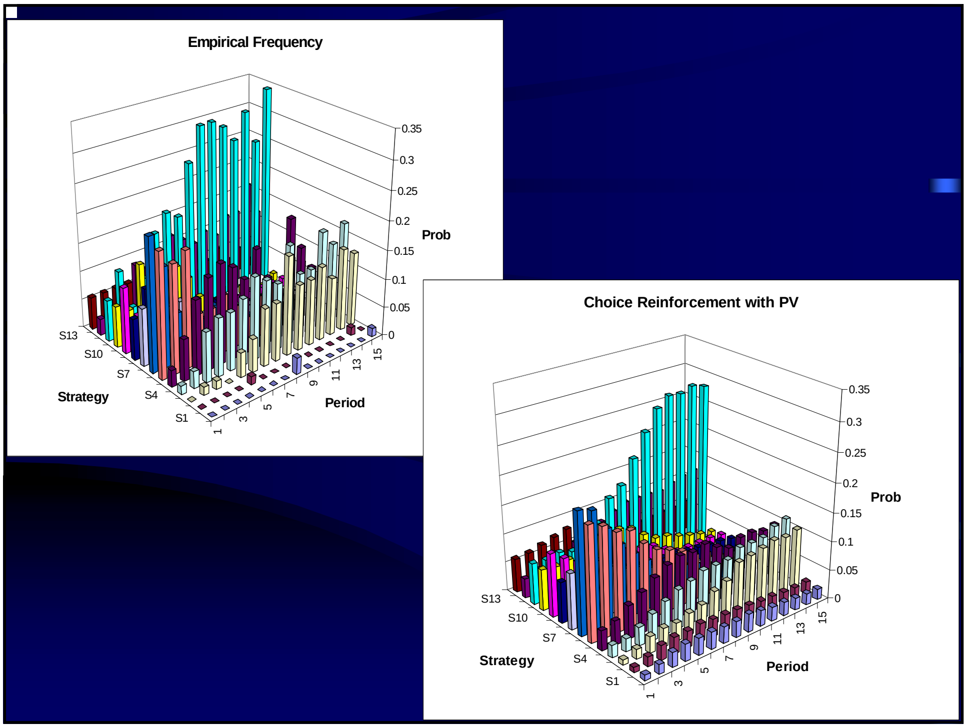
Model Fit in 7 Games (Hit Rate, BIC and Log Likelihood)										
In-sample (Hit Rate/BIC)		N	f EWA (1)		Reinforcement (2)		Beliefs(fict. play) (3)		EWA (5)	
Pooled (common param.s)		10573	**52%**	**-15306**	48%	-17742	*43%*	*-18880*	46%	-17742
Total (game-specific param.s)		10573	52%	-15306	51%	-16758	*46%*	*-17031*	**52%**	**-15090**
Out-of-sample (Hit Rate/LL)		N	f EWA		Reinforcement		Beliefs (fict. play)		EWA	
Pooled		4674	**52%**	**-6862**	49%	-7764	*44%*	*-8406*	46%	-7929
Total		4674	52%	-6862	52%	-7426	*46%*	*-7474*	**52%**	**-6738**
Note: Bold denotes **best** fits; italics denotes *worst* fits										

Continental divide game payoffs

your choice	Median Choice 1	2	3	4	5	6	7	8	9	10	11	12	13	14
1	45	49	52	55	56	55	46	-59	-88	-105	-117	-127	-135	-142
2	**48**	53	58	62	65	66	61	-27	-52	-67	-77	-86	-92	-98
3	**48**	**54**	**60**	66	70	74	72	1	-20	-32	-41	-48	-53	-58
4	43	51	58	65	**71**	**77**	80	26	8	-2	-9	-14	-19	-22
5	35	44	52	60	69	**77**	**83**	46	32	25	19	15	12	10
6	23	33	42	52	62	72	82	62	53	47	43	41	39	38
7	7	18	28	40	51	64	78	75	69	66	64	63	62	62
8	-13	-1	11	23	37	51	69	83	81	80	80	80	81	82
9	-37	-24	-11	3	18	35	57	88	89	91	92	94	96	98
10	-65	-51	-37	-21	-4	15	40	**89**	**94**	98	101	104	107	110
11	-97	-82	-66	-49	-31	-9	20	85	**94**	**100**	105	110	114	119
12	-133	-117	-100	-82	-61	-37	-5	78	91	99	**106**	**112**	**118**	**123**
13	-173	-156	-137	-118	-96	-69	-33	67	83	94	103	110	117	**123**
14	-217	-198	-179	-158	-134	-105	-65	52	72	85	95	104	112	120



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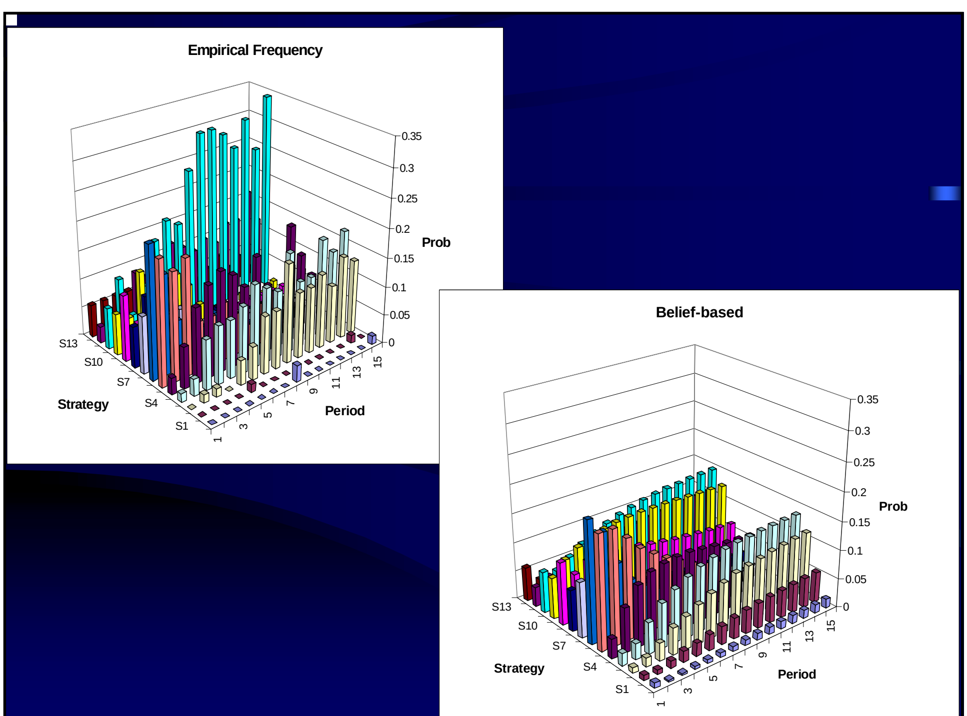
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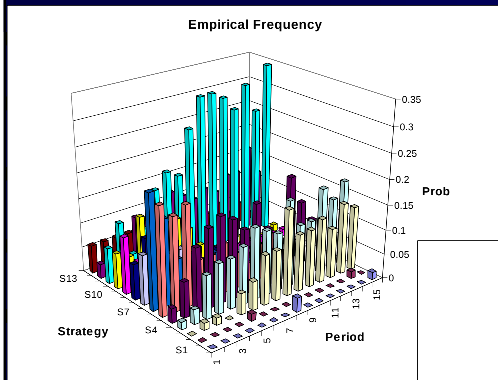
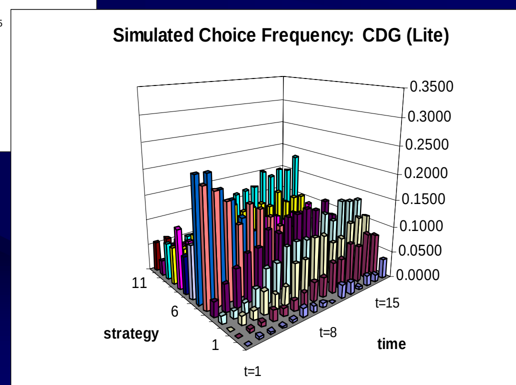
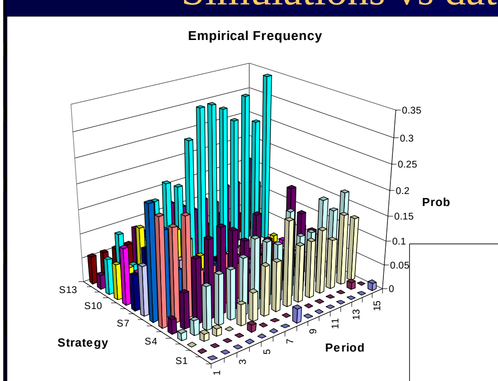
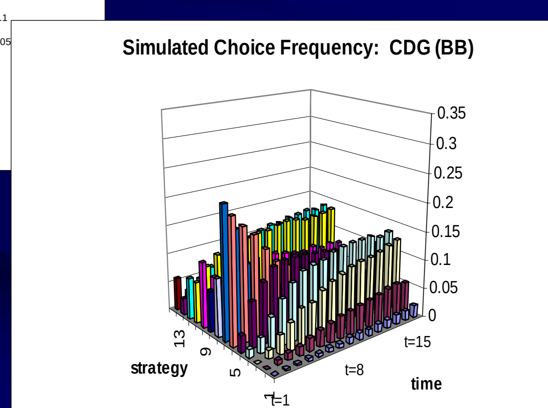
Simulations vs data: fEWA



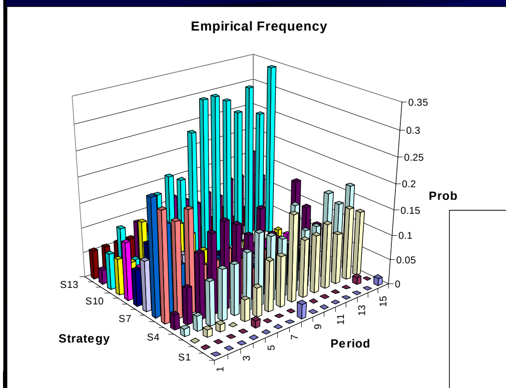


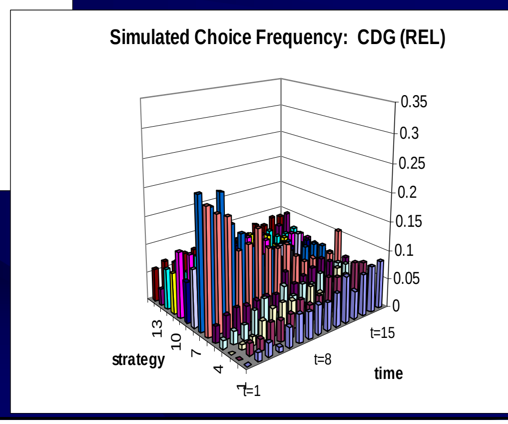
Simulations vs data: belief learning



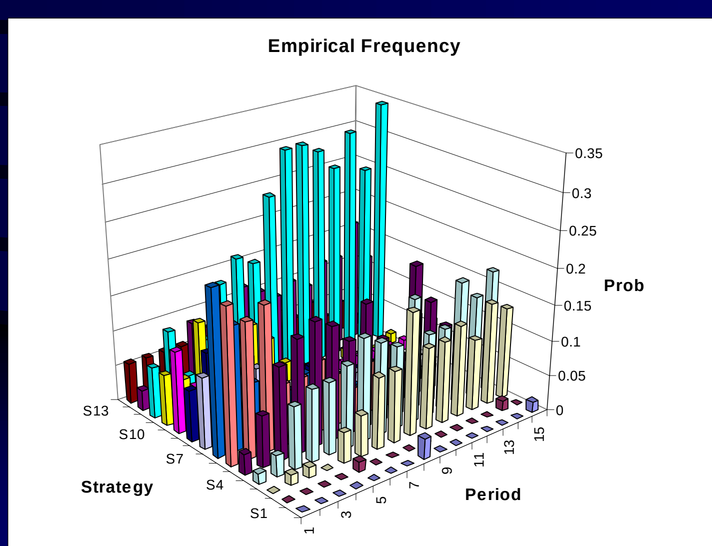


Simulations vs data: reinforcement





Behavior in “continental divide game”



Functional fEWA (one parameter κ)

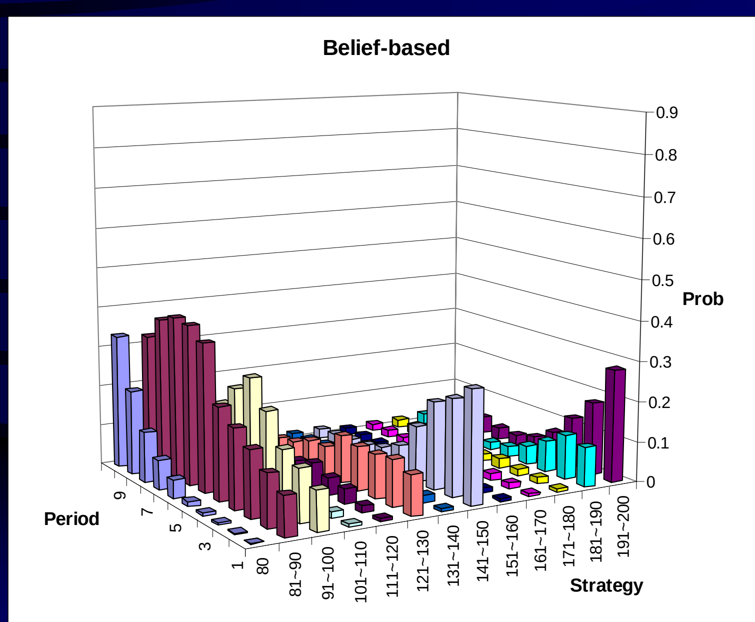
- Substitute functions for parameters

  Easy to estimate

  Allows change within game
- "Change detector" for decay rate ϖ

  $\varpi(i,t)=1-.5[\Sigma_k \ (S_{-i}^k(t) - \Sigma_{\tau=1}^{t} S_{-i}^k(\tau)/t)^2]$

  ϖ close to 1 when stable, dips to 0 when unstable
- $\delta(i,t)=\varpi(i,t)/W$ (W=support of Nash equil'm)



Teaching in repeated games

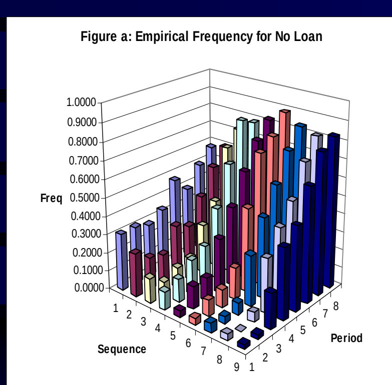
- Finitely-repeated trust game (Camerer & Weigelt E'metrica '88)

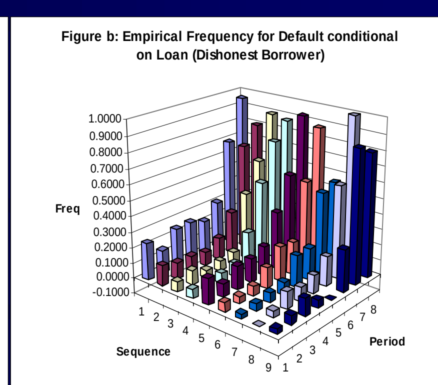
		borrower action	
		repay	default
lender	loan	40,***60***	-100,***150***
	no loan	10,***10***	

- 1 borrower plays against 8 lenders

  A fraction (p(honest)) borrowers *prefer* to repay (controlled by experimenter)

Empirical results (conditional frequencies of no loan and default)



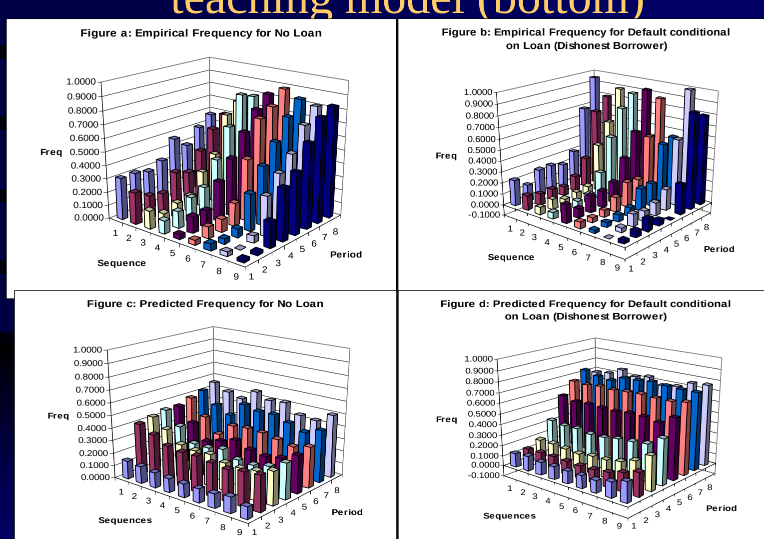


Teaching in repeated trust games

- Some (α=89%) borrowers know lenders learn by fEWA.
  Actions in t "teach" lenders what to expect in t+1
  (Fudenberg and Levine, 1989)

- Teaching: *Strategies* have reputations
- QR Equilibrium: *Borrowers* have reputations (types)

Empirical results (top) and teaching model (bottom)



Teaching in repeated trust games

- Some (α=89%) borrowers know lenders learn by fEWA.
  Actions in t "teach" lenders what to expect in t+1
- ρ (=.93) is "peripheral vision" weight

16 1 2 3 4 5 6 7 8

Repay Repay Repay Default .....

↑

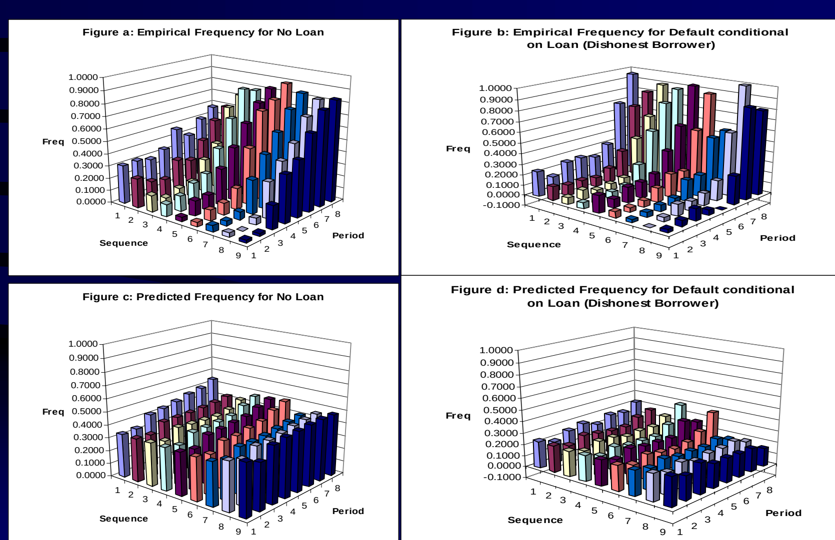
look "peripherally" (ρ weight)

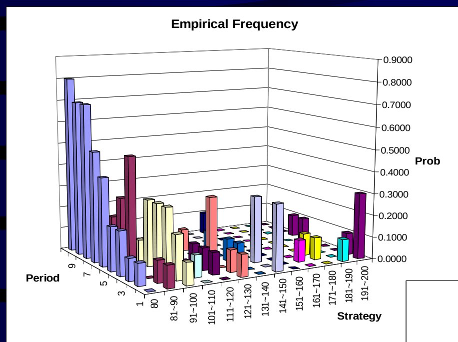
17 1 2 3 ← look back

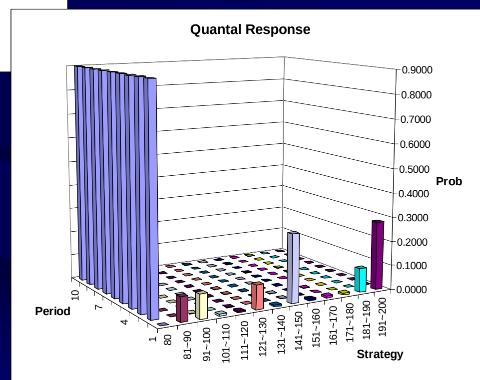
Repay No loan Repay

- Teaching: *Strategies* have reputations
- QREequilibrium: *Borrowers* have reputations (types)

Empirical results vs AQRE fits







Boxscore (out-of-sample calibration)

	hit %	LL	parameters
• AQRE	72%	-1543	η=.91
• Teaching	76%	-1467	τ=.93 α=.89

• # sessions teaching wins 7 of 8 6 of 8

Why do this?

- Models make precise predictions
- Predict effects of p(continuation) (horizon T), payoff, P(nice)
- Potential applications:

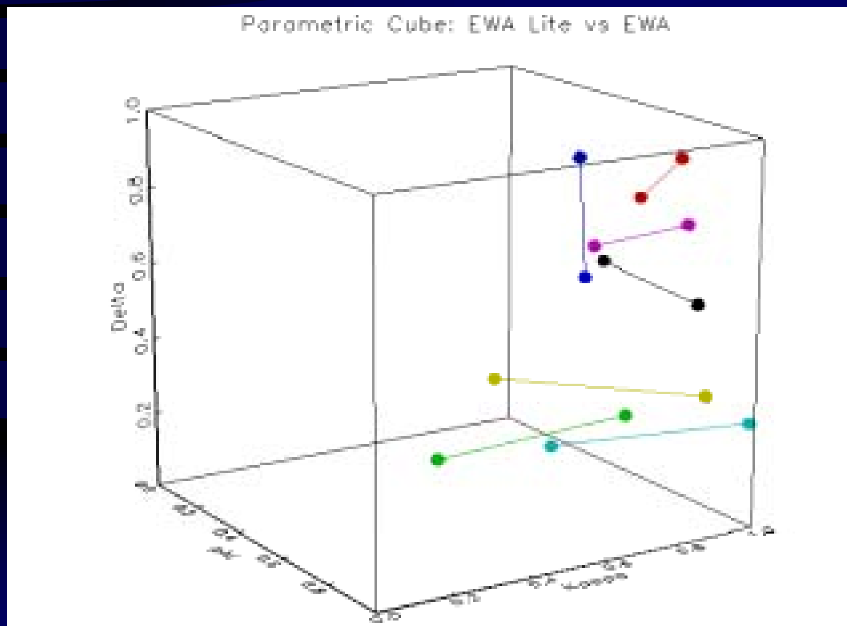
  Contracting & strategic alliances

  Politics (lame duck effects, e.g. Clinton pardons)

  Macroeconomic time-consistency problem
  (Does gov't "teach" public to expect low inflation?)

Table 2: Parameter Estimate τ and fit of thinking steps and QRE

		Self-awareness		Over-confident		
		Projection Bias	Relative Proportion	Opponent Level k-1	Opponent Levels k-1 to 0	QRE
Stahl and Wilson (1995) [3]						
cross game	min	0.00	0.03	0.00	0.00	
(12 games)	median	0.88	0.87	1.23	3.45	
	max	8.46	3.81	2.56	24.11	
	Pooled [1]	13.46	2.68	136.69	3.37	
	fit(sqrt(MSD))	0.18	0.15	0.15	0.15	0.18
	LL	-1176	-1118	-1107	-1106	-1176
Cooper and Van Huyck (2001)						
	min	0.61	0.20	0.08	0.20	
(8 games)	median	1.15	1.13	1.25	1.10	
	max	5.01	1.73	1.87	1.75	
	Pooled	0.79	0.91	0.92	0.81	
	fit(sqrt(MSD))	0.16	0.15	0.11	0.12	0.16
	LL	-193	-192	-185	-186	-197
Costa-Gomes, Crawford and Broseta (2001)						
	min	0.48	1.44	1.23	1.04	
(13 games)	median	0.54	1.81	1.92	1.87	
	max	1.08	2.96	2.42	2.37	
	Pooled	0.65	1.79	1.74	1.94	
	fit(sqrt(MSD))	0.17	0.09	0.09	0.08	0.13
	LL	-649	-565	-553	-555	-599



Predictive fit of various models

	Sample	Thinking		EWA Lite		EWA		Belief-based		Reinforcement with PV		QRE	
		Out-of-sample Validation											
	Size	%Hit	LL	%Hit	LL	%Hit	LL	%Hit	LL	%Hit	LL	%Hit	LL
Mixed Strategies	960	*35%*	-1387	**36%**	**-1382**	*36%*	-1387	*34%*	-1405	*33%*	-1392	*35%*	-1400
Patent Race	1760	*64%*	-1931	*65%*	-1897	**65%**	-1878	53%	-2279	*65%*	**-1864**	40%	-2914
Continental Divide	315	43%	-485	*47%*	-470	**47%**	**-460**	25%	-565	44%	-573	6%	-805
Median Action	160	68%	-119	74%	-104	*79%*	**-83**	**82%**	-95	74%	-105	49%	-187
Pot Games	739	67%	**-431**	**70%**	-436	**70%**	-437	66%	-471	*70%*	-432	65%	-505
Traveller's Dilemma	160	*41%*	-484	**46%**	-445	*43%*	**-443**	36%	-465	*41%*	-561	27%	-720
p-Beauty Contest	580	*6%*	**-2022**	**8%**	-2119	*6%*	-2042	*7%*	-2051	*7%*	-2494	3%	-2502
Pooled	4674	49%	-6860	**51%**	**-6852**	49%	-7100	40%	-7935	46%	-9128	36%	-9037

Feeling: How adding social preferences helps

- Social prefs: $u_1(x_1,x_2)=x_1+\alpha x_2$ (Edgeworth 1898+)
- game 6

game 6	L	R	data	fit(.19)	fit(0)	equil'm
t	<u>6,3</u>	2,1	.38	.45	.66	1
b	4,5	4,5	.62	.55	.34	0
data	.89	.11				
fit(α=.19)	.69	.31				
fit(α=0)	.73	.27				
equil'm	1	0				

- social preference makes (2,1) unattractive, increases unpredicted choice of b

Thinking and learning: Why?

- Cognitive limits on iterated thinking
- Why?

  Limited working memory

  Doubts about rationality or payoffs of others (and doubts about doubts...)
- Why learning?

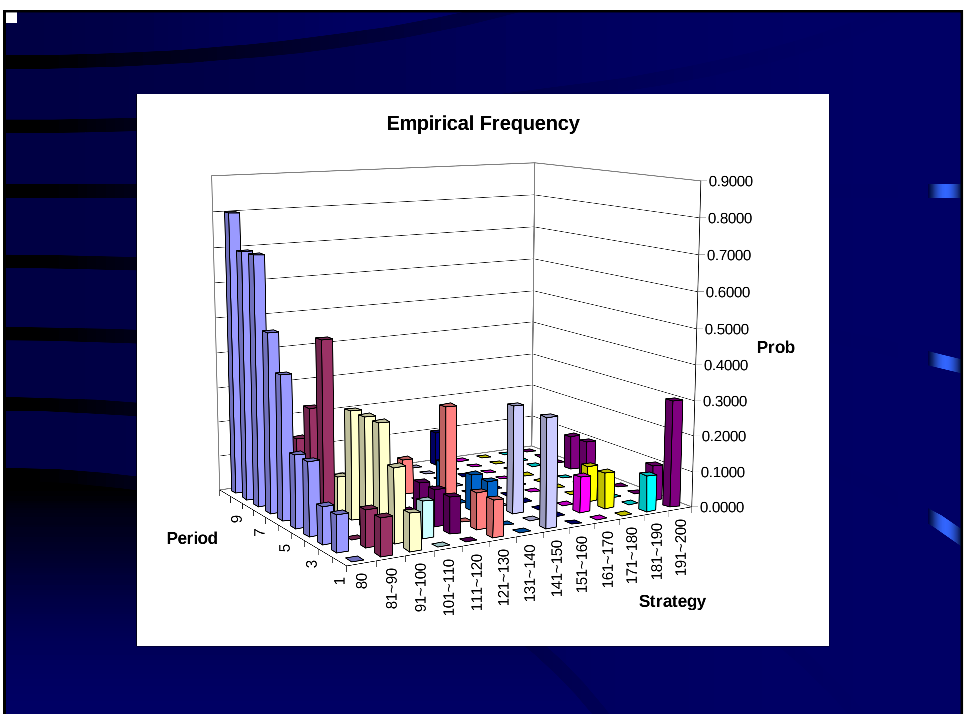
  Efficient compared to thinking

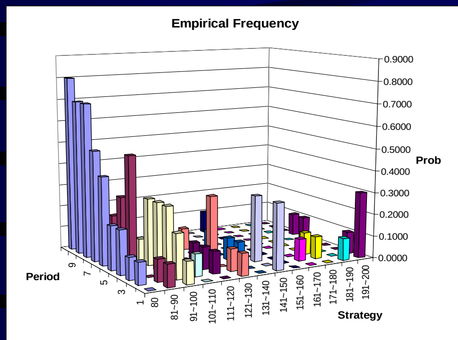
  "Only academics learn by thinking and reading..." (Vernon Smith '94)

Table 4: Economic Value of Advice from Different Learning Theories

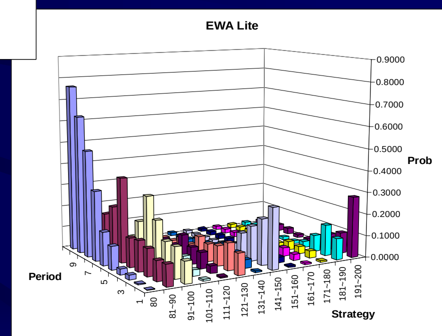
	Total Payoff and Percentage Improvement for Bionic Subjects[1]								
	Observed	EWA Lite		Belief-based		Reinforcement		EWA	
Continental Divide[2]	837	861	2.9%	856	2.3%	738	-11.8%	867	**3.5%**
Median Action[2]	503	510	**1.4%**	507	0.9%	508	1.1%	509	1.3%
Mixed Strategies	334	321	-4.0%	325	**-2.8%**	324	-3.0%	315	-5.7%
Patent Race	467	474	**1.5%**	473	1.2%	472	1.1%	473	1.2%
p-Beauty Contest[2]	519	625	20.4%	625	20.4%	606	16.9%	642	**23.8%**
Pot Games	4244	4964	**17.0%**	4800	13.1%	4642	9.4%	4633	9.2%
Traveller's Dilemma	540	589	9.1%	571	5.8%	556	3.1%	592	**9.8%**
total	7444	8343	**12.1%**	8157	9.6%	7848	5.4%	8031	7.9%

Note: **Bold** is "best practice" in row/measure; underline is worst





Empirical Frequency
Prob
Period
Strategy
0.0000
0.1000
0.2000
0.3000
0.4000
0.5000
0.6000
0.7000
0.8000
0.9000
1
3
5
7
9
80
81~90
91~100
101~110
111~120
121~130
131~140
141~150
151~160
161~170
171~180
181~190
191~200



EWA Lite
Prob
Period
Strategy
0.0000
0.1000
0.2000
0.3000
0.4000
0.5000
0.6000
0.7000
0.8000
0.9000
1
3
5
7
9
80
81~90
91~100
101~110
111~120
121~130
131~140
141~150
151~160
161~170
171~180
181~190
191~200