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The demand for local telephone service: a fully discrete model of residential calling patterns and service choices

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We present an empirical model of households' choices among local telephone service options (for example, between flat-rate and measured service) and the interrelation of these choices with the number and average duration of local calls households make at each time of day to each geographical zone. Using a nested logit model with estimation performed on a randomly selected subset of the households' calling patterns, we calculate elasticities of demand for each local service option, number of calls, average duration, and revenues with respect to the fixed monthly charges and the usage charges for calling under each option. We find moderate price elasticities of number of calls with respect to usage charges for households subscribing to measured service. Nevertheless, raising usage charges has a negligible effect on revenues, since a sufficient number of households either originally subscribe to flat-rate service or convert to flat-rate service in response to higher usage charges. We find a high elasticity of demand for each service option with respect to its fixed monthly fee. This indicates high substitutability among service options. The shift among service options induces new calling patterns, which we find to be a small but not negligible indirect effect.

1. Introduction

■ Most telephone operating companies offer residential customers several options for local service. There are two general categories of service: flat-rate service, under which a household can, for a fixed monthly charge, make an unlimited number of calls within a specified geographical area, and measured service, for which the household pays a lower fixed monthly fee but can make only a specified number (or dollar value) of calls at no extra charge, after which charges are incurred for additional calls. Various flat-rate services differ in the size of the geographical area in which calling is at no extra charge, with higher monthly fees for larger areas. Measured services differ with respect to the threshold number (or dollar value) of calls beyond which the customer is charged.

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The service option that a household chooses depends in general on the household's calling pattern, i.e., on the number and duration of calls the household makes by time of day and distance. Households that make numerous calls or relatively expensive calls (e.g., at high-cost times of day or to relatively distant locations) tend to choose flat-rate service, while households that make few, inexpensive calls tend to choose measured service. Causation also runs in the other direction. Once a household has obtained a particular service, the household's calling pattern is conditional upon that choice, since the marginal price that the household faces for calls is then given.

Previous studies have examined how a shift from flat-rate to measured service affects economic welfare and households' calling patterns. Theory suggests that measured service enhances economic efficiency and that the price elasticity of demand determines the extent of the welfare gain (Alleman, 1977; Mitchell, 1978). Empirical studies have found that the effect of the shift on demand is fairly small and depends on the demographics of the household (Pavarini, 1979; Wilkinson, 1983; Park, Wetzel, and Mitchell, 1983; Park *et al.*, 1983).

We extend the previous empirical work by allowing for: (1) voluntary choice of service by the household (to capture the opposite direction of causality, from calling patterns to service choice); (2) a wider variety of service options; and (3) a more detailed delineation of households' calling patterns that includes the number and average duration of calls by time of day and destination zone. This third extension is particularly important, given the first two, since the detailed calling pattern of a household determines the least costly service option. Extending the theoretical analysis to examine the welfare implications of numerous voluntary service options is an important direction for future research.

Modelling service option choice depends critically on being able to model accurately individual households' calling patterns. One method is to divide calls into several categories on the basis of the time and distance of the call and to estimate regression equations for the number and average duration of calls within each category. The difficulties encountered with this approach are well known and numerous.

(1) For any reasonable number of distinct times of day and distance bands one must estimate a large number of regression equations. Allowing for a full set of cross elasticities entails a generally unmanageable number of parameters.

(2) The dependent variables in these equations are truncated at zero, and there may be zero calls for many of the time and zone categories for any particular household. Consequently, estimating the regression equations by ordinary least squares leads to classic truncation bias (Amemiya, 1974; Heckman, 1976; Lee, 1981). Correction for this bias is complex, particularly given the number of equations and the interrelations among the equations.

(3) The concept of price in these models is problematical. Under measured service the household incurs extra charges for calls beyond a threshold, but not for those below it. As a result, the marginal price for a call in any category depends on the number of calls in that and other categories that the household has previously made during the billing period. Marginal price for any category of calls is thus endogenous to the individual household, not only with the outcome of the regression for that category, but also with the combined outcome for all call categories.

Our model describes households' interrelated choices of local service option and monthly calling pattern in a way that avoids the above-described difficulties. We characterize each household as choosing a particular service option and a particular calling portfolio, where we define a portfolio of calls as a particular number and average duration of calls at each time of day to each distance zone. We specify our model as nested logit, which is a type of probabilistic choice model especially designed for handling interrelated choice situations (McFadden, 1978; Ben-Akiva and Lerman, 1985). With this specification the probability of choosing a particular service option depends on the household's expected portfolio (reflecting, for example, the tendency of households that place many calls to choose flat-rate service). The portfolio that the customer chooses in a given month depends on the chosen service option (reflecting the fact that households' calling patterns depend on the cost per call under their chosen service option).[1]

The set of portfolios among which the household chooses is immense. We therefore perform estimation on a sample of portfolios randomly selected according to a probability distribution that is similar to the distribution of observed portfolios in a sample of households. We include a correction factor in estimation to preserve consistency in the face of this sampling.

Sampling of alternatives to estimate nested logit models has previously been used in several situations for which the choice set is very large. Examples include households' choices of make and model or class of automobile (Manski and Sherman, 1980; Berkovec and Rust, 1985; Mannering and Winston, 1985; Train, 1986), households' choices of dwelling location and unit (Friedman, 1975; Weisbrod, Lerman, and Ben-Akiva, 1980), and travelers' choices of destination (Silman, 1980; Daly, 1982). These studies have used sampling procedures that allow the correction factor to reduce either to a constant, such that it does not affect estimation, or to a very simple function. We build upon this earlier work by using a more flexible sampling procedure that Ben-Akiva and Lerman (1985) call "importance sampling," a procedure previously used in empirical work by Cambridge Systematics, Inc. (1984).

Details of specification and estimation appear in Sections 2 and 3. In Section 4 we report results obtained from data for a sample of residential customers from a local telephone operating company.

2. Specification

■ Divide the times of day into distinct categories labelled $t = 1, \ldots, T$. The geographic areas, or zones, to which a person can call under local service are labelled $z = 1, \ldots, Z$. The number of calls that a household makes to zone z at time t is N_{tz}, and the average duration of these calls is D_{tz}. We define a "portfolio" of calls as a particular number and average duration of calls to each zone during each time of day. More precisely, a portfolio is a particular value of the vector with elements $(N_{11}, \ldots, N_{TZ}, D_{11}, \ldots, D_{TZ})$.[2] Label the set of all possible portfolios as A and a particular portfolio as $i \in A$. Finally, index the available service options by $s = 1, \ldots, S$.

In our application three service options are available to all households and two additional services are available to some households. We describe these options in Table 1. A portfolio is defined on the basis of the 21 time and zone categories given in Table 2.

We observe a household choosing service option s and making portfolio of calls i during a time period.[3] We assume that the probability of our observing a particular (s, i) combi-

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[1] Nested logit models have been used in numerous other contexts to capture the interrelations among discrete choices. In particular, empirical models have been estimated for the number and make and model or class of vehicles that a household chooses (Hensher and Le Plastrier, 1983; Hocherman, Prashker, and Ben-Akiva, 1983; Mannering and Winston, 1985; Train, 1986); the number of vehicles to own and the mode of travel to work, such as automobile or bus (Lerman and Ben-Akiva, 1976; Train, 1980); the mode of travel and the destination (Ben-Akiva, 1973); related appliance choices, such as gas, electric, or oil space heating and central or room air conditioning (Goett, 1984; Goett and McFadden, 1984); and housing location and type (Lerman, 1977). The specification of our model is within the tradition of these studies.

[2] Note that if data are available on the duration of each call, then we can define duration categories, $d = 1, \ldots, D$, and can define a portfolio as a particular value of the vector $(N_{111}, \ldots, N_{TZD})$.

[3] A household can also choose not to acquire access to the phone system. That is, one option is "no service" with the only portfolio available under this option being no calls. The specification in the text allows for this possibility. It might be more reasonable, however, to specify a three-level nested logit model with the "highest" level being whether to acquire any service and portfolio. We do not investigate this issue since our estimation sample consists only of households that have acquired service. The estimation is consistent under either method of handling the choice of access: it is consistent on a subset of alternatives or as the "lower" two levels of a three-level nested logit. The estimated model does not, however, incorporate the factors and parameters that relate to the household's decision of whether to access the system. For simulation of situations in which access is relevant, we can conceivably combine the model with results from an empirical model of access choice with appropriate renormalization of parameters.

TABLE 1 Service Options

Service	Availability	Charges for Calls to Nearest Zone	Charges for Calls to Other Zones
Budget Measured	Available to all customers.	Each call charged seven cents.	Each call charged at rate that varies with time, distance, and duration of call.
Standard Measured	Available to all customers.	$4.00 worth of calling is at no extra charge, then each call is charged as under budget measured service.	
Local Flat-Rate	Available to all customers.	No extra charge.	Each call charged at rate that varies with time, distance, and duration of call.
Extended Local Flat-Rate	Available to customers in only some exchanges.	No extra charge.	No extra charge for calls to some exchanges; charged as under local flat-rate service for calls to other exchanges.
Metropolitan Area Flat-Rate	Available to nonrural customers.	No extra charge.	No extra charge.

nation, given the available options and possible portfolios, is nested logit. Under this assumption the appropriate nesting of alternatives (where, in this case, an alternative is an (s, i) combination) depends on the correlations across alternatives of unobserved factors. In particular, one must group together alternatives that are similar in unobserved factors.

We do not observe most of the specific factors that relate to portfolio choice, such as

TABLE 2 Time and Zone Categories

Zones		Times	
Number	Description	Times of Week during Which Different Rates Are Charged for Calls to That Zone	Rates during That Time
1	Zone immediately surrounding the household's residence.	9 A.M.–9 P.M., Monday–Sunday	Full tariffs
		7 A.M.–9 P.M. and 9 P.M.–Midnight, Monday–Sunday	50% off
		Midnight–7 A.M., Monday–Sunday	86% off
2–6	Geographic bands successively more distant from household's residence.	9 A.M.–9 P.M., Monday–Friday	Full tariffs
		All other times	50% off
7	Specific exchanges outside of zones 1–6, applicable only to households in certain exchanges within a metropolitan area.	9 A.M.–9 P.M., Monday–Friday	12% off, on average
		7 A.M.–9 A.M., 9 P.M.–Midnight, Monday–Sunday	48% off, on average
		Midnight–7 A.M., Monday–Sunday	60% off, on average
		9 A.M.–9 P.M., Saturday–Sunday	56% off, on average
8	Remainder of metropolitan area in which household resides, applicable only to households in certain exchanges within a metropolitan area.	9 A.M.–9 P.M., Monday–Friday	12% off, on average
		7 A.M.–9 A.M., 9 P.M.–Midnight, Monday–Sunday	48% off, on average
		Midnight–7 A.M., Monday–Sunday	60% off, on average
		9 A.M.–9 P.M., Saturday–Sunday	56% off, on average

where the household's friends and relatives live, the time and location of activities that require telephone use, and so on. These factors are, however, similar over all service options for any portfolio. Consequently, we nest together all alternatives with the same portfolio but different service options. On the other hand, the primary factor affecting the choice of service option is cost, which we do observe. Since alternatives with the same service option but different portfolios can be similar with respect to observed factors, but not, at least relatively, with respect to unobserved factors, we do not nest them together.

Under this nesting pattern the nested logit probability of observing option s and portfolio i is

$$P_{is} = \frac{e^{Y_{is}}(\sum_{s'} e^{Y_{is'}})^{\lambda-1}}{\sum_{j} (\sum_{s'} e^{Y_{js'}})^{\lambda}},$$

where Y_{is} is a parametric function of observed factors relating to service option s and portfolio i. This expression for P_{is} is convenient since we can rewrite it as the product of two logit probabilities. Without loss of generality we can decompose Y_{is} into two parts, one that varies over both i and s and another that varies only over i:

$$Y_{is} = W_{is} + V_i/\lambda.$$

We can write P_{is} as the product of the marginal probability of portfolio i and the conditional probability of service option s, given portfolio i:

$$P_{is} = P_i P_{s|i},$$

where

$$P_i = \exp(V_i + \lambda I_i)/\sum_{j \in A} \exp(V_j + \lambda I_j) \tag{1}$$

with $I_i \equiv \ln\left(\sum_{s'=1}^{S} \exp(W_{is'})\right)$ and

$$P_{s|i} = \exp(W_{is})/\sum_{s'=1}^{S} \exp(W_{is'}). \tag{2}$$

The term I_i is the "inclusive price" of portfolio i.

Note that the direction of conditionality in this specification is from portfolio to service option. This does not imply, however, that the household makes choices sequentially in this manner.[4] As in all nested logit models, the direction of conditionality reflects correlations among unobserved factors across alternatives; as such it arises from patterns in the researcher's lack of information, rather than from the households' decision processes.[5]

The coefficient of inclusive value, λ, measures substitutability across alternatives. If substitution is greater within than among nests, then $0 < \lambda < 1$; whereas, if substitution among nests exceeds substitution within nests, then $1 < \lambda$. Given our nesting pattern, the parameter is less than one if households shift to different service options more readily than they shift to different portfolios. It is greater than one if households shift to different portfolios more readily than they shift to different service options.[6]

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[4] If anything, the household probably does the opposite by choosing service option first and portfolio conditional upon service option.

[5] We can generalize the specification by describing the service option choice or portfolio choice as itself nested logit such that the complete model is multilevel nested. Taylor (1979) proposed a similar approach for choice of service option by using an elimination-by-aspects model (Tversky, 1972).

[6] A value of λ exceeding one can, depending on the range of data used in estimation, be inconsistent with a particular description of consumer behavior called the random utility model. There are, however, dynamic aspects of the choice of service option and portfolio that the random utility model does not represent (i.e., the observed portfolio is "built up" over a period of time as households choose to make additional calls, and the service option is chosen before the portfolio is revealed). From a purely statistical perspective, the value of λ indicates relative substitutability within and among nests, and neither possibility can be ruled out *a priori*.

This specification has several advantages. First, since P_s and $P_{i|s}$ are both logit, the model is relatively inexpensive to estimate and easy to interpret. Second, the cost of any portfolio under any service option is simply the bill that the household would receive if it made that portfolio of calls and chose that service option.[7] Threshold values for calling at no extra charge, which are based on either the number or the dollar value of calls, enter the calculation of the cost of a portfolio under a service option in the same way as in the telephone company's calculation of bills. Consequently, we can readily and consistently examine the impact of changes in tariffs and thresholds. Finally, this specification incorporates interrelations among calling patterns and service option choices. The probability of a household's choosing any particular portfolio changes as the tariffs or thresholds associated with any service option change and depends on the portfolio of calls that the household makes.

We now describe the specification of W_{is} and V_i. The only difference among service options is the billing procedure. Consequently, we specify W_{is} to depend only on the cost to the customer of portfolio i under option s (C_{is}) and option-specific constants. The specification of V_i is more complex. A portfolio provides benefits through the information transmitted in calls and extracts opportunity costs through the time spent on the phone.[8] Suppressing differences in coefficients across time and zone categories and households, we specify V_i for a portfolio with N_i calls of average duration D_i (with subscripts deleted to simplify notation) as:

$$V = \theta N \log \phi D - \alpha N D = \theta N \log D - \gamma N - \alpha N D,$$

where $\gamma = -\theta \log \phi$.

We can interpret the specification in the following way.[9] First, each call made by the household provides benefits $\theta \log \phi D$. Thus, calls of zero duration provide no benefits. The benefits of a call increase with its duration (more time for information transfer), but each extra minute spent talking has decreasing marginal utility. We can think of the parameter ϕ as measuring the rate of information transfer. We expect it to be positive, but it could be either above or below one. Consequently, γ could be either negative or positive. We can think of the parameter θ as measuring the benefits from the information that can potentially be transferred by a call. This is necessarily positive. The model assumes that the benefits from N calls are simply N times the benefits from a single call. Second, the time spent talking on the telephone has an opportunity cost αND. Thus, the opportunity cost depends on the total duration ND of time on the phone, not on the separate values of N and D. Presumably α is positive, so that households perceive increasing opportunity cost to the time spent talking.

We allow the parameters to vary in two ways. The benefits from a call depend critically on its destination, while the opportunity cost depends on the time of day the call is placed. To reflect this we allow θ to vary over zones and α to vary over time periods. In addition, earlier studies have consistently found that household demographics affect the households' calling patterns (Taylor, 1980; Infosino, 1980; Brandon, 1981; Park *et al.*, 1983). We specify the benefits from information θ to vary with income and the number of phone users in the household, and the opportunity cost of calling α to depend on income.

3. Estimation procedure

■ Estimation of the parameters entering P_i is complicated by the fact that the number of possible portfolios (elements of A) is immense. Thus, enumeration of every component in

[7] Since cost varies over i and s, it enters an element of W_{is}. We estimate its impact on choice of portfolio and service option in $P_s|_i$.

[8] We thank Charles Manski for suggesting these concepts to us.

[9] From a purely functional-form perspective, we can view this specification without interpretation as simply linear in N and nonlinear in D with interaction allowed between N and D.

the denominator of P_i is infeasible. Instead, we use a sample of portfolios for each household. The sample includes the household's chosen portfolio and a subset of the portfolios that it did not choose.

We construct the sample of portfolios for each household by drawing from the set of all portfolios according to a prespecified probability distribution and by adding the household's chosen alternative. Denote by B the sample of portfolios constructed for a particular household. Denote by $\pi(B|i)$ the conditional probability of constructing the subset B, given that the chosen portfolio is i.[10] The joint probability of drawing a chosen alternative i and a subset of alternatives B is

$$\pi(i,B) = \pi(B|i)P_i.$$

Thus, the conditional probability of alternative i's being chosen, given a sample of portfolios B, is

$$\pi(i|B) = \frac{\pi(B|i)P_i}{\sum_{j\in B} \pi(B|j)P_j}, \tag{3}$$

which exists if $\pi(B|j) > 0$ for all $j \in B$. McFadden (1978) established this condition, called the positive conditioning property, as a condition for a consistent estimator for the logit model with samples of alternatives.

Rewrite (3) in logit form by using (1) for P_i:

$$\pi(i|B) = \frac{\exp(V_i + \lambda I_i + \ln \pi(B|i))}{\sum_{j\in B} \exp(V_j + \lambda I_j + \ln \pi(B|j))}. \tag{4}$$

McFadden (1978) has shown that under normal regularity conditions, maximizing the conditional log likelihood function,

$$\sum_{h=1}^{H} \ln \pi_h(i|B), \tag{5}$$

(where subscript h denotes particular households within a sample of H households) yields consistent estimates of the unknown parameters. Note that the logit model (4) is the same as P_i except that the summation in the denominator is over all portfolios in the constructed set B rather than in the entire set A, and the exponentiated terms include an additive alternative-specific correction for the bias introduced by the sampling of alternatives; we constrain the coefficient of this variable to be one.

We specify the sampling distribution for portfolios and give the value of $\ln \pi(B|i)$ that results from this distribution in the Appendix. We construct a subset of ten portfolios, consisting of the household's chosen portfolio plus nine other sampled portfolios, for each household.

4. Empirical results

■ We observed the number and average duration of local calls made in November of 1984 in each time and zone category for a sample of residential customers of a local telephone operating company on the East Coast. The sample is stratified random on the basis of households' locations. But it is not representative for two reasons: (1) some geographical areas are not included because households' local calling records were not available; and (2) socioeconomic characteristics of the sampled households were obtained through a survey, for which the nonresponse rate was fairly high. We rely in the analysis on the fact that estimation of logit models on nonrepresentative samples is consistent if the sample is drawn on the basis of exogenous factors (Manski and McFadden, 1981.)

[10] Since B necessarily includes the chosen portfolio, $\pi(B|j) = 0$ for $j \notin B$.

Service option choice conditional on portfolio. Table 3 gives estimated parameters for each of three specifications of the cost function that enters $P_{s|i}$. With each specification cost has a significantly negative impact, in that the probability of choosing a service option decreases as the cost of that option increases (with the cost of other options held constant). We obtain the best fit with ln (C_{is}), which yields a coefficient of about two. This implies that the ratio of probabilities for any two service options is inversely proportional to the square of the ratio of their costs:

$$\frac{P_{s|i}}{P_{s'|i}} = k\left(\frac{C_{is'}}{C_{is}}\right)^{2.08},$$

such that the relative probabilities change at an increasing rate as the relative costs change. This result gives credence to the popular notion that households are relatively insensitive to small cost differences when they select service options and that they become increasingly sensitive as the differences become larger.

We find that the option-specific constants are highly significant. Mechanically, the estimated values for these constants are those that result in the average probability (i.e., predicted share) for each option being equal to the actual (i.e., observed) share in the sample.

TABLE 3 Logit Model of Service Choice, Given Portfolio

Service Options	Number of Customers with Alternatives Available		Number of Customers Who Chose the Alternative
Budget Measured	2963		579
Standard Measured	2963		855
Local Flat-Rate	2963		1120
Extended Local Flat-Rate	84		20
Metropolitan Area Flat-Rate	1873		389
	Estimated Parameters (t-statistics in parentheses)		
	Model 1: ln C_{is}	Model 2: C_{is}	Model 3: C_{is} Divided by Income of Household in Thousands of Dollars
C_{is}: cost of portfolio under designated service option (includes monthly fixed fee and charges for calls, in 1984 dollars; specified differently in each model).	−2.081 (23.87)	−.09111 (17.42)	−.4538 (14.51)
Option-Specific Constants:			
Standard Measured	1.228 (17.83)	.6135 (11.04)	.5079 (9.282)
Local Flat-Rate	2.635 (24.74)	1.576 (20.94)	1.081 (18.07)
Extended Local Flat-Rate	2.254 (7.880)	1.123 (3.922)	.8614 (3.279)
Metropolitan Area Flat-Rate	3.757 (21.82)	3.474 (17.23)	1.317 (13.60)
Number of Households	2963	2963	2963
Initial Log Likelihood	−3812.4	−3812.4	−3812.4
Log Likelihood at Convergence	−3356.0	−3487.8	−3562.3

Intuitively, these constants capture the average effect of all unincluded variables. Perhaps the most important unincluded variable relating to each service option is the insurance quality of the option. Under flat-rate services, for an additional fixed charge the household is provided an upper limit on charges for calling within a certain geographic area. The wider the area of calling at no extra charge, the more valuable is the insurance the option provides.

The estimated constants are consistent with this concept. Budget measured service provides no insurance, since there is a charge for each call: its constant, which is zero by normalization, is lower than those estimated for all the other service options. Standard measured service insures against cost variation within a range of calls (i.e., below the threshold), while local flat-rate service provides complete insurance for all calls in a local area; the constant for local flat-rate service exceeds that for the standard measured service. Finally, metropolitan area flat-rate service provides insurance for a wider area than local flat-rate service and local extended flat-rate services, and its constant is consequently greater.[11]

□ **Portfolio choice.** Table 4 presents the estimated parameters of the model of portfolio choice, with inclusive value based on service-choice model 1 in which cost enters in logarithmic form.[12] All of the parameters enter with the expected signs and reasonable relative

TABLE 4 **Logit Model of Service Option Choice Conditional on Portfolio**

Alternative Set: The household's chosen portfolio plus nine portfolios selected randomly from the set of all available portfolios.

Explanatory Variable	Coefficient	t-Statistic
Benefits of Information (θ):		
$N \log D$ for calls to zone 1	.0239	11.8
$N \log D$ for calls to zones 2–6	.0410	13.10
$N \log D$ for calls to zones 7–8	.0474	11.11
$\sum_{\text{zones}}$ (Population of zone in millions) $N \log D$ for zone	.00752	14.58
(Income of household in thousands of \$)($N \log D$ for all zones)	$.213 \times 10^{-3}$	3.13
(Number of telephone users in household)($N \log D$ for all zones)	$-.755 \times 10^{-3}$	4.035
Rate of Information Transfer ($-\gamma$):		
Total number of calls (N to all zones)	-.890	8.525
Opportunity Cost of Conversation Minutes ($-\alpha$):		
Total duration at 9 A.M.–9 P.M. to zone 1	−.00472	10.11
Total duration 7 A.M.–9 A.M. and 9 P.M.–midnight to zone 1	−.00438	8.475
Total duration midnight–7 A.M. to zone 1	−.00504	5.158
Total duration for 9 A.M.–9 P.M. to zones 2–8	−.00141	1.866
Total duration for 9 P.M.–9 A.M. to zones 2–8	−.0111	12.56
(Household income in thousands of \$)(Total duration all zones and times)	$-.0606 \times 10^{-3}$	3.22
Other Variables		
Inclusive value of service option choice (using model 1 of Table 3)	4.178	13.68
Sampling correction factor (coefficient is constrained to 1.0)	1.0	-·-
Number of Households	3038	
Log Likelihood at Zero	−7125.9	
Log Likelihood at Convergence	−6242.8	

[11] We cannot compare extended local flat-rate service with local flat-rate service since the former is only available to some households, so that the unincluded variables are averaged over different populations.

[12] The estimated parameters of the portfolio choice model are essentially the same under all three specifications of the service choice model given in Table 2.

magnitudes. Their interpretation is generally straightforward at this point. There are, however, two aspects of the parameter estimates that warrant discussion.

First, the coefficient of inclusive value exceeds one. This implies that substitution among nests (i.e., from one portfolio to another) occurs more readily than substitution within nests (i.e., from one service option to another.) Stated more directly, households respond to price changes more readily by adjusting their calling patterns than by shifting to different service options.

Second, we estimate that the function V_i, and hence the probability of choosing portfolio i, decrease in the number of calls. We expect this result for two reasons.

(1) Recall that there are multiple time-of-day and destination zone categories. As the total number of calls increases, the number of portfolios that are possible with that number of calls increases. For example, there is only one portfolio of no calls, but there are 21 portfolios—when we ignore duration—associated with making a total of one call—a portfolio for each time and zone category in which that one call could be made. Therefore, if the probability of making a certain total number of calls increases with the number of calls, but increases less rapidly than the number of portfolios that are possible with that number of calls, then the probability of each portfolio must decrease in the number of calls. This is what is occurring in the estimated model.

(2) It is reasonable to expect that the probability of a household's making a particular number of calls reaches a maximum at some finite number of calls. The number of portfolios that are possible with a certain number of calls increases with the number of calls, but does so at a continuously decreasing rate. Consequently, if V_i decreases linearly with the number of calls (as the estimates in Table 4 indicate), then the probability of making a certain number of calls first increases with the number of calls (with the expansion of the number of possible portfolios dominating the decrease in V_i minus the cost of each portfolio), but eventually decreases (when the decrease in V_i, which is linear in the number of calls, starts to dominate the diminishing expansion in the number of portfolios).

5. Estimated aggregate price elasticities

■ Price elasticities in the model vary over customers and, for any customer, depend on current prices and all other factors entering the model. Correspondingly, aggregate price elasticities are specific to a particular population at a particular time. To show the order of magnitude of the elasticities implied by the model, we calculate price elasticities for the estimation sample.[13] These appear in Table 5.

We find that own-price elasticities for the monthly fixed charge of each service are fairly high. This indicates a high degree of substitutability among the services.[14] The cross price elasticities reflect initial market shares. For example, since the demand for local flat-rate service is very large compared with the demand for the other services, the shift in demand that results from an increase in the fixed monthly charge of local flat-rate service

[13] We adjusted the alternative specific constants in the service choice model to reflect actual shares in the telephone company's service area. See Train (1986, Section 2.6) for a discussion of the consistency of such an adjustment.

[14] These high elasticities do not contradict the earlier conclusion (based on the estimated value of the inclusive value coefficient) that substitution among portfolios is higher than that among services. (Or, more precisely, they do not imply that the elasticity of portfolio choice is even higher.) An inclusive price coefficient that exceeds one indicates that if the cost of a particular portfolio under a service option increases, and the costs of all other portfolio-service option combinations remain the same, then households will switch to other portfolios more readily than they will shift to other service options. In this situation households can avoid the extra charge by changing either portfolio or service option. The elasticities with respect to fixed monthly charges, given in Table 5, represent a different situation. A household can avoid the additional fixed charge for a service only by changing service options; if it changes portfolio and not service option, then it must still bear the additional charge.

TABLE 5 **Estimated Price Elasticities**

	Monthly Fixed Charge for Service (initial shares in parentheses)							
	Budget Measured (.05)	Standard Measured (.15)	Local Flat-Rate (.74)	Extended Local Flat-Rate (.01)	Metropolitan Area Flat-Rate (.05)	Charge for Calls to Zone 1	Charge for 1st Min. for Calls to Zones 2–8	Charge for Addl. Mins. for Calls to Zones 2–8
Number of Households Choosing:								
Budget measured	−1.06	.16	1.52	.02	.11	−.41	−.04	−.04
Standard measured	.05	−1.38	1.36	.01	.14	−.26	−.04	−.03
Local flat	.07	.25	−.46	.01	.11	.08	−.00	−.01
Extended local flat	.11	.11	.45	−.91	.34	.08	.06	.06
Metro flat	.04	.22	1.07	.04	−2.19	.07	.21	.21
Number of Calls	.14	.14	−.29	−.00	−.06	−.02	−.00	−.00
Average Duration	.00	.00	.00	.00	.00	.00	.01	−.01
Total Revenues	.04	.18	.56	.01	.02	.05	.07	.06

constitutes a relatively high percentage increase in the demand for other services. Conversely, the extended local flat-rate service option captures only a very small share of the market. Therefore, an increase in the fixed charge has a relatively small effect on the demand for the other services. This occurs even though the own-price elasticity for extended local flat-rate service is about twice that of local flat-rate service.

Our results show that increasing the fixed charge for each service affects the number of calls made by households in the expected way. Increases in the fixed charge of measured services shifts customers to flat-rate services, under which customers make more calls since the marginal price of calling is lower. Conversely, increasing the fixed charge of flat-rate services decreases the amount of calling since customers shift to measured services, under which calling is relatively expensive.

As expected, increasing the charges for marginal calls—particularly for zone 1 calls, which constitute about 80% of all calls—shifts customers from measured to flat-rate services. These elasticities are generally fairly low, however.

We find that charges for marginal calls have a very small effect on the total number of calls households make. This is so largely because in the great majority of cases customers are not charged for marginal calls. Zone 1 calls are at no extra charge to four-fifths of customers (those choosing flat-rate services). All zone 2–8 calls are at no extra charge for customers with metropolitan area flat-rate service, and many of these calls are at no extra charge under extended local flat-rate service. Customers with standard measured service do not pay for marginal calls if they are below the threshold of calling at no extra charge.

To determine the price response of customers who face charges for marginal calls, we consider those customers with budget measured service. The own-price elasticity of zone 1 calls for these customers is about one-half (−.45 to be precise). For customers with standard measured service, some of whom make calls beyond the threshold for calling at no extra charge and some of whom do not, the elasticity is −.38, which is consistent with an elasticity of somewhat less than one-half for customers who actually face charges for calling on the margin.[15]

[15] Note that for households with budget or standard measured service the elasticity of option shares with respect to the charge for calls is lower than the elasticity of the number of calls made (−.41 compared with −.45 for households with budget measured service and −.26 compared with −.38 for households with standard measured service). These relative magnitudes are consistent with the earlier statement (based on the inclusive value coefficient) that if households can avoid a price increase by either changing portfolios or changing services, the households will respond by shifting to different portfolios more readily than by changing service options.

These estimates are considerably higher than those obtained by Park, Wetzel, and Mitchell (1983) for households with mandatory measured service. They estimate that the own-price elasticity of local calls is -3.0 times the price for each call (in 1979 dollars) and point out that when households have the option to switch to flat-rate services, the elasticity will be lower. With the prices in our study, their formula gives an elasticity of $-.15$. Direct comparison is difficult, however, since our estimates of $-.45$ and $-.38$ are for households that chose measured services, while those of Park, Wetzel, and Mitchell are for all households under mandatory measured service.

We find that the average duration of a call is essentially unaffected by charges associated with calling. The charge for calls to zone 1 and the initial period charge for calls to zones 2–8 do not depend on the call's duration. Consequently, except for the effect of shifts to different services, we do not expect the average duration of a call to respond to these charges. We do, however, expect the cost of additional minutes of conversation for calls to zones 2–8 to affect the average duration of a call. Only a fifth of all calls are to zones 2–8, however, and many of these calls are made by customers who use standard measured service and are below the threshold for calling at no extra charge or who used metropolitan area flat-rate service. Given the few calls for which duration charges are actually levied, the effect of changes in such charges on the average duration of all calls is necessarily small.

We find that changes in calling charges also have a small effect on telephone company revenues. Since there is no extra charge for most calls, price increases translate into relatively small revenue gains independently of how responsive or nonresponsive customers are. The elasticities in Table 5 are for each of the three separate calling charges: for calls to zone 1 and for initial and additional minutes of conversation on calls to zones 2–8. Consequently, a 1% increase in one type of charge represents a less than 1% increase in all charges for calling. It is perhaps more natural for us to consider the elasticity with respect to increases in all charges for calling; this is simply the sum of the elasticities with respect to each individual charge: .18. This figure is still fairly small. This indicates that raising charges for calls, while maintaining a set of service offerings that allow numerous opportunities for calling at no extra charge, is not a particularly effective method for raising revenues.

6. Conclusions

■ Using a fully discrete model of calling patterns and service choice allows us to estimate households' responses to changes in fixed charges for service options, calling charges for initial and additional minutes by time of day and distance, and thresholds for calling at no extra charge. The responses we estimate incorporate the interdependence of the choice of service option with the number and duration of calls made by the household by time of day and distance. Furthermore, the estimation procedure is fairly simple. Had we used standard regression approaches and appropriately corrected for endogeneity and truncation, our analysis would have been more complex. Even with such corrections, we could not have readily handled the changes in the thresholds for calling at no extra charge.

Our analysis provides substantive information on how households respond to price changes for service options and calls. If households face a price increase that they can avoid by changing either their calling patterns or service options, they tend to change their calling patterns more readily than their service options. When households face positive prices for calls (e.g., under measured services), they respond to an increase in the price by reducing moderately the number of calls made (an elasticity in the range of one-half); in addition, although to a lesser extent, they shift to services that allow calling at no extra charge. On average, however, the elasticity of the number of calls made with respect to calling charges is very small, since extra charges are not levied for most calls. For price increases that can be avoided only by changing service options (e.g., fixed charges for service options), households switch services fairly readily. This indicates, as we would expect, that households consider the various options close substitutes.[16] Furthermore, as the price of one service increases, households shift from that service to other services at an *increasing* rate; that is, the own-price elasticity of each option share increases with its price.

Appendix

The task is to specify a sampling distribution for portfolios and to calculate $\ln \pi(B|i)$ on the basis of this distribution. A straightforward way to sample portfolios is to specify independent probability distributions for the number of calls and the average duration of calls to each zone during each time of day. Let $G_{tz}(\cdot)$ by a cumulative distribution, specified by the researcher, for number of calls N_{tz} to zone z during time t. Denote the corresponding density function as $g_{tz}(\cdot)$. Let $H_{tz}(\cdot)$ be a cumulative distribution for average duration D_{tz} of calls to zone z during time t, with corresponding density $h_{tz}(\cdot)$. We sample a portfolio of calls by drawing an N_{tz} and D_{tz} from distributions $G_{tz}(\cdot)$ and $H_{tz}(\cdot)$ for each tz. The probability of constructing subset B for a household that chose alternative i is therefore

$$\pi(B|i) = k \prod_{\substack{j \in B \\ j \neq i}} \prod_{t=1}^{T} \prod_{z=1}^{Z} g_{tz}(N_{tz}^{j}) h_{tz}(D_{tz}^{j})$$

for B containing i and $\pi(B|i) = 0$ for B not containing i, where N_{tz}^{j} and D_{tz}^{j} are the number of calls and the average duration of calls to zone z at time t in portfolio j, and k is a normalizing constant defined such that $\pi(B|i)$ sums to one over elements of B. Taking logs, we obtain

$$\ln \pi(B|i) = \tilde{k} - \sum_{t=1}^{T} \sum_{z=1}^{Z} \ln g_{tz}(N_{tz}^{i}) - \sum_{t=1}^{T} \sum_{z=1}^{Z} \ln h_{tz}(B_{tz}^{i}). \tag{A1}$$

This is the correction term that enters the conditional likelihood function. Note that since $\tilde{k}$ is constant over elements of B, it has no effect on $\pi(i|B)$ in (4), and consequently it need not be included in the correction term.

Our final task, therefore, is to specify $G_{tz}(\cdot)$ and $H_{tz}(\cdot)$. Since N is discrete and D continuous, we give below a candidate distribution of each type and its corresponding correction factors.

☐ **Exponential distribution for the number of calls made.** The exponential distribution is

$$G_{tz}(N) = 1 - e^{-N/\mu_{tz}},$$

where μ_{tz} is the mean number of calls at time t to zone z (chosen to match, perhaps, the observed means for the sample of households). We draw a random number q uniformly from the unit interval, and define

$$N_{tz} = INT(-\mu_{tz} \ln q),$$

where INT denotes the nearest integer value. The associated density value is

$$\ln g_{tz}(N_{tz}) = \ln (G_{tz}(N_{tz} + 1) - G_{tz}(N_{tz})) = (N_{tz}/\mu_{tz}) + \ln (1 - e^{-1/\mu_{tz}}).$$

☐ **Truncated alpha-Erlang distribution for the duration of calls.** For duration we consider an alpha-Erlang distribution, truncated at maximum duration MAX_{tz} and renormalized. The alpha-Erlang distribution is

$$H_{tz}(D) = 1 - \exp(-D/\beta_{tz}) \left(\sum_{i=0}^{\alpha_{tz}-1} \frac{(D/\beta_{tz})^{i}}{i!} \right),$$

where α_{tz} and β_{tz} are the parameters of the distribution such that the mean equals $\alpha_{tz}\beta_{tz}$ and the variance equals $\alpha_{tz}(\beta_{tz})^2$.

Let η_{tz} be the mean and ω_{tz} be the standard deviation of the duration of calls to zone z at time t. We set $\alpha_{tz} = INT(\eta_{tz}/\omega_{tz})^2$ and $\beta_{tz} = \eta_{tz}/\alpha_{tz}$, truncate the distribution at MAX_{tz}, and renormalize. The procedure is as follows. First, we calculate $H_{tz}(MAX_{tz})$. Then we draw α_{tz} random numbers from the unit interval and label these q_i for $i = 1, \ldots, \alpha_{tz}$. Next we define $D_{tz} = -\beta_{tz} \ln \prod_{i=1}^{\alpha_{tz}} q_i$. Finally, we determine whether $D_{tz} \geq MAX_{tz}$. If so, we return to the second step and repeat the procedure. If not, we use D_{tz} as the selected value of average duration for time t and zone z. The associated density value is:

$$\ln h_{tz}(D_{tz}) = \ln \{ \beta_{tz}^{-\alpha_{tz}} D_{tz}^{\alpha_{tz}-1} \exp(-D_{tz}/\beta_{tz}) / [(\alpha - 1)! H_{tz}(MAX_{tz})] \}.$$

[16] Households nevertheless switch among services considerably less than we would expect under pure cost minimization.

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