LECTURE / DISCUSSION

Mixed Logit

Mixed Logit

- Allows for complete relaxation of IIA
- Incorporates fact that different customers have different preferences
- Accounts for correlation in unobserved factors over repeated choices by each customer

Specification

$$U_{in} = \beta_n X_{in} + \varepsilon_{in}$$

$$\beta_n \sim g(\beta \mid b, W)$$

where b is mean of β_n's
W is covariance of β_n's

Conditional of β_n, model is logit:

$$L_{in}(\beta_n) = \frac{e^{\beta_n X_{in}}}{\sum_j e^{\beta_n X_{jn}}} \quad .$$

However, each person's β_n is not known. Integrate β_n over all its possible values:

$$P_{in} = \int L_{in}(\beta) \cdot g(\beta|b,W)d\beta \quad .$$

P_{in} is the mixed logit probability.

Example
Customer's Choice of Energy Supplier

Supplier attributes:

Fixed price in cents per kWh
Length of contract
Local utility
Well-known company
Time-of-day rates (11¢ in day, 5¢ at night)
Seasonal rates (10¢ in summer, 8¢ in winter, 6¢ in spring / fall)

Simple Logit Model: Same Coefficients for All Customers

	Coefficient
Price	-0.625
Contract	-0.108
Local utility	1.44
Well-known company	0.996
Time-of-day rates	-5.46
Seasonal rates	-5.84

Willingness to pay for:

1 extra year of contract	-0.17
Local utility	2.3
Well-known company	1.6
Time-of-day rates	-8.7
Seasonal rates	-9.3

Mixed Logit: Distribution of Coefficients in Population

	Estimated Parameter*	Standard Errors*
Price	-.8827	.050
Contract mean	-0.2125	.026
Std. deviation	.3865	.028
Local utility mean	2.2277	.127
Std. deviation	1.7514	.137
Well-known company mean	1.5906	.100
Std. deviation	0.9621	.098
Time-of-day rates	2.1328	.054
Std. deviation	0.4113	.040
Seasonal rates	2.1577	.051
Std. deviation	0.2812	.022

Mixed Logit Fits Much Better Than Standard Logit

Log likelihood:

at zero	-5972.
standard logit	-4959.
mixed logit	-3619.

Likelihood ratio test of standard logit versus mixed logit:

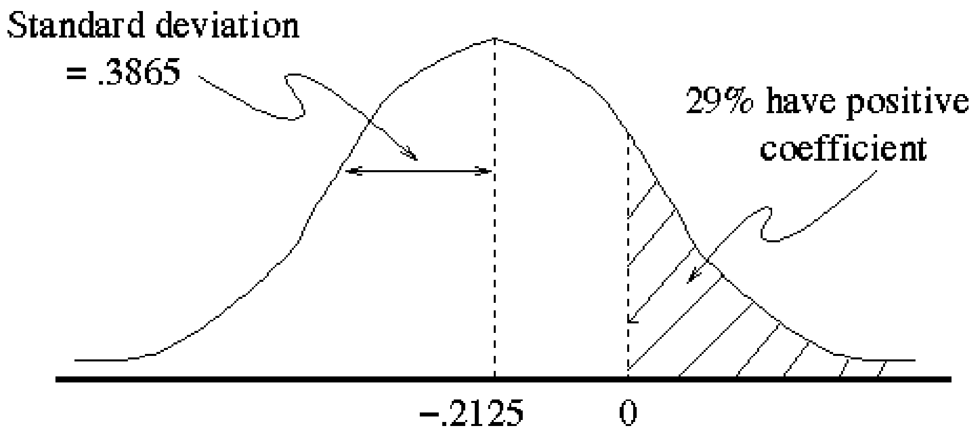
2(4959 - 3619) = 2680

5 extra parameters

Critical chi-squared with 5 d.f. = 11.07

Reject standard logit relative to mixed logit

Distribution of Coefficients



Contract Length

Note: $$z = \frac{\beta - \text{mean}}{\text{std. dev.}} \sim N(0,1)$$

So:

$$\text{Prob}(\beta > 0) = \text{Prob}\left(z > \frac{-\text{mean}}{\text{std. dev.}}\right.$$

$$= \text{Prob}\left(z > \frac{.2125}{.3865}\right.$$

$$= \text{Prob}(z > .5498)$$

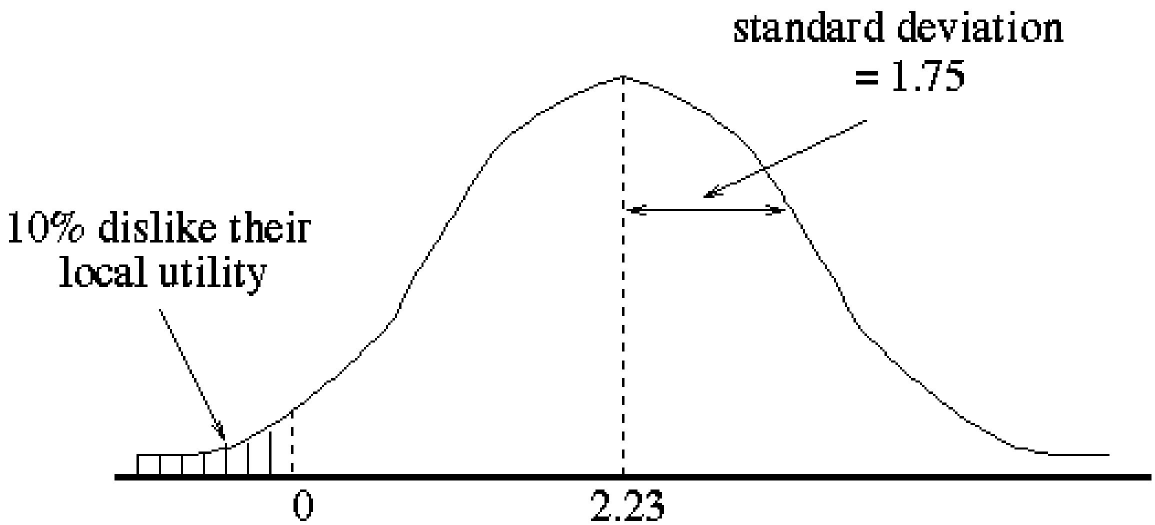
$$= 1 - \text{Prob}(z < .5498)$$

$$= 1 - .79 = .29$$

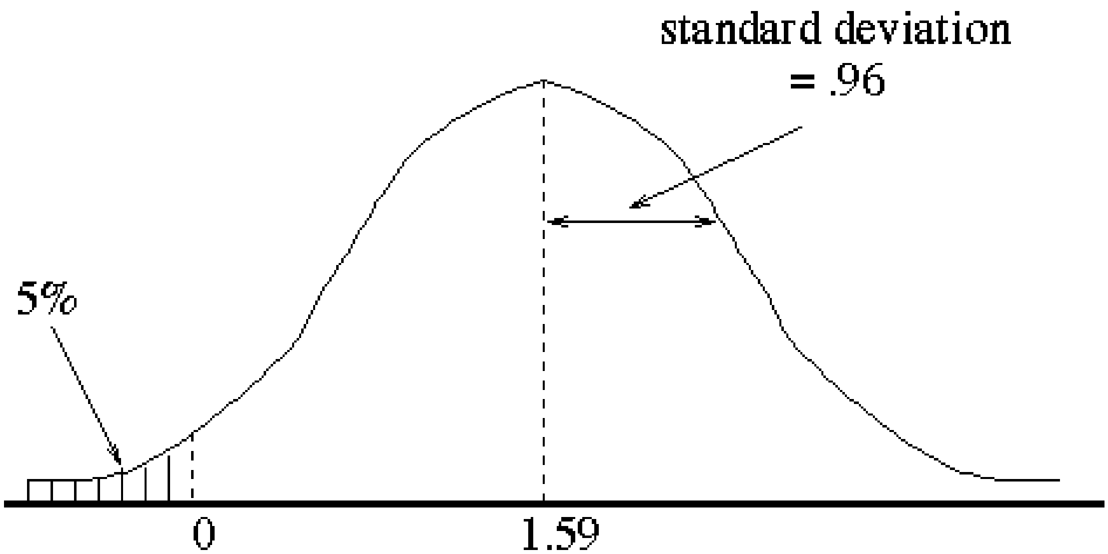
TABLE B.1 Cumulative Normal Distribution (Table Entry is $\Phi(z) = Prob(Z \le z)$)

z	*0.00*	*0.01*	*0.02*	*0.03*	*0.04*	*0.05*	*0.06*	*0.07*	*0.08*	*0.09*
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Local Utility



Well-Known Company



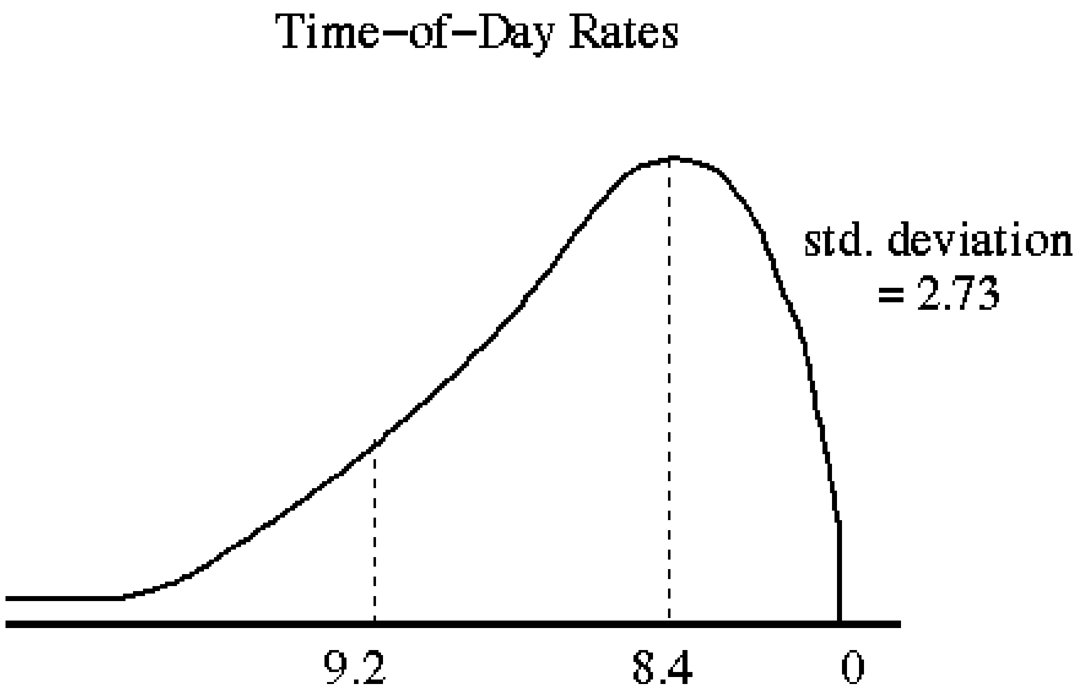
Log-Normal Distributions

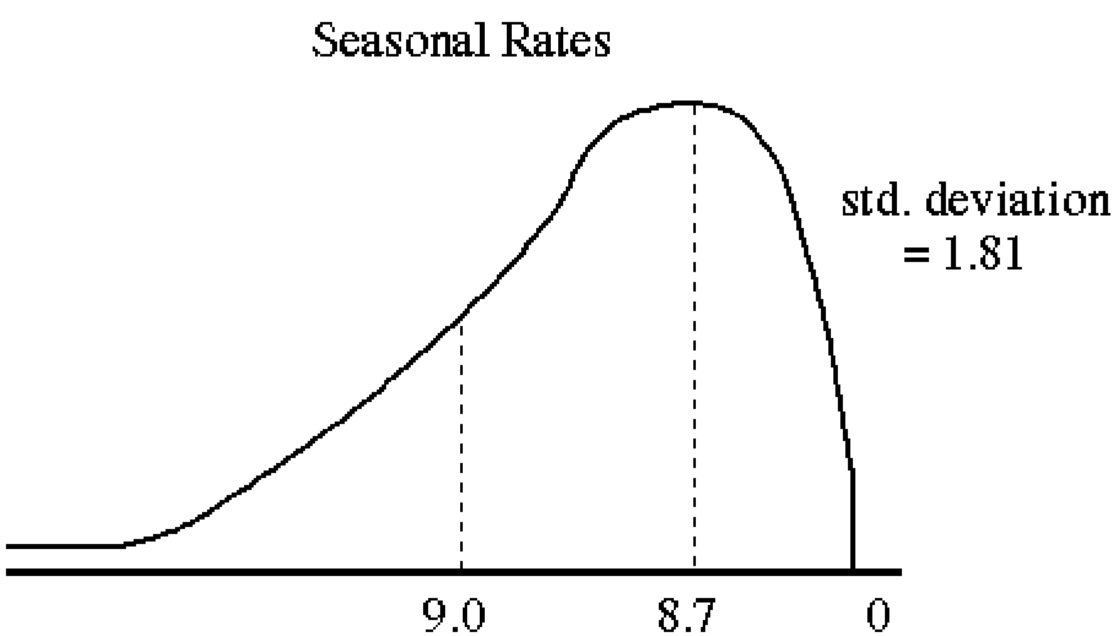
log (β) ~ Normal with mean m and standard deviation s

mode: exp (m)

mean: exp $(m + s^2/2)$

std. dev.: $\exp\left(m + \frac{s^2}{2}\right) \cdot \sqrt{\exp\left(\frac{s^2}{2}\right) - 1}$


Time-of-Day Rates
std. deviation
= 2.73
9.2
8.4
0


Seasonal Rates
std. deviation
= 1.81
9.0
8.7
0

$$\text{Willingness to pay} = \frac{\text{Attribute}'\text{s coefficient}}{\text{Price coefficient}}$$

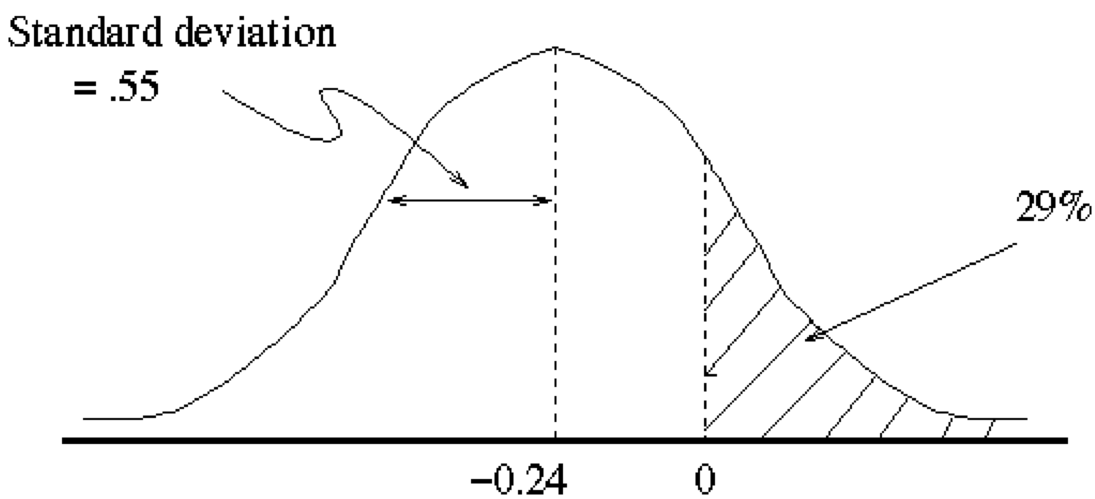
If price coefficient is fixed, WTP follows the same distribution as coefficient, but divided by price coefficient.

Example:

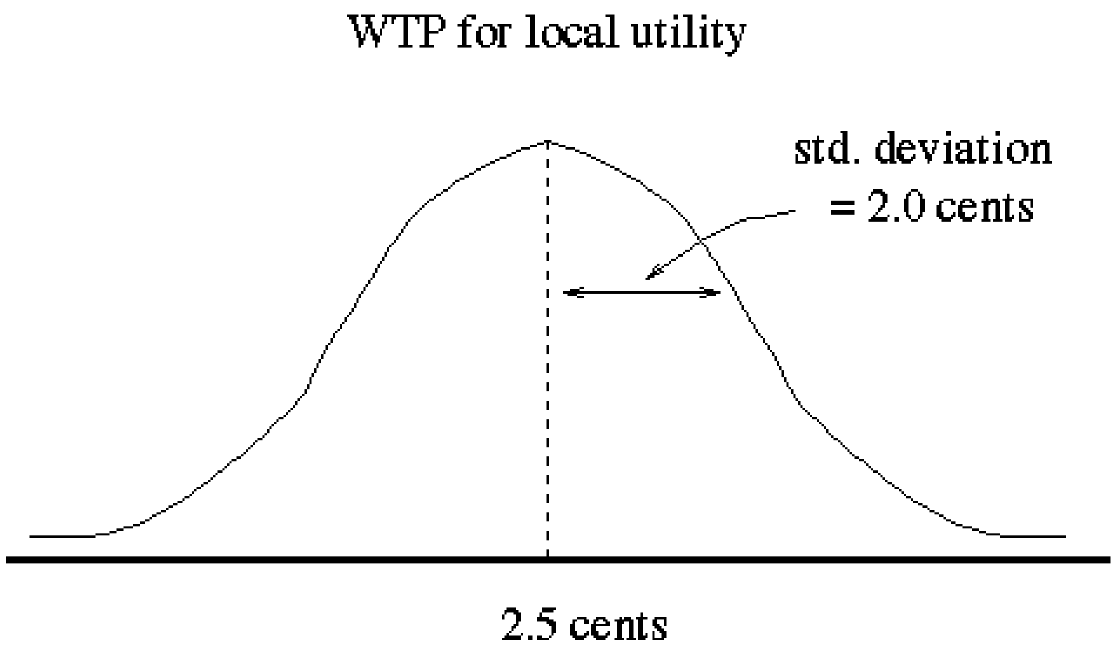
Contract length coefficient ~ Normal with
mean = -.2125
std. dev. = .3865

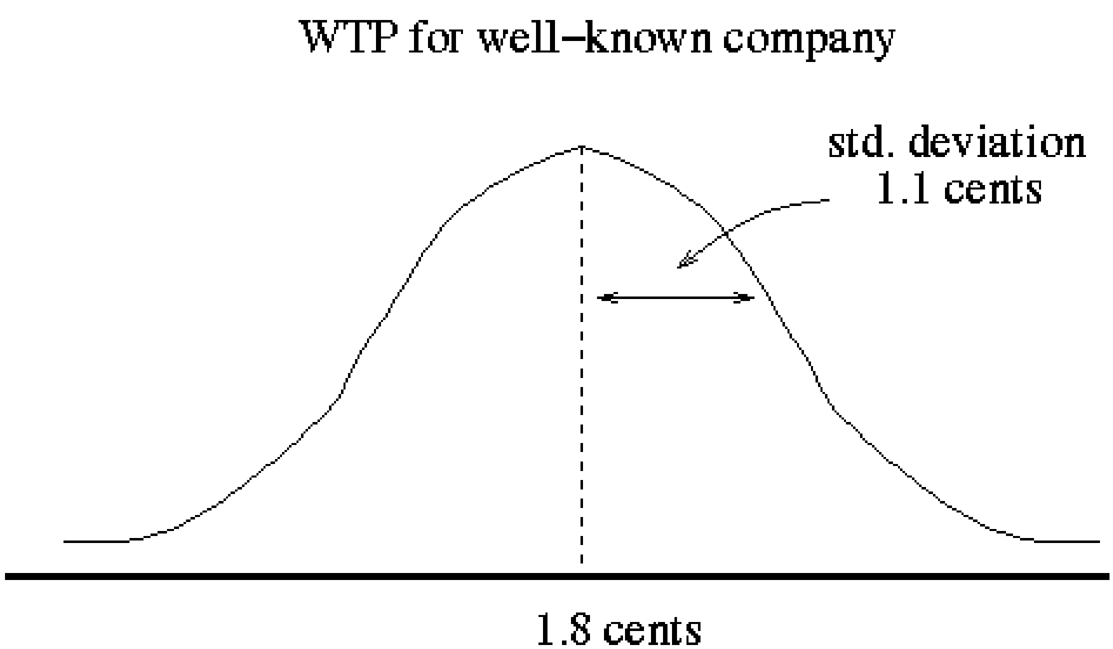
WTP for 1 extra year of contract ~ Normal with
mean = -.2125 / .8827 = -.24
std. dev. = .3865 / .8827 = .55

Distribution of Willingness to Pay



WTP for 1 extra year of contract


WTP for local utility
std. deviation
= 2.0 cents
2.5 cents


WTP for well–known company
std. deviation
1.1 cents
1.8 cents

Substitution Patterns

Base

Local utility at 8¢	.86
Enron at 7¢	.14

Scenario 1: Add another new supplier with seasonal rates

	Logit	Mixed Logit
Local utility at 8¢	.78	.84
Enron at 7¢	.13	.09
Sears with seasonal rate	.09	.07

Scenario 2: Local utility counters with seasonal rates

	Logit	Mixed Logit
Local utility at 8¢	.69	.81
Enron at 7¢	.11	.08
Sears with seasonal rate	.08	.02
Local utility with seasonal rate	.12	.09

Calculation of Choice Probabilities

$$P_{in} = \int L_{in}(\beta)\, g(\beta \mid b,W)\, d\beta$$

Approximate the integral through simulation.

1. Draw β from $g(\beta \mid b,W)$
2. Calculate logit probability:

$$L_{in} = \frac{e^{\beta X_{in}}}{\sum_j e^{\beta X_{jn}}}$$

3. Repeat steps 1-2 many times and average the results:

$$\tilde{P}_{in} = \frac{1}{R} \sum L_{in}(\beta^r)$$

How to draw β from $g(\beta \mid b,W)$?

Suppose β has one element and is distributed normal with mean b and standard deviation s .

1a. Draw z from a random number generator for standard normal

1b. Create $\beta = b + s \cdot z$

Generalize for β with multiple elements and / or for non-normal distributions.

Software:

1. At Kenneth Train's website

   http://elsa.berkeley.edu/~train

   Click on "software".
   Includes manual and sample runs.
   Free, but written in Gauss, so you need to buy Gauss
   from Aptech, Inc.

- LIMDEP has mixed logit for cross-sectional data and will soon have it for panel data.

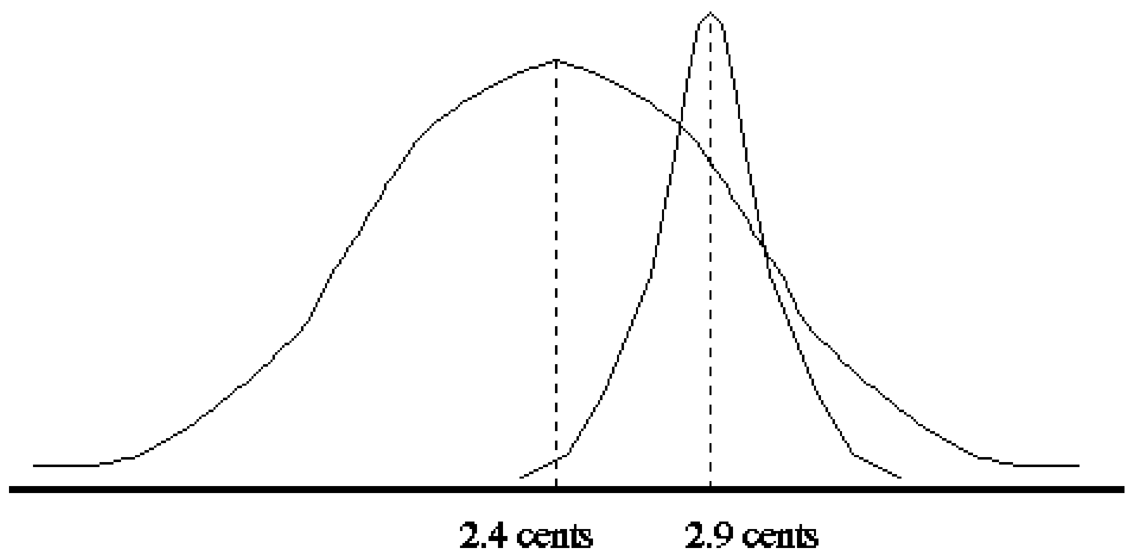
LECTURE / DISCUSSION

Customer-Specific Coefficients with Mixed Logit

"ML / COIT" = Maximum Likelihood with Conditioning of Individual Tastes

Mixed logit gives the distribution of coefficients in the population.

Where does each sampled customer's coefficients lie in this distribution?



Local Utility

$g(\beta \mid b,W)$ is distribution of β in the population.

Distribution of β conditional on sampled customer's choices:

$$h(\beta \mid y_n, b, W) = \frac{P(Y_n \mid \beta) \cdot g(\beta \mid b,W)}{P(y_n \mid b,W)}$$

$$= \frac{L_{ni}(\beta)\, g(\beta \mid b,W)}{\int L_{ni}(\beta)\, g(\beta \mid b,W) d\beta}$$

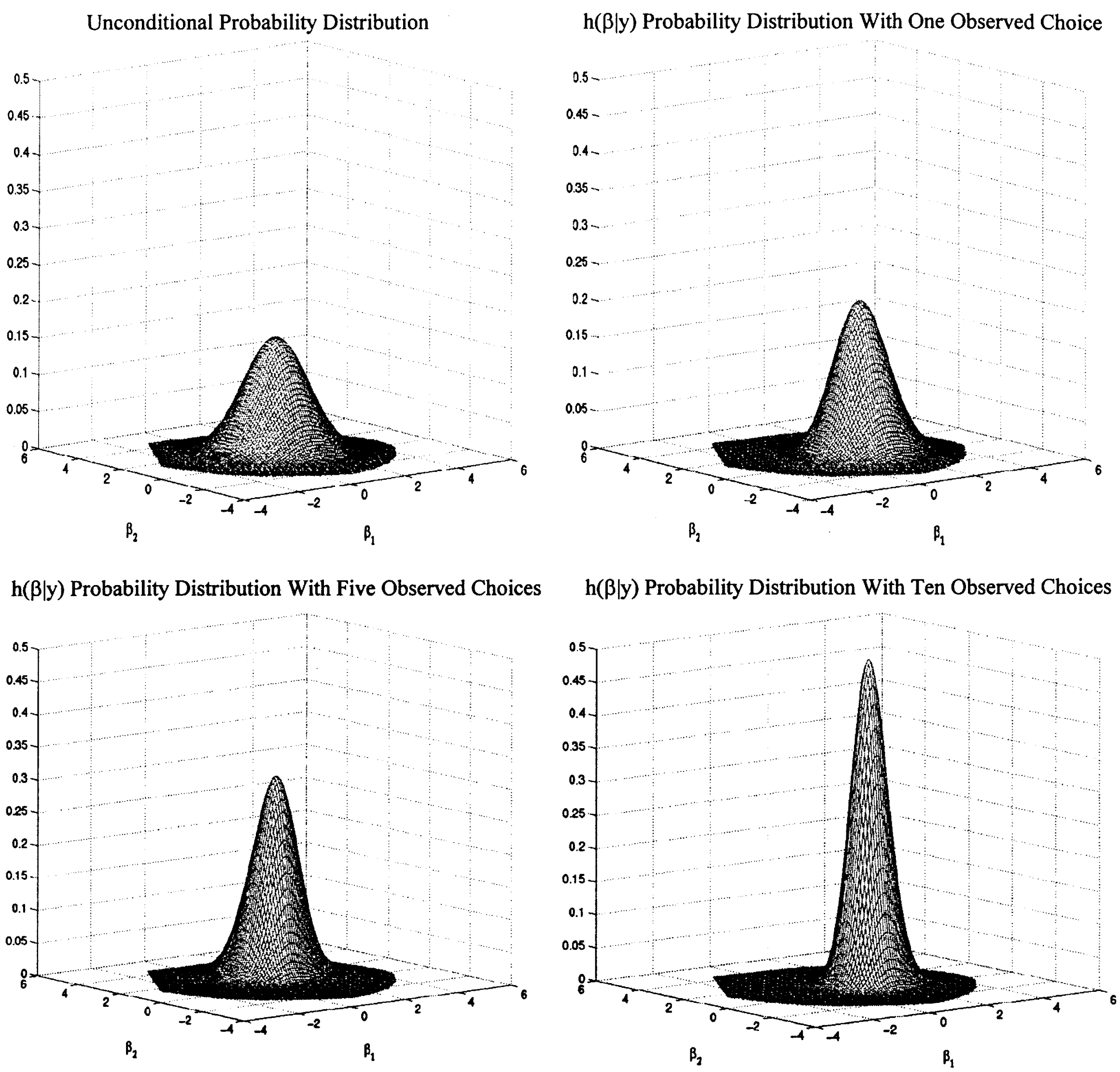
Expected β for each customer:

$$E(\beta \mid y_n, b, W) = \int \beta h(\beta \mid y_n, b, W) d\beta$$

$$= \frac{\int \beta L_{ni}(\beta)\, g(\beta \mid b, W) d\beta}{\int L_{ni}(\beta)\, g(\beta \mid b, W) d\beta}$$

Approximate each integral through simulation.

Figure 1: Conditional Distributions for One Respondent in Monte Carlo Experiment



Monte Carlo Results

1 choice situation		
	Standard deviation of $E(\beta)$	.41
	Absolute diff. of $E(\beta)$ from β	.73
10 choice situation		
	Standard deviation of $E(\beta)$	.83
	Absolute diff. of $E(\beta)$ from β	.42

Expected WTP for Each Customer

	Population Mean	Customer A's Conditional Mean
Contract length	-0.24	2.20
Local utility	2.50	3.30
Well-known company	1.80	2.00
Time-of-day rates	-10.40	-6.30
Seasonal rates	-10.20	-6.60

- **Customer likes long-term contract, local utility, and non-fixed rates.**
- **Local utility can retain and make profit from this customer by offering a long-term contract with time-of-day or seasonal rates.**

Expected WTP for Each Customer

	Population Mean	Customer B's Conditional Mean
Contract length	**-0.24**	**-4.50**
Local utility	**2.50**	**0.77**
Well-known company	**1.80**	**1.40**
Time-of-day rates	**-10.40**	**-14.50**
Seasonal rates	**-10.20**	**-12.30**

- **Customer greatly dislikes non-fixed rate and long-term contracts, prefers a well-known company to the local utility.**
- **A well-known company can capture this customer by offering a fixed price without a contract.**

Comparison of Population Distribution of β's with Distribution of E(β)'s

	β	E(β)
Contract length		
Mean	-.213	-.215
Standard deviation	.387	.326
Local utility		
Mean	2.23	2.21
Standard deviation	1.75	1.38
Known company		
Mean	1.59	1.60
Standard deviation	.96	0.68
Time-of-day rate		
Mean	9.18	9.26
Standard deviation	2.73	3.11
Seasonal rate		
Mean	9.00	9.13
Standard deviation	1.80	2.06

Prediction of Last Choice Situation

	Average Probability of Chosen Alternative
Population distribution (no conditioning)	.35
Conditional mean of coefficients	.56
Conditional distribution of coefficients	.52

➥ COIT improves forecasts considerably

Procedure Gives Same Kind of Information as Hierarchical Bayes

But:

- easily accommodates non-normal distributions
- allows for classical specification and hypothesis testing for population distribution
- does not require derivation and drawing from conditional posterior distribution of population parameters
- requires calculation of likelihood function for population parameters

Important Note:

When model is specified the same, HB and ML/COIT give the same results asymptotically.

Paper at:

http://elsa.berkeley.edu/wp/train0999.pdf