LECTURE / DISCUSSION

Hypothesis Testing

T-test

Use to test value of one parameter.

I. Most common application: to test whether a variable enters significantly.

Model: $V_{TOU} = -\alpha BTOU + \beta INC + c$

$V_{ST} = -\alpha BST$

where BTOU = time-fo-use billing;
BST = standard billing; and
INC = income.

Hypothesis: $\alpha = 0$

$$t = \frac{\text{Estimated } \alpha}{\text{Standard error of estimated } \alpha}$$

If $|t|$ exceeds critical value (usually 1.96), then REJECT hypothesis.

⇒ α is not zero. BTOU has a significant effect on person's choice.

If $|t|$ is less than critical value, then CANNOT REJECT hypothesis.

⇒ True α might be zero. BTOU does not enter significantly.

II. More general application: to test whether the coefficient of a variable equals some particular value.

More generally, hypothesis: $\alpha = a$

$$t = \frac{\text{Estimated } \alpha - a}{\text{Standard error of estimated } \alpha}$$

If $|t|$ exceeds critical value, then hypothesis that $\alpha = a$ is rejected.

If $|t|$ is less than critical value, then hypothesis that $\alpha = a$ cannot be rejected.

Likelihood Ratio Test

1. Hypothesis is a restriction. Specify the unrestricted and the restricted models.

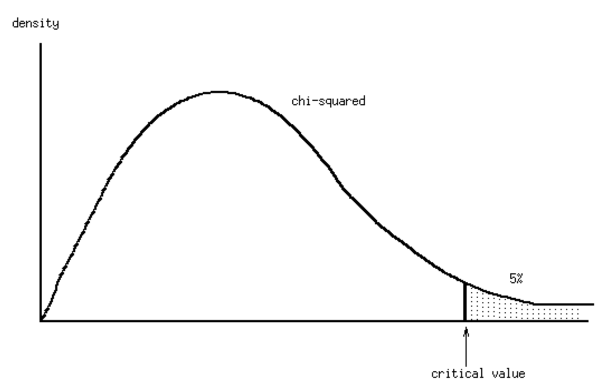
2. Estimate the unrestricted model and note the value of the log likelihood function, say LLU.

3. Estimate the restricted model, constraining parameters to their hypothesized values, and note the value of the log likelihood function, say LLR.

4. Compute the test statistic LRT = 2(LLU - LLR). This must be positive because LLU $\geq$ LLR.

5. Under the null hypothesis that the restrictions are true, the LRT statistic is distributed as a chi-squared random variable with degrees of freedom equal to the number of restrictions in the hypothesis.

6. Compare the statistic with the critical value of the chi-squared distribution.



Critical Values of Chi-Squared

Degrees of Freedom	Significance Level	
	10%	5%
1	2.71	3.84
2	4.61	5.99
3	6.25	7.81
4	7.78	9.49
5	9.24	11.07
6	10.64	12.59
7	12.02	14.07
8	13.36	15.51
9	14.68	16.92
10	15.99	18.31
15	22.31	25.00
20	28.41	31.41
40	51.81	55.76

Example

$$V_{TOU} = -\alpha BTOU + \beta INC + c$$

$$V_{ST} = -\alpha BST$$

1. Test whether a parameter equals zero.

Hypothesis: $\alpha = 0$.

- Run model with all explanatory variables:

```
ivalt [bill: btou bst \
     inctou: inc zero \
     toudum: one zero]
```

This is an unrestricted model. LLU = -376.4 .

- Run model with BILL omitted; that is, with:

```
ivalt [inctou: inc zero \
          toudum: one zero]
```

This is the restricted model. LLR = -384.2 .

- Test statistic 2(384.2 - 376.4) = 2(7.8) = 15.6 .
- Critical value of chi-squared with one degree of freedom is 3.84 (at 95% confidence).
- Because test statistic exceeds critical value, hypothesis is REJECTED.

2. Test whether two parameters both equal zero.

Hypothesis: $\alpha = \beta = 0$

- Run model with BILL, INCTOU, and TOUDUM as explanatory variables. This is the unrestricted model. LLU = -376.4 .

- Run model with BILL and INCTOU omitted; that is, with only TOUDUM. This is the restricted model.
LLR = -397.2 .

- Test statistic: 2(397.2 - 376.4) = 20.8 .

- Critical value of chi-squared with two degrees of freedom is 5.99 .

- Because the test statistic exceeds the critical value, the hypothesis is REJECTED.

3. Test whether two parameters are equal to each other.

Unrestricted model:

$$V_{TOU} = \alpha_{TOU}\, BTOU + \beta INC + c$$

$$V_{ST} = \alpha_{ST}\, BST$$

Hypothesis: $\alpha_{TOU} = \alpha_{ST}$

- Run model with BTOU, BST, INCTOU, and TOUDUM as explanatory variables:

```
ivalt [btou: btou zero \
     bst: zero bst \
     inctou: inc zero \
     toudum: one zero]
```

Get: LLU = -372.1 .

- Run model with BILL, INCTOU, TOUDUM:

```
ivalt [ bill: btou bst \
     inctou: inc zero \
     toudum: one zero]
```

Get: LLR = -376.4 .

- Test statistic: 2(376.4 - 372.1) = 8.6 .
- Critical value: 3.84 .
- REJECT hypothesis.

4. Test whether parameters are the same for two subpopulations.

Run model on subsample A; get LLA = -162.1.

Run model on subsample B; get LLB = -206.1.

Calculate the log likelihood for the unrestricted model:

$$LLU = LLA + LLB = -162.1 - 206.1 = -368.2$$

Run model on full sample. This is restricted model.
Get LLR = -372.1.

There are four parameters in the restricted model and eight parameters in the unrestricted model. The number of restrictions is therefore four.

Test statistic is 2(372.1 - 368.2) = 7.8.

The critical value of chi-squared with four degrees of freedom is 9.49.

ACCEPT hypothesis.

WORKSHOP ON HYPOTHESIS TESTING

Heating and Cooling System Choice

Alternatives:

1. Gas central heat with cooling.
2. Electric central resistance heat with cooling.
3. Electric room resistance heat with cooling.
4. Electric heat pump (heating and cooling).
5. Gas central heat without cooling.
6. Electric central resistance without cooling.
7. Electric room resistance without cooling.

Dependent variable: *depvar*

Data set: nested1.sav

Explanatory variables:

ich1,...,ich7 installed cost for heating portion of system

icca installed cost for cooling portion of system
(Note: ic1 = ich1 + icca = installed cost for alt. 1, etc. for alts. 2-4. ic5 = ich5 = installed cost for alt. 5, etc. for alts. 6-7, since these systems have no cooling.)

och1,...,och7 annual operating cost for heating portion of system

occa annual operating cost for cooling portion of system
(Note: operating cost for heating and cooling combined is calculated analogously to installation cost.)

income income of household

1. Estimate a basic model that includes:

- full installation cost of alternative (including heating, and, if available, cooling)
- full operating cost of alternatives
- income entering the room systems (alts. 3 and 7)
- income entering the systems with cooling (alts. 1-4)
- a constant for the systems with cooling (alts. 1-4)

The file **hypo.cmd** will estimate this model.

How do you interpret the two income variables and their estimated coefficients? That is, what do these variables seem to capture or indicate?

2. Test the hypothesis that the discount rate is 0.20.

Recall that LF = IC + (1 / r)OC where *r* is the discount rate. With r = .20, we have

$$LF = IC + 5 * OC \quad .$$

So: a test of the hypothesis that r = .20 is equivalent to a test of the hypothesis that the coefficient of operating cost is five times greater than that of installation cost.

3. Test the hypothesis that people consider the costs for heating and cooling equally in their choice of HVAC systems.

That is, consider a model

$$V_i = \beta ICH_i + \lambda ICCA_i + \theta OCH_i + \phi OCCA_i + \text{other terms}$$

The hypothesis to be tested is that $\beta = \lambda$ and $\theta = \phi$.

DISCUSSION OF WORKSHOP RESULTS

LECTURE / DISCUSSION

Independence from Irrelevant Alternatives

Independence from Irrelevant Alternatives

$$\frac{P_{in}}{P_{kn}} = \frac{\dfrac{e^{V_{in}}}{\sum e^{V_{jn}}}}{\dfrac{e^{V_{kn}}}{\sum e^{V_{jn}}}} = \frac{e^{V_{in}}}{e^{V_{kn}}} = e^{V_{in}-V_{kn}}$$

This ratio does not depend on existence or attributes of alternatives other than *i* or *k*, as long as each V_{in} depends only on the attributes of alternative i and / or the characteristics of the subject, and ***not*** on the attributes of other alternatives.

Practical Advantages of IIA

1. Allows sampling of alternatives for estimation.

   When the choice set is large, data collection and computation costs can be reduced by sampling a subset of alternatives. For many sampling procedures, the parameters of an MNL model for the sample of alternatives are the same as the parameters of an MNL model for the full choice set.

2. Allows forecasting demand for a new alternative.

   The demand for a new alternative can be forecast in the MNL model simply by adding an e^v term for the new alternative to the denominator of the probability. (It must be the case that the estimated parameters are sufficient to calculate the v for the new alternative.)

Further reading: The following two pages give further details.

Sampling of Alternatives for Estimation

Suppose C_n is the choice set for subject n, $P_{in} = \dfrac{e^{V_{in}}}{\sum_{j\in C_n} e^{V_{jn}}}$ is the MNL choice probability

for this subject, and i_n is the observed choice. Suppose the analyst selects a subset of alternatives A_n for this subject which will be used as a basis for estimation; i.e., if i_n is contained in A_n, then the subject is treated *as if* the choice were actually being made from A_n, and if i_n is not contained in A_n, the observation is discarded. In selecting A_n, the analyst may use the information on which alternative i_n was chosen, and may also use information on the variables that enter the determination of the V_{in}. The rule used by the analyst for selecting A_n is summarized by a *probability* $\pi(A|i_n,V\text{'s})$ that subset A of C_n is selected, given the observed choice i_n and the observed V's (or, more precisely, the observed variables behind the V's). The selection rule is a *uniform conditioning* rule if for the selected A_n, it is the case that $\pi(A_n|j,V\text{'s})$ is the same for all j in A_n. Examples of uniform conditioning rules are (1) select (randomly or purposively) a subset A_n of C_n *without* taking into account what the observed choice or the V's are, and keep the observation if and only if i_n is contained in the selected A_n; and (2) given observed choice i_n, select m-1 of the remaining alternatives at random from C_n, without taking into account the V's . An implication of uniform conditioning is that in the sample containing the pairs (i_n,A_n) for which i_n is contained in A_n, the probability of observed response i_n conditioned on A_n is

$$P_{in|A_n} = \frac{e^{V_{in}}}{\sum_{j\in A_n} e^{V_{jn}}} .$$

This is just a MNL model that treats choice *as if* it were being made from A_n rather than C_n, so that maximum likelihood estimation of this model for the sample of (i_n,A_n) with $i_n \in A_n$ estimates the same parameters as does maximum likelihood estimation on data from the full choice set. Then, this sampling of alternatives cuts down data collection time (for alternatives not in A_n) and computation size and time, but still gives consistent estimates of parameters for the original problem.

Forecasting Demand for a New Alternative

Suppose choice among alternatives in a set C_n is observed for a sample, and that choice in this sample can be described by a MNL model. Suppose this sample is used to estimate the parameters of the logit model, so that one has an estimated model

$$P_{in} = \frac{e^{V_{in}}}{\sum_{j \in C_n} e^{V_{jn}}}$$

that describes the historical choice behavior.

Now suppose a new alternative "0" were to be added to C_n , so that the subject's choice set B_n would be the union of C_n and the new alternative "0", $B_n = \{0\} \cup C_n$. Suppose that the parameters estimated on the historical data are sufficient so that you can calculate the representative utility V_0 of the new alternative. If IIA holds, then the choice probabilities from B_n would be

$$P_{in|B_n} = \frac{e^{V_{in}}}{e^{V_{0n}} + \sum_{j \in C_n} e^{V_{jn}}} \equiv \frac{e^{V_{in}}}{e^{V_{0n}} + e^{I_n}} \equiv \begin{cases} P_{in|C_n} \cdot P(C_n|B_n) & \text{if } i = C_n \\ 1 - P(C_n|B_n) & \text{if } i = 0 \end{cases}$$

where

$$I_n = \log\left(\sum_{j \in C_n} e^{V_{jn}}\right) \qquad \text{(inclusive value for set } C_n\text{)}$$

$$P(C_n|B_n) = \frac{e^{I_n}}{e^{V_{0n}} + e^{I_n}} \qquad \text{(probability of choosing } C_n \text{ from } B_n\text{)}$$

Note that if V_{0n} cannot be calculated completely from the historically estimated parameters, then additional information or assumptions must be introduced to make the forecast. This is a problem if there are alternative-specific intercepts or slope coefficients in the model. Hence, for forecasting, one should try first to describe historical behavior using only *generic* (i.e., non-alternative-specific) variables, and introduce alternative-specific variables *only* if they are essential to explain observed behavior and the attributes of the new alternative.

Red Bus / Blue Bus Problem

Two alternatives orginially: car, blue bus

Suppose: $V_c = V_{bb}$

Then:

$$P_c = P_{bb} = \frac{1}{2}$$

$$\frac{P_c}{P_{bb}} = 1$$

Now add a third alternative, red bus, which is exactly like the blue bus.

Logit predicts: $P_c = P_{bb} = P_{rb} = 1/3$

In reality: $P_c = 1/2,\quad P_{bb} = P_{rb} = 1/4$

$$\frac{P_{bb}}{P_{rb}} = 1 \text{ , because bb and rb are the same}$$

$$\frac{P_c}{P_{bb}} = 1 \text{ , by IIA}$$

The validity of IIA in an application is an empirical question that can be answered by a specification of the following hypothesis: The IIA property is a sufficiently good approximation to the true data generation process so that patterns of choice are consistent with the MNL model.

Substitution Patterns

IIA implies proportional substitution.

Example: Impact of rebates on heat pumps.

Alternative	Original Probability	Logit Forecasts	
1	.600	.54	(-10%)
2	.055	.0495	(-10%)
3	.002	.0018	(-10%)
4	.033	.133	
5	.080	.072	(-10%)
6	.010	.009	(-10%)
7	.220	.198	(-10%)
8			
P_5 / P_6	8	8	

In reality, substitution is rarely proportional.