Model Trending Real Exchange Rates

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The title was supposed to be "Modeling Trending Real Exchange Rates," which would have made more sense, but the first instance of the fragment "ing" somehow disappeared in the working paper production process. C'est la vie.

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Abstract

The multilateral real exchange rates of major industrial countries often contain deterministic time trends. This note develops a simple stochastic model of a small open economy with a deterministically trending real exchange rate. Real exchange rate trends are caused by differential productivity growth in tradables and nontradables. Although the model assumes complete price flexibility, it can produce a correlation between the real exchange rate and the international real interest-rate differential similar to the one that arises in sticky-price overshooting models dominated by monetary shocks.

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The theoretical touchstone for Marianne Baxter's stimulating inquiry into the real interest-rate parity relation is the famous Dornbusch (1976)-Mussa (1977) overshooting model of exchange rates, which has as main building blocks the assumptions of short-run price-level stickiness and nominal interest parity. In the end Baxter concludes that this model helps little in understanding empirical comovements of real exchange rates and real interest differentials. She offers no competing model better at explaining the data.

Central to the investigation is the hypothesis that real exchange rates contain stochastic trends. *Permanent* components of a country's real exchange rate, Baxter reminds us, need bear no particular relation to the difference between home and foreign expected real interest rates. To isolate more clearly any connection between *transitory* real exchange rate components and real interest rates, while avoiding the econometric difficulties inherent in working with unit roots, data on real exchange rates must be passed through an appropriate filter. The paper can be viewed as an exploration into the statistical relationship between filtered real exchanges rates and real interest differentials.

It is difficult to dissent from the view that real exchange rates undergo apparently permanent changes. In line with this informal evidence, econometric studies of the post-1973 floating-rate era suggest that unit roots are present in real exchange rates. Looking over a longer time horizon, however, one is struck by a different empirical regularity: some real exchange rates contain a pronounced *deterministic* trend. Such trends suggest an alternative class of exchange-rate models, the empirical performance of which could serve as a benchmark for judging how well overshooting models perform.

Overshooting models, notably Mussa's (1977) version, certainly can accommodate nonstochastic trends. They would result from assumed secular change in the exogenous factors underlying the long-run demand for or supply of domestic output. Baxter's equation (10) contains a drift term that might be due to such factors. But a model better suited to capture long-term real exchange rate trends would account for the intertemporal budget constraints limiting the growth of demand and for the factor accumulation and productivity growth underlying supply. This type of model might take on added relevance if, as Baxter contends, most information resides in medium- to low-frequency components of real exchange rates and real interest differentials.

In these comments I document the evidence on deterministic trends in real exchange rate measures for Japan and the United States. Then I present an intertemporal small-country model that is consistent with such trends. A stochastic version of the model can produce the positive covariation between real exchange rates and real interest rate differentials that Baxter seeks, despite perfectly flexible prices and wages. I end with some observations on the econometric detection and economic interpretation of a real exchange rate-real interest differential relationship.

1. Real exchange rates over the long run

Long-run trends in real exchange rates cannot be detected without long data series. Here I search for deterministic time trends in the real exchange rates of the Japanese yen and the United States dollar over the 1950-88 period. A country's real exchange rate, q, is defined as its price level in dollars divided by an equally-weighted geometric average of the dollar price levels in a reference group of twelve countries.[1] As in Baxter's notation, a rise in q is a real currency appreciation and a fall is a real depreciation. The price-level data come from Summers and Heston (1991), and thus the real exchange rates I use can be interpreted as relative prices of identical national output baskets consisting of tradables and nontradables.

Figures 1 and 2 display annual data on the yen and dollar real exchange rates. To the unaided eye the Japanese data seem clearly to disclose a nonstochastic trend. The U.S. data are more problematic, however, since the dollar's more or less steady real decline through the late 1970s is interrupted by a massive and ultimately transitory real appreciation during the 1980s. Here, too, the presence of a deterministic trend seems plausible. The next step is to assess the size and statistical significance of the suspected time trends.

The data generating process I consider is of the univariate form

$$\ln q_t = \gamma + \mu t + z_t, \quad (1 - \phi_1 B - \phi_2 B^2) z_t = \varepsilon_t,$$

where μ is the unconditional deterministic trend in the real exchange rate's natural logarithm, B is the backward-shift operator, and ε_t is white noise.[2]

A key question the data must resolve is how to allocate the trend in real exchange rates between stochastic and deterministic components. Over every sample period, I examine two versions of the above

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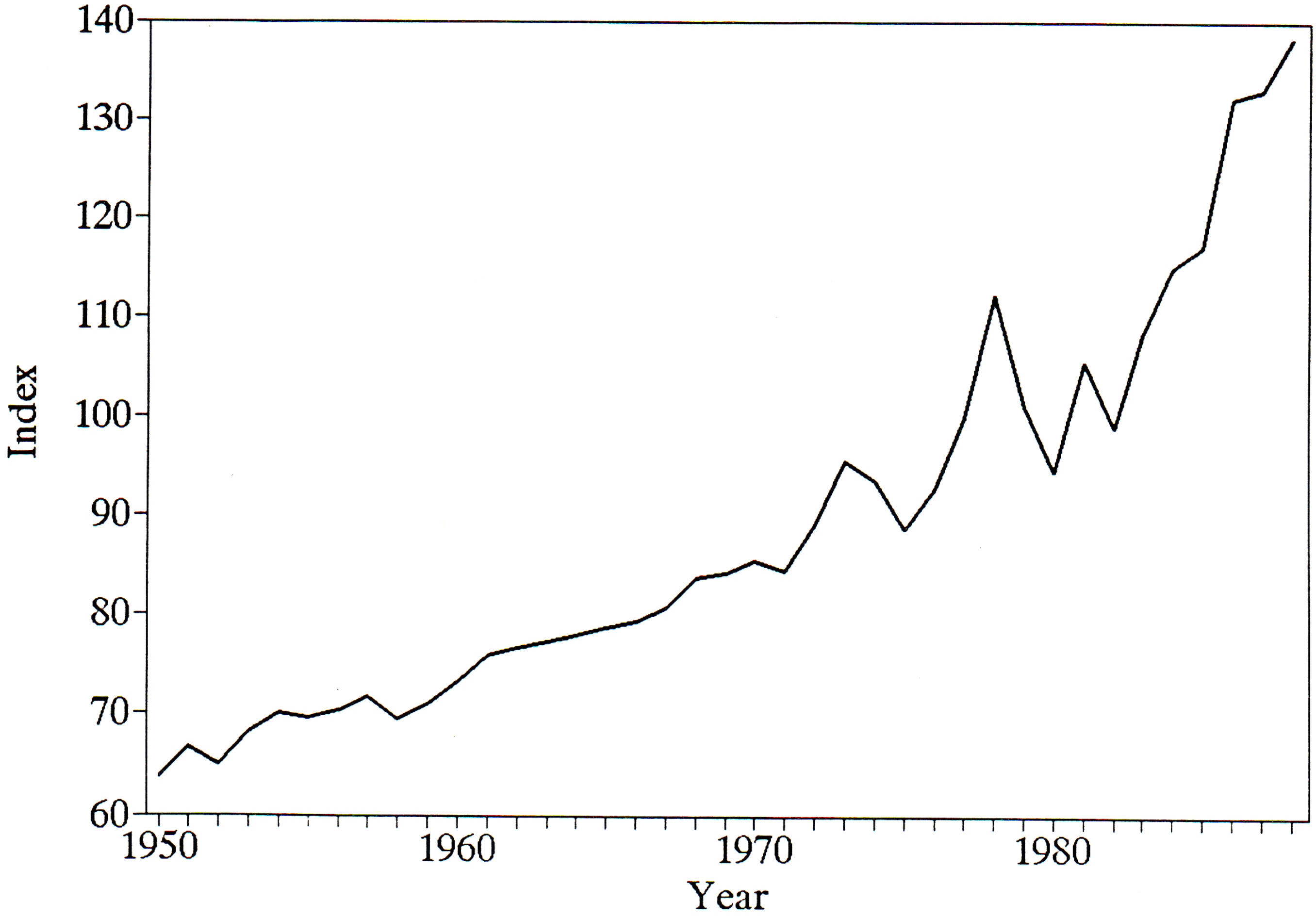
[1]The group members are Australia, Belgium, Canada, France, Germany, Italy, Japan, the Netherlands, Sweden, Switzerland, the United Kingdom, and the United States.

[2]The roots of the polynomial equation $1 - \phi_1 B - \phi_2 B^2 = 0$ are assumed to lie outside or on the unit circle.

FIGURE 1
Real Exchange Rate of Yen 1950-88



data-generating process, one of which imposes a unit root ex ante.

Table 1 analyzes the Japanese data. For every sample period, the top row of statistics comes from the non-unit root specification while the second row imposes a unit root. Preliminary estimates placed ϕ_2 very close to zero in all cases, so the restriction $\phi_2 = 0$ was assumed. Given this restriction, $\ln q$ contains a unit root if and only if $\phi_1 = 1$. Standard errors are adjusted for heteroskedasticity of unknown form, as a hopeful correction for time-varying real exchange rate variances.

Over the unified sample period 1951-88 the unconditionally expected trend rate of real yen appreciation is around 1.9 or 2.0 percent per year and statistically significant regardless of the specification adopted. The data fail to give strong evidence against the hypothesis that the log of the real yen rate follows a random walk.

This picture changes once the data are separated into eras of fixed (1951-72) and floating (1973-88) nominal exchange rates. It is once again true, over both subperiods, that the specification one chooses makes little difference for the point estimate of the time trend. The 1973-88 estimate of a 2.7 percent per year unconditional expectation of real yen appreciation is nearly twice as high as the corresponding 1951-88 estimates. These point estimates are highly significant, except in the unit-root specification after 1973.

Subsample Dickey-Fuller tests reject the unit-root hypothesis at the 5 percent level or below. Indeed, over 1951-72 the yen real exchange rate is essentially white noise around a time trend. The findings in table 1 contradict the view that real exchange rates, especially under floating, always contain stochastic trends.

The results for the dollar, reported in table 2, show that ϕ_2 cannot be set to zero for that currency. In the nonstationary case one

FIGURE 2
Real Exchange Rate of Dollar 1950-88

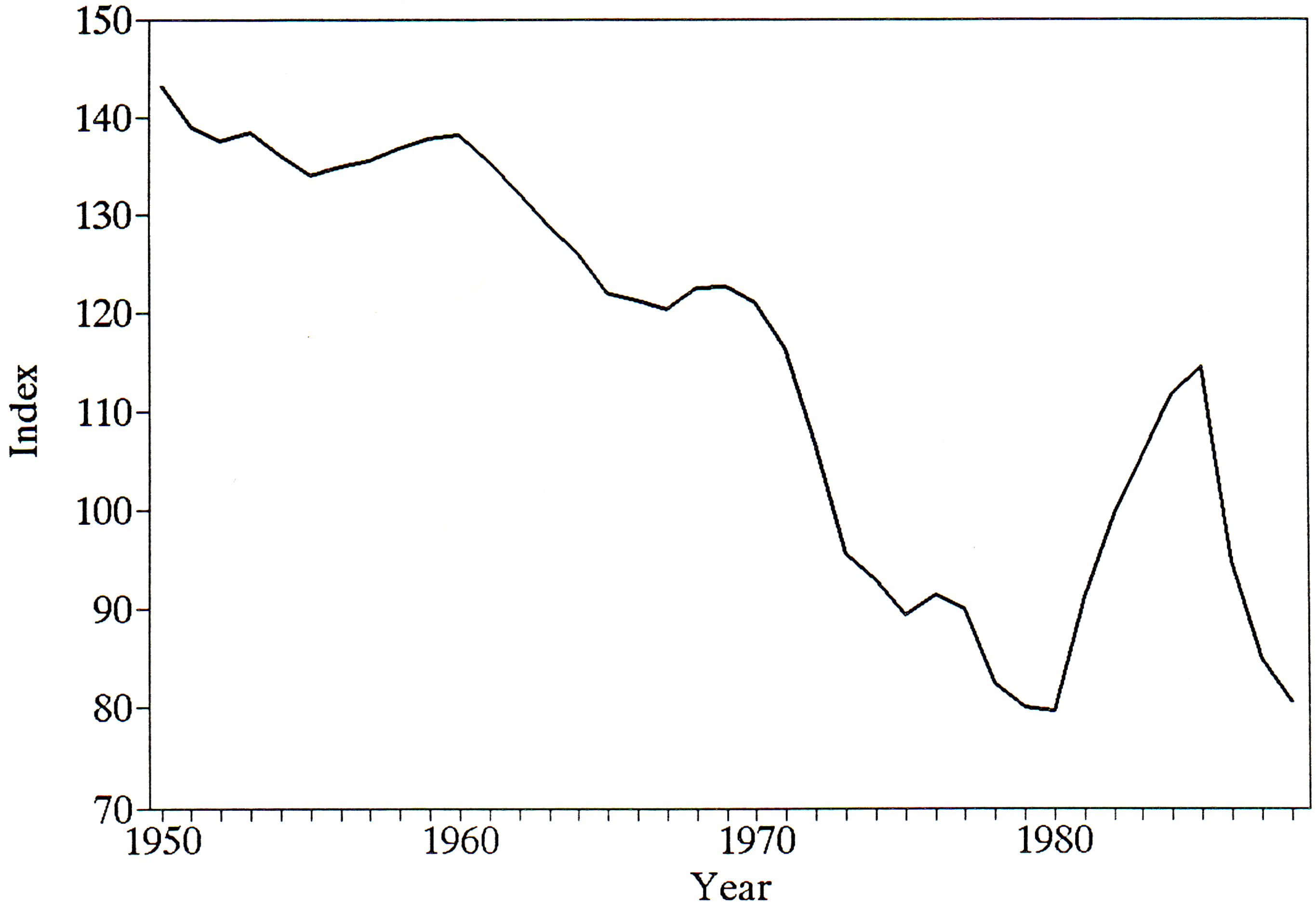


Table 1

Estimates of $\ln q_t = \gamma + \mu t + (1 - \phi_1 B - \phi_2 B^2)^{-1} \varepsilon_t$

Japanese annual data

Sample	$\hat{\mu}$	$\hat{\phi}_1$	$\hat{\phi}_2$	Q-test	$\bar{R}^2$
1951-88	0.019 (0.002)	0.618 (0.151)	0	0.968	0.95
	0.020 (0.008)	1	0	0.631	0.00
1951-72	0.014 (0.001)	0.064 (0.250)	0	0.775	0.96
	0.015 (0.005)	1	0	0.796	0.00
1973-88	0.027 (0.004)	0.421 (0.147)	0	0.659	0.78
	0.027 (0.017)	1	0	0.326	0.00

Notes: Standard errors of point estimates are reported in parentheses. Q-stat is the significance level of the Box-Ljung Q-statistic. Estimates by nonlinear least squares with standard errors corrected for heteroskedasticity of unknown form.

Table 2

Estimates of $\ln q_t = \gamma + \mu t + (1 - \phi_1 B - \phi_2 B^2)^{-1} \varepsilon_t$

United States annual data

Sample	$\hat{\mu}$	$\hat{\phi}_1$	$\hat{\phi}_2$	Q-test	$\bar{R}^2$
1952-88	-0.016 (0.003)	1.394 (0.140)	-0.644 (0.108)	0.979	0.95
	-0.015 (0.016)	1.523 (0.098)	-0.523	0.829	0.25
1952-72	-0.014 (0.005)	1.681 (0.276)	-0.912 (0.258)	0.984	0.94
	-0.028 (0.062)	1.852 (0.338)	-0.852	0.874	0.31
1973-88	-0.000 (0.006)	1.251 (0.171)	-0.651 (0.092)	0.626	0.69
	-0.015 (0.035)	1.507 (0.111)	-0.507	0.925	0.21
1952-79	-0.027 (0.012)	1.321 (0.194)	-0.436 (0.232)	0.887	0.98
	-0.020 (0.010)	1.472 (0.206)	-0.472	0.956	0.19

Notes: Standard errors of point estimates are reported in parentheses. Q-stat is the significance level of the Box-Ljung Q-statistic. Estimates by nonlinear least squares with standard errors corrected for heteroskedasticity of unknown form.

root of the equation $1 - \phi_1 B - \phi_2 B^2 = 0$ is 1, and so $\phi_1 + \phi_2 = 1$. Thus in each sample period's second row the unit root hypothesis is imposed by setting $\phi_2 = 1 - \phi_1$ and estimating the parameters of the ARIMA(1,1,0) process $(1-B)\ln q_t = \mu(2-\phi_1) + (\phi_1-1)(1-B)\ln q_{t-1} + \varepsilon_t$.

The U.S. data provide no evidence against the unit-root hypothesis.[3] In the U.S. case, though, a unit root does affect one's views about deterministic trends. For both the full sample and the fixed-rate subsample, the unconditional expectation of the dollar's annual real depreciation is on the order of 1.5 percent under a trend-stationary specification. The time trends are statistically insignificant under the unit-root specification, not because the point estimates are much smaller--indeed, the 1952-72 point estimate is -2.8 percent per year--but because the standard errors blow up. Over the floating-rate period, neither specification yields a statistically significant time trend, although the point estimate in the unit-root specification is an economically significant -1.5 percent yearly.

The 1973-88 results may be due to the dollar's behavior over the 1980s (figure 2), which arguably was the result of an aberrant policy mix. It is therefore of interest to examine a subsample that ends in 1979. In this sample the deterministic trend is significant at the 5 percent level regardless of the specification chosen. Under trend-stationarity the dollar's unconditionally expected annual real depreciation rate is estimated at 2.7 per cent per year. Under a unit root the estimate is 2 percent. The obvious question is whether a model explaining the time trends in the data can also throw light on

[3] Using data stretching from 1869 to 1984, Frankel (1986) was able to reject the hypothesis of a unit root in the univariate process for the dollar-sterling real exchange rate.

the comovement of real exchange rates and real interest differentials.

2. Modeling deterministic trends in real exchange rates

The simplest setting for thinking about deterministically trending real exchange rates is a model of differential productivity growth across sectors, in the Balassa (1964) tradition. There has been curiously little theoretical effort to embody Balassa's empirical regularities in models that account for optimal consumption and saving behavior in the presence of integrated world asset markets.[4]

To make life simple I describe a model in which preferences and technologies are Cobb-Douglas and the representative consumer's elasticity of intertemporal substitution is unity. All the special assumptions of the model could be relaxed substantially without materially changing its predictions. Initially the model is developed without stochastic features, which are added at the end.

A small open economy uses capital and labor to produce tradable goods priced in world markets and nontradables priced at home. Capital is internationally mobile, and one unit of the tradable good can be transformed at no cost into one unit of installed capital in either sector.[5] While mobile across sectors, labor cannot cross national

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[4]An exception is Rogoff (1991), who also reports empirical tests of his model. In contrast, there have been a number of important empirical inquiries--starting with Balassa himself and including Hsieh (1982), Kravis and Lipsey (1987), Marston (1987), Yoshikawa (1990), and Bergstrand (1991). My model is special in its prediction that the real exchange rate may be determined entirely on the economy's production side. (This result is due to intersectoral factor mobility, international capital mobility, and the two-factor, two-good structure.) In Rogoff's model (1991) demand-side factors dominate because productive factors are fixed in supply and sector specific.

[5]Nontradables, in contrast, cannot be invested.

borders. The domestic labor force L grows at the proportional rate n:

(1) $\hat{L}(t) = n.$

In (1) and below, a "hat" above a variable denotes a rate of percentage change. The total labor force at any time is fully employed in tradables (L_T) and in nontradables (L_N), so that

(2) $L = L_T + L_N.$

Production of tradables and nontradables requires capital inputs, K_T and K_N, as well as labor inputs. Capital does not depreciate in use. The production functions are

(3) $Y_T = \theta_T K_T^{\alpha} L_T^{1-\alpha} \equiv \theta_T L_T f(K_T/L_T)$

and

(4) $Y_N = \theta_N K_N^{\beta} L_N^{1-\beta} \equiv \theta_N L_N g(K_N/L_N)$

for tradables and nontradables, respectively. The factor-productivity parameters θ_T and θ_N are functions of time and grow at the constant nonnegative proportional rates $\hat{\theta}_T$ and $\hat{\theta}_N$. Capital-labor ratios in the two sectors are denoted by by $k_T \equiv K_T/L_T$ and $k_N \equiv K_N/L_N$.

I identify the price of nontradables in terms of tradables with the real exchange rate and use q as before to denote this price. A rise in q is again a real appreciation, a fall a real depreciation.

The world capital market confronts the economy with a parametric rate of return on capital employed in tradables, r. Given that the price of capital in terms of tradables is fixed at 1, asset-market arbitrage ensures a domestic rental rate for capital equal to r. Production efficiency then requires that this rental equal capital's marginal value product in either sector:

$$(5) \qquad r = \theta_T f'(k_T) = \theta_T \alpha k_T^{\alpha-1},$$

$$(6) \qquad r = q\theta_N g'(k_N) = q\theta_N \beta k_N^{\beta-1}.$$

Equation (5) ties down k_T; the factor-price frontier (zero-profit condition) in tradables then determines the tradables wage, w:

$$(7) \qquad w = \theta_T[f(k_T) - f'(k_T)k_T] = \theta_T(1 - \alpha)k_T^{\alpha}.$$

Combination of (5) and (7) leads to the wage equation

$$(8) \qquad w = \theta_T(1 - \alpha)(\theta_T\alpha/r)^{\frac{\alpha}{1-\alpha}} = (1 - \alpha)\theta_T^{\frac{1}{1-\alpha}}(\alpha/r)^{\frac{\alpha}{1-\alpha}}.$$

Given international prices, the wage w is determined *entirely* by factor productivity in tradables.

Behind this result is the assumption that the economy actually *does* produce some tradables. In principle the economy could produce nothing but nontradables, financing its consumption of tradables out of foreign-asset holdings. If the economy were to specialize in nontradables--and I will have to check later whether nonspecialization is dynamically sustainable--the tradables wage would depend on factors other than those appearing in (8). For the moment I will simply assume nonspecialization.

A zero-profit condition for nontradables yields the equilibrium real exchange rate (relative price of nontradables), q. Equation (6) gives a capital-labor ratio of

$$(9) \quad k_N = (q\theta_N\beta/r)^{\frac{1}{1-\beta}} .$$

But under competitive conditions $q\theta_N g(k_N) = rk_N + w$. So (4), (8), and (9) show that (as long as r is constant) q has the dynamics:

$$(10) \quad \hat{q} = (1-\beta)\hat{w} - \hat{\theta}_N = \frac{1-\beta}{1-\alpha}\,\hat{\theta}_T - \hat{\theta}_N.$$

Turn next to the economy's consumption side. There is a representative dynasty that grows at rate n, $n < r$. Its members maximize the discounted value of current and future generations' utility from consumption of tradables and nontradables,

$$\int_t^\infty [\nu\log c_T(s) + (1-\nu)\log c_N(s)]e^{(n-\delta)(s-t)}ds,$$

where c_T and c_N are *per capita* consumption levels and the subjective discount rate δ is assumed to exceed n. One first-order condition for maximizing this objective function is that per capita tradables consumption grow at rate $r - \delta$:

$$(11) \quad \hat{c}_T = r - \delta.$$

A second is the static tangency condition $\frac{(1-\nu)/c_N}{\nu/c_T} = q$, which, together with (11), gives the dynamics of c_N as

$$(12)\quad \hat{c}_N = r - \delta - \hat{q}.$$

The real interest rate in this economy is just

$$(13)\quad r - (1-\nu)\hat{q} = r - (1-\nu)\left(\frac{1-\beta}{1-\alpha}\,\hat{\theta}_T - \hat{\theta}_N\right).$$

This expression leads to the important conclusion that national real interest rates--when defined, as above and in Baxter's paper, in terms of the domestic consumption basket--need not, as a matter of theory, converge. For example, a permanent fall in productivity growth in nontradables, $\hat{\theta}_N$, entails a permanent rise in the equilibrium rate of increase in q, and thus a fall in the domestic real interest rate. Because there has been no accompanying change abroad, the foreign-domestic real interest differential widens permanently.

The economy's equilibrium growth path is not "balanced." If productivity growth is faster in tradables than in nontradables and $\beta < \alpha$, as is typical, then (10) and (12) show that the ratio of tradables to nontradables consumption will rise over time.

3. Factor markets and the possibility of specialization

The preceding discussion was predicated on the assumption that the economy remains nonspecialized in production, with some tradables always produced. To check whether this will be so, a closer look at the economy's factor markets is necessary.

International capital mobility ensures that the supply and allocation of capital will accommodate the implied consumption paths. How does the labor market adjust over time? The equilibrium condition in the market for nontradables is

$$c_N = (L_N/L)\theta_N g(k_N),$$

from which it follows that

$$(14)\quad \hat{c}_N - \hat{\theta}_N - \beta\hat{k}_N = \hat{L}_N - \hat{L}.$$

The left-hand side of (14) is the excess per capita demand for nontradables that would emerge if the labor-force share of nontradables remained constant over time: it is the percentage growth in per capita demand for nontradables, less the increase in supply due to growing factor productivity, less the increase in supply from growing employment of capital, given L_N. [Observe the implication of (8)-(10) that $\hat{w} = \hat{k}_N = \hat{\theta}_T/(1 - \alpha)$.] Thus the right-hand side of (14) is the growth in the nontradable sector's labor-force share that maintains goods-market equilibrium.[6]

Define ω_T to be L_T/L and ω_N to be $L_N/L = 1 - \omega_T$. By (1), (9), (10), and (12), (14) implies

$$(15)\quad \hat{\omega}_N = r - \delta - \frac{\hat{\theta}_T}{1-\alpha}.$$

Given the unit substitution elasticities I've assumed, the growth of relative employment in nontradables equals the growth rate of tradables consumption less that of the tradables wage. Eq. (15) implies that

$$(16)\quad \hat{\omega}_T = -(\omega_N/\omega_T)\hat{\omega}_N.$$

[6]As signaled earlier, the country can satisfy growth in its demand for tradables by running down its net foreign assets. Provided an equilibrium exists (which it will under the parameters assumed here), initial consumption levels adjust to place the economy within its intertemporal budget constraint.

Eqs. (15) and (16) show that the share of nontradables in employment can grow, shrink, or remain constant over time. If $r = \delta$, for example, $\hat{\omega}_{\mathrm{N}}$ is negative and real wage growth leads to a secular exodus of labor from nontradables. Note from (16) that because $\omega_{\mathrm{N}}(t)$ asymptotes to 0 whenever $\hat{\omega}_{\mathrm{N}} < 0$, $\hat{\omega}_{\mathrm{T}}(t)$ asymptotes to 0 as well: the shift of employment shares toward tradables proceeds at an ever-decelerating pace.

In an economy with growing tradables consumption, however, $\hat{\omega}_{\mathrm{N}}$ can be positive if productivity growth in tradables is modest enough. This case appears problematic, for equation (15) now implies that in finite time the economy will specialize in producing nontradables. Because the nonspecialization assumption maintained so far patently is contradicted, we have to think about the dynamics of q in an economy specialized in nontradables.

In such an economy the rental on capital is still r and the real exchange rate still satisfies the zero-profit condition $q\theta_{\mathrm{N}}g(k) = rk + w$, with k given by (9). (Now $k = K/L = K_{\mathrm{N}}/L_{\mathrm{N}}$, of course.) Thus, we can think of equilibrium q as an invertible function of w. The equilibrium wage implies a value of q such that supply equals demand in the nontradables market, given that $L_{\mathrm{N}} = L$. Wage dynamics can be understood by combining (9), (12), the factor-demand equation $\hat{k} = \hat{w}$, and the goods-market equilibrium condition $c_{\mathrm{N}} = \theta_{\mathrm{N}}k^{\beta}$: the result is $\hat{w} = r - \delta$. Since the zero-profit condition again implies [as in (10)] that price increases must cover the increase in factor costs net of productivity improvements, the conclusion is that

$$(17) \quad \hat{q} = (1-\beta)\hat{w} - \hat{\theta}_{\mathrm{N}} = (1-\beta)(r - \delta) - \hat{\theta}_{\mathrm{N}}$$

when the economy is specialized.

Equation (17) implies a domestic real interest rate of

$$(18)\quad r - (1-\nu)\hat{q} = r - (1-\nu)\Big[(1 - \beta)(r - \delta) - \hat{\theta}_N\Big].$$

Notice the difference between the present case and the case in which tradables are produced at home. Tradables will be produced at zero profit only if $\hat{w} = \hat{\theta}_T/(1-\alpha)$ [recall (8)]; equation (10) follows immediately from this relation and the zero-profit condition for nontradables. When tradables aren't produced, however, $\hat{w}$ is no longer determined by the factor-price frontier in tradables, and instead is ultimately determined from the economy's demand side. Equation (17) reflects that higher growth in the consumption of tradables would be accompanied, at given relative prices, by equiproportionate growth in the demand for nontradables. With all domestic labor already employed in tradables, this demand growth can be satisfied only if the capital intensity of nontradables production rises over time. Employers thus bid up the wage over time, and q must rise with it.

4. Implications of stochastic productivity growth

The preceding model can be extended to a stochastic setting. To simplify I assume that θ_N only is random. It is given by

$$(19)\quad \theta_N(t) = \kappa e^{\hat{\theta}_N t - z(t)},$$

where κ is a constant and $z(t)$ is a random variable--an adverse productivity shock in nontradables. (I am abusing the notation by now letting $\hat{\theta}_N$ stand for the deterministic time trend in θ_N.) The shock $z(t)$ is in the time-t information set and evolves according to a Gaussian diffusion process:

$$(20) \quad dz = -\rho z dt + \sigma d\zeta, \qquad \rho \geq 0.$$

This equation means that $z(t)$ can be written as the stochastic integral

$$(21) \quad z(t) = e^{-\rho t} z(0) + \int_0^t e^{-\rho(t-s)} d\zeta(s),$$

so that $z(t)$ is a distributed lag on past innovations $d\zeta(s)$. If $\rho = 0$, $z(t)$ follows a random walk; otherwise (20) describes a mean-reverting process under which the influence of past innovations decays at a positive rate.

Since $z(t)$ is known at time t, factors will move immediately to equate ex post marginal value products between sectors. (For brevity I discuss only the nonspecialization case.) Equation (9) will still hold at each moment and by (19) the real exchange rate will be:

$$(22) \quad q(t) = (\chi/\kappa) r^{\frac{\beta-\alpha}{1-\alpha}} \theta_T(t)^{\frac{1-\beta}{1-\alpha}} e^{-[\hat{\theta}_N t - z(t)]},$$

where χ is a constant function of α and β. Taking natural logarithms of (22) leads to the univariate model

$$(23) \quad \ln q(t) = \gamma + \mu t + z(t),$$

where $\gamma \equiv \ln(\chi/\kappa) + \frac{\beta-\alpha}{1-\alpha}\ln r + \frac{1-\beta}{1-\alpha}\theta_T(0)$ and μ, the deterministic trend, is given as in (10) by

$$\mu \equiv \frac{1-\beta}{1-\alpha}\hat{\theta}_T - \hat{\theta}_N.$$

Now consider two points in time, t and $t-1$. say, and define $\phi \equiv e^{-\rho}$, $\varepsilon(t) \equiv \int_{t-1}^{t} e^{-\rho(t-s)}d\zeta(s)$. Then (21) and (23) imply

$$\ln q(t) = [(1-\phi)\gamma + \phi\mu] + \mu(1-\phi)t + \phi\ln q(t-1) + \varepsilon(t),$$

which, because $E_{t-1}\varepsilon(t) = 0$ and $\phi < 1$, is the same as the stationary process found in section 1 to be a good characterization of Japan's real exchange rate (table 1). If $\rho = 0$, $\phi = 1$ and the log real exchange rate follows the random walk

$$\text{(24)} \quad \ln q(t) = \mu + \ln q(t-1) + \varepsilon(t)$$

with $\varepsilon(t) = \int_{t-1}^{t} d\zeta(s)$. Eq. (24) was the alternative, nonstationary, characterization of Japan's real exchange rate.[7]

The final step is to characterize the domestic real interest rate. Itô's lemma, applied to (22), shows that

$$\frac{dq}{q} = \left(\mu + \frac{\sigma^2}{2}\right)dt + dz.$$

Accordingly (20) implies that the domestic real interest rate is

$$\text{(25)} \quad r - (1-\nu)\frac{E_t dq}{q dt} = r - (1-\nu)\left(\frac{1-\beta}{1-\alpha}\hat{\theta}_T - \hat{\theta}_N + \frac{\sigma^2}{2}\right) + (1-\nu)\rho z.$$

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[7] In Rogoff's model (1991) the real exchange rate follows a random walk when there are no productivity shocks in nontradables, or when those shocks themselves follow a random walk. In the present model, however, shocks to the tradable and nontradable sectors play symmetric roles in determining q.

Eq. (25) is comparable to eq. (13) apart from two modifications. First, the equation contains a variance term that reflects Jensen's inequality. Second, and more important, is the dependence of the real interest rate on the current value of the shock z. It is this term that induces a positive correlation between the log real exchange rate [eq. (23)] and the real interest rate.[8]

The intuition is clear. According to (23) an adverse productivity shock in nontradables raises their price q. By (20), however, this shock is expected to decay over time, and as a result, q is expected to fall. This expected fall in q implies a relatively high domestic real interest rate. Thus q and the real interest rate are positively correlated, as they may be in the class of models Baxter describes. As already noted, the Japanese case (table 1) fits this picture.

If z follows a random walk $\rho = 0$ and this correlation disappears: permanent productivity disturbances induce no definite comovements in real exchange rates and real interest rates. This result does not mean, of course, that some relation will not reemerge under more complicated unit-root processes, such as the ARIMA(1,1,0) that appears to fit the real exchange rate of the U.S. dollar (table 2). Notice, however, that the estimated autoregressive terms are significantly positive, indicating forward momentum in the U.S. real exchange rate. If the univariate integrated model in table 2 is a good approximation to agents' forecasting rule, then we'd expect a *negative* correlation between the U.S. real exchange rate and real interest rate.

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[8]Here the correlation actually is perfect, although this tight link could be broken by making r stochastic.

5. The real exchange rate-real interest rate link: Detection and interpretion

I conclude with some observations on the two main issues Baxter addresses, the use of econometrics to detect the real exchange rate-real interest rate link and the bearing of that evidence on the validity of competing exchange-rate theories.

Even if real interest rate differentials need not converge to zero, Baxter is still correct in arguing that they should be statistically stationary.[9] Log real exchange rates can plausibly be nonstationary, as in the last section's model. If they are stationary no special pre-filtering is necessary; but if they are not, one must take a stand on how to remove the unit root.

Baxter takes earlier researchers to task for analyzing first-differenced real exchange rates, a procedure she claims amounts to discarding important low-frequency information. To assess this claim, consider a nonstationary process such as the one the dollar's real exchange rate apparently follows (table 2),

$$(1 - B)\ln q_t = \phi(1 - B)\ln q_{t-1} + \varepsilon_t,$$

where $0 < \phi < 1$ and the constant is suppressed. The spectral density of the AR(1) process $(1 - B)\ln q_t$ at frequency λ is

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[9]The easiest way to see this is to note that when uncovered interest-rate parity holds, $\ln q_{t+1} - \ln q_t \approx r_t^* - r_t + \varphi_{t+1}$ where φ_{t+1} is an I(0) forecast error. Thus $r_t^* - r_t$ can't be I(1) unless $\ln q_t$ is I(2), which is hard to imagine. An I(1) risk premium cointegrated with $r_t^* - r_t$ would in principle allow $r_t^* - r_t$ to be I(1) too, but this hypothesis seems almost equally far fetched.

$$f(\lambda) = \frac{\sigma_\varepsilon^2}{2\pi}\left(\frac{1}{1 - 2\phi\cos\lambda + \phi^2}\right),$$

which is decreasing over $[0,\pi]$. The differenced variable $(1 - B)q_t$ therefore has relatively more spectral power at *low* frequencies, not at high ones. So differencing $\ln q_t$ won't necessarily prevent the detection of a medium- to low-frequency real exchange rate-real interest rate relationship if one is present.

When Baxter argues for focusing on the Beveridge-Nelson (BN) transitory component of $\ln q_t$ she may in essence be advocating an approach not too distant from simple differencing. Continuing with the present example, the BN transitory component of $\ln q_t$ [defined, following Baxter's eq. (9), as $\ln q_t$ minus its permanent component] is

$$\frac{-\phi}{1-\phi}\sum_{i=0}^{\infty}\phi^i\varepsilon_{t-i} = \frac{-\phi}{1-\phi}(1 - B)\ln q_t.$$

Thus, aside from a proportionality constant, the BN transitory component of $\ln q_t$ *is* the first difference of that variable, given the form of nonstationarity that I have posited.[10]

Notice, however, that we should not now expect to find a positive correlation between the BN transitory component of $\ln q_t$ and the real interest differential because the former is perfectly negatively correlated with $\ln q_t$ itself. Indeed, under interest parity the BN component and the real-interest differential now are *negatively*

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[10] At this point I emphasize that the real exchange rate data I use differ from Baxter's and have somewhat different time-series properties. (In particular time-averaging is probably an issue.) My general point is that for some nonstationary processes the BN filter will have an effect on the data similar to that of the first-difference filter.

correlated for the reason explained at the end of the last section. Baxter's multivariate calculations are of course more complex than my example, but the example's results do raise the question of how to interpret her findings in terms of competing economic models.[11] The intertemporal model I discussed earlier suggests that it may be difficult to choose between classical and Keynesian models, for example, merely by testing their implications concerning real exchange rates and real interest-rate differentials.

Baxter's exploratory attempt to link real interest differentials to policy variables is therefore welcome as a preliminary step in throwing structural light on the correlations in the data. But the results leave wide open the question of which class of models can best explain the empirical record. Much more needs to be done, in particular, before we conclude that monetary policy does not have the short-run effects on interest and exchange rates that policymakers confidently expect. I, for one, would have to be convinced that a realistically calibrated sticky-price model would be very unlikely to produce the empirical results reported here. Baxter has applied this type of methodology successfully to other questions in international macroeconomics. Why not apply it to this one?

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[11] It should also be remarked that the BN decomposition into permanent and transitory components is only one of many possible decompositions.

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