# Export Variety and Country Productivity: <br> Estimating the Monopolistic Competition Model With endogenous productivity 

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#### Abstract

This paper provides evidence on the monopolistic competition model with heterogeneous firms and endogenous technology. We show that this model has a well-defined GDP function where relative export variety enters positively. The GDP function is estimated on data for 44 countries from 1980 to 2000. Export variety to the United States increases by about $10 \%$ per year, or eight times over these two decades. Instruments such as tariffs and distance are shown to affect export variety, but the fall in U.S. tariffs explains only a small part of export variety growth. The eightfold increase in export variety is associated with a $10 \%$ productivity improvement for exporters. Overall, the model can explain $40 \%$ of the within-country variation in productivity, but only a small fraction of the between-country variation in productivity.


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## 1. Introduction

Empirical research in international trade (as well as other fields) has made it clear that productivity levels differ a great deal across countries. ${ }^{1}$ This conclusion begs the question of where the technology differences come from. While various explanations have been proposed that do not depend on international trade, ${ }^{2}$ our interest here is whether trade itself can explain the productivity differences across countries. This conclusion is suggested by recent models of monopolistic competition and trade in which productivity levels are endogenous.

Two examples of this recent literature are Eaton and Kortum (2002) and Melitz (2003). Eaton and Kortum allow for stochastic differences in technologies across countries, with the lowest cost country becoming the exporter of a product variety to each location. In that case, the technologies utilized in a country will depend on its distance and trade barriers with other countries. Melitz allows for stochastic draws of technology for each firm, and only those firms with productivities above a certain cutoff level will operate. A subset of these firms - the most productive - also become exporters. Melitz shows how the average productivity in a country is affected by changes in trade barriers and transport costs. A reduction in trade barriers, for example, draws less efficient firms into exporting, but they are still more efficient than the marginal domestic firm. It follows that average country productivity rises.

Empirical testing of this class of models can proceed by utilizing firm-level data and inferring the productivity levels of firms. That approach is taken by Bernard et al (2003) for U.S. firms; Eaton, Kortum and Kramarz (2003, 2004) for French firms; and Helpman, Melitz and Yeaple (2004) for U.S. multinationals operating abroad. When firm level data are available, it is

[^1]highly desirable to make use of it like these authors do. But for many countries such data are not available, and in those cases, we are still interested in determining the extent to which openness to trade can explain country productivity. That is our objective in this paper, using a broad cross-section of advanced and developing nations and disaggregating across sectors.

In section 2 we review the monopolistic competition model with heterogeneous firms, from Melitz (2003), Bernard, Redding and Schott (2004) and Chaney (2005). We emphasize some features of that model that these authors do not: for example, the "cutoff" productivity of firms producing for either the domestic or export markets are both at the socially optimal level. This means that we can use a GDP function for the economy, similar to the competitive case. In each sector, only a subset of firms become exporters, and these are the most productive firms. It follows that when the share of exporting firms rises, or equivalently, the share of exported varieties rises, then average productivity and GDP increase. Therefore, relative export variety enter the GDP function positively. ${ }^{3}$

Our empirical specification is developed in section 3. We draw on Harrigan (1997), who estimates a translog GDP function in a competitive model allowing for industry productivity differences across countries. In our case, we allow for exogenous country-wide differences in productivity, which are country fixed effects in the translog function. At the industry level, we assume that the distribution function for firm productivities is the same across countries, so productivity is determined by the endogenous "cutoff" levels for firms. ${ }^{4}$ A goal of the empirical work is to determine what amount of the productivity differences across countries and over time

[^2]are determined by endogenous versus exogenous factors.
In section 4 we estimate the GDP function using data on 44 countries from 1980 to 2000, distinguishing the sectoral outputs of seven sectors. Export variety is treated as an endogenous variable, and the instrumental variables used to determine export variety are those suggested by our model: tariffs, trade agreements and distance. The index of export variety we use draws on Feenstra (1994), and has been employed recently by Broda and Weinstein (2005) and Hummels and Klenow (2005). Using this measure, average export variety to the U.S. has increased by about $10 \%$ per year over 1980-2000, so export variety expands eight times over the two decades. Most of this increase in export variety is explained by time fixed-effects, however, rather than by observed tariff cuts or trade agreements.

Parameter estimates of the GDP function show that a doubling of export variety leads to a $3.4 \%$ increase in country productivity, on average, so the eight-fold expansion of export variety over 1980-2000 explains more than a 10\% increase in exporters' productivity. This is an estimate of the endogenous portion of productivity gains, though as just noted, the variety increase itself is not well-explained by tariff cuts. In the time series we are able to explain a substantial portion of productivity gains across countries: over all 44 countries, export variety explains $40.4 \%$ of within-country variation in productivity, and $60.8 \%$ of the within-country variation for just the OECD countries. But this linkage between export variety and productivity is not enough to explain the enormous cross-country differences in productivity. In the total sample, export variety can explain only $2.4 \%$ of the variation in country productivity, or $10.6 \%$ for the OECD countries. We conclude that the monopolistic competition model with endogenous productivity differences is quite effective at accounting for the time-series variation within countries, but cannot account for the large absolute differences in productivity between them.

## 2. Monopolistic Competition with Heterogeneous Firms

We assume some familiarity with the monopolistic competition model of Melitz (2003), and outline here a two-country, multi-factor, multi-sector version that also draws upon Bernard, Redding and Schott (2005) and Chaney (2005). We focus on the home country H, while denoting foreign variables with the superscript F .

In each sector $\mathrm{i}=1, \ldots, \mathrm{~N}$ at home, there is a mass of $\mathrm{M}_{\mathrm{i}}$ firms operating in equilibrium. Each period, a fraction $\delta$ of these firms go bankrupt and are replaced by new entrants. Each new entrant pays a fixed cost $\mathrm{f}_{\mathrm{ie}}$ to receive a draw $\varphi_{\mathrm{i}}$ of productivity from a cumulative distribution $\mathrm{G}_{\mathrm{i}}\left(\varphi_{\mathrm{i}}\right)$, which gives rise to the marginal cost of $\mathrm{m}_{\mathrm{i}} / \varphi_{\mathrm{i}}$. Only those firms with productivity above a cutoff level $\varphi_{i}^{*}$ find it profitable to actually produce (the cutoff level will be determined below). Letting $M_{i e}$ denote the mass of new entrants in sector $i$, then $\left[1-G_{i}\left(\varphi_{i}^{*}\right)\right] M_{i e}$ firms successfully produce. In a stationary equilibrium, these should replace the firms going bankrupt, so that:

$$
\begin{equation*}
\left[1-\mathrm{G}_{\mathrm{i}}\left(\varphi_{\mathrm{i}}^{*}\right)\right] \mathrm{M}_{\mathrm{ie}}=\delta \mathrm{M}_{\mathrm{i}} . \tag{1}
\end{equation*}
$$

Conditional on successful entry, the distribution of productivities for firms in sector $i$ is then:

$$
\mu_{\mathrm{i}}\left(\varphi_{\mathrm{i}}\right)=\left\{\begin{array}{c}
\frac{\mathrm{g}_{\mathrm{i}}\left(\varphi_{\mathrm{i}}\right)}{\left[1-\mathrm{G}_{\mathrm{i}}\left(\varphi_{\mathrm{i}}^{*}\right)\right]} \text { if } \varphi_{\mathrm{i}} \geq \varphi_{\mathrm{i}}^{*},  \tag{2}\\
0 \quad \text { otherwise },
\end{array}\right.
$$

where $g_{i}\left(\varphi_{i}\right)=\partial G_{i}\left(\varphi_{i}\right) / \partial \varphi_{i}$.
Home and foreign consumers both have CES preferences that are symmetric over product varieties. Given home expenditure of $E_{i}^{H}$ in sector $i$, it follows that the revenue earned by a home firm from selling in the domestic sector $i$ at the price $p_{i}\left(\varphi_{i}\right)$ is:

$$
\begin{equation*}
r_{i}\left(\varphi_{i}\right)=p_{i}\left(\varphi_{i}\right) q_{i}\left(\varphi_{i}\right)=\left[\frac{p_{i}\left(\varphi_{i}\right)}{P_{i}^{H}}\right]^{1-\sigma_{i}} E_{i}^{H}, \quad \sigma_{i}>1, \tag{3}
\end{equation*}
$$

where $\mathrm{q}_{\mathrm{i}}\left(\varphi_{\mathrm{i}}\right)$ is the quantity sold and $\mathrm{P}_{\mathrm{i}}^{\mathrm{H}}$ is the home CES price index for sector i :

$$
\begin{equation*}
P_{i}^{H}=\left[\int_{\varphi_{\mathrm{i}}^{*}}^{\infty} p_{\mathrm{i}}\left(\varphi_{\mathrm{i}}\right)^{1-\sigma_{\mathrm{i}}} \mathbf{M}_{\mathrm{i}} \mu_{\mathrm{i}}\left(\varphi_{\mathrm{i}}\right) \mathrm{d} \varphi_{\mathrm{i}}+\int_{\varphi_{\mathrm{i}}^{\mathrm{F}}}^{\infty} \mathrm{p}_{\mathrm{i}}^{\mathrm{F}}\left(\varphi_{\mathrm{i}}\right)^{1-\sigma_{\mathrm{i}}} \mathbf{M}_{\mathrm{i}}^{\mathrm{F}} \mu_{\mathrm{i}}^{\mathrm{F}}\left(\varphi_{\mathrm{i}}\right) \mathrm{d} \varphi_{\mathrm{i}}\right]^{\frac{1}{1-\sigma_{\mathrm{i}}}} . \tag{4}
\end{equation*}
$$

The first integral in this expression is taken over home firms, with prices $p_{i}\left(\varphi_{i}\right)$ and mass $M_{i}$, selling in the domestic market. The second integral is taken over foreign firms, with prices $p_{i}^{F}\left(\varphi_{i}\right)$ and mass $M_{i}^{F}$, exporting to the home market.

In our analysis below it will be convenient to treat revenue as a function of quantity. Using the second equality in (3) to solve for price as a function of quantity, and substituting this back into (3), we obtain:

$$
\begin{equation*}
r_{i}\left(\varphi_{i}\right)=A_{i d} q_{i}\left(\varphi_{i}\right)^{\frac{\sigma_{i}-1}{\sigma_{i}}} \text {, where } A_{i d} \equiv P_{i}^{H}\left(\frac{E_{i}^{H}}{P_{i}^{H}}\right)^{\frac{1}{\sigma_{i}}} \tag{3'}
\end{equation*}
$$

We introduce the notation $\mathrm{A}_{\mathrm{id}}$ as shift parameter in the demand curve facing home firms for their domestic sales. It depends on the CES price index $P_{i}^{H}$, and also on domestic expenditure $E_{i}^{H}$ in sector i. Home firms takes both these as given when maximizing profits.

While firms with productivities $\varphi_{i} \geq \varphi_{i}^{*}$ find it profitable to produce for the domestic market, those with productivities $\varphi_{i} \geq \varphi_{i x}^{*} \geq \varphi_{i}^{*}$ find it profitable to export (we will determine $\varphi_{\mathrm{ix}}^{*}$ below). ${ }^{5}$ A home firm exporting in sector i faces the iceberg transport costs of $\tau_{\mathrm{i}} \geq 1$ meaning that $\tau_{i}$ units must be sent in order for one unit to arrive in the foreign country. Letting

[^3]$\mathrm{p}_{\mathrm{ix}}\left(\varphi_{\mathrm{i}}\right)$ and $\mathrm{q}_{\mathrm{ix}}\left(\varphi_{\mathrm{i}}\right)$ denote the price received and quantity shipped at the factory-gate, ${ }^{6}$ the revenue earned by the exporter is:
\[

$$
\begin{equation*}
\mathrm{r}_{\mathrm{ix}}\left(\varphi_{\mathrm{i}}\right)=\mathrm{p}_{\mathrm{ix}}\left(\varphi_{\mathrm{i}}\right) \mathrm{q}_{\mathrm{ix}}\left(\varphi_{\mathrm{i}}\right)=\left[\frac{\mathrm{p}_{\mathrm{ix}}\left(\varphi_{\mathrm{i}}\right) \tau_{\mathrm{i}}}{\mathrm{P}_{\mathrm{i}}^{\mathrm{F}}}\right]^{1-\sigma_{\mathrm{i}}} \mathrm{E}_{\mathrm{i}}^{\mathrm{F}} \tag{5}
\end{equation*}
$$

\]

where $P_{i}^{F}$ is the aggregate CES price for sector $i$ in the foreign country, and $E_{i}^{F}$ is foreign expenditure in sector i. Once again, it is convenient to treat revenue as a function of quantity, which is determined from (5) as:

$$
\begin{equation*}
r_{i x}\left(\varphi_{i}\right)=A_{i x} q_{i x}\left(\varphi_{i}\right)^{\frac{\sigma_{i}-1}{\sigma_{i}}}, \text { where } A_{i x} \equiv\left(\frac{P_{i}^{F}}{\tau_{i}}\right)\left(\frac{\tau_{i} E_{i}^{F}}{P_{i}^{F}}\right)^{\frac{1}{\sigma_{i}}} . \tag{5'}
\end{equation*}
$$

We introduce the notation $\mathrm{A}_{\mathrm{ix}}$ as shift parameter in the demand curve facing home firms for their export sales. It depends on the foreign CES price index $P_{i}^{F}$, foreign expenditure $E_{i}^{F}$, and the transport costs in sector i.

Integrating ( $3^{\prime}$ ) and ( $5^{\prime}$ ), we obtain revenue from domestic and export sales in sector i:

$$
\begin{align*}
& \mathrm{R}_{\mathrm{id}} \equiv \mathrm{M}_{\mathrm{i}} \mathrm{~A}_{\mathrm{id}} \int_{\varphi_{\mathrm{i}}^{*}}^{\infty} \mathrm{q}_{\mathrm{i}}\left(\varphi_{\mathrm{i}}\right)^{\frac{\sigma_{\mathrm{i}}-1}{\sigma_{\mathrm{i}}}} \mu_{\mathrm{i}}\left(\varphi_{\mathrm{i}}\right) \mathrm{d} \varphi_{\mathrm{i}}  \tag{6}\\
& \mathrm{R}_{\mathrm{ix}} \equiv \mathrm{M}_{\mathrm{i}} \mathrm{~A}_{\mathrm{ix}} \int_{\varphi_{\mathrm{ix}}^{*}}^{\infty} \mathrm{q}_{\mathrm{ix}}\left(\varphi_{\mathrm{i}}\right)^{\frac{\sigma_{\mathrm{i}}-1}{\sigma_{\mathrm{i}}}} \mu_{\mathrm{i}}\left(\varphi_{\mathrm{i}}\right) \mathrm{d} \varphi_{\mathrm{i}} \tag{7}
\end{align*}
$$

Notice that the mass of domestic firms or varieties is $\int_{\varphi_{i}^{*}}^{\infty} M_{i} \mu_{i}\left(\varphi_{i}\right) d \varphi_{i}=M_{i}$, which we are aggregating over in (6). But the mass of exporting firms or varieties that we are aggregating over

[^4]in (7) is $\int_{\varphi_{\mathrm{ix}}^{*}}^{\infty} \mathrm{M}_{\mathrm{i}} \mu_{\mathrm{i}}\left(\varphi_{\mathrm{i}}\right) \mathrm{d} \varphi_{\mathrm{i}} \leq \mathrm{M}_{\mathrm{i}}$. It will be convenient to denote the range of export varieties relative to domestic varieties by:
\[

$$
\begin{equation*}
\chi_{\mathrm{i}} \equiv\left(\int_{\varphi_{\mathrm{ix}}^{*}}^{\infty} \mu_{\mathrm{i}}(\varphi) \mathrm{d} \varphi\right)=\left(\frac{1-\mathrm{G}\left(\varphi_{\mathrm{ix}}^{*}\right)}{1-\mathrm{G}\left(\varphi_{\mathrm{i}}^{*}\right)}\right) \leq 1 . \tag{8}
\end{equation*}
$$

\]

On the resource side, having $\mathrm{M}_{\mathrm{i}}$ firms produce output $\mathrm{q}_{\mathrm{i}}\left(\varphi_{i}\right)$ for the domestic market has a cost of $h_{i}\left[v_{i}\left(\varphi_{i}\right)\right]=M_{i}\left[\left(q_{i}\left(\varphi_{i}\right) / \varphi_{i}\right)+f_{i}\right]$, where $f_{i}$ is the fixed cost of production, $v_{i}\left(\varphi_{i}\right)$ is a K-dimensional vector of factor demand, and $h_{i}: R^{K} \rightarrow R^{1}$ is a homogeneous of degree one and strictly quasi-concave mapping from the vector of factor demands to a scalar. The total resources used for domestic production are then:

$$
\begin{equation*}
\int_{\varphi_{\mathrm{i}}^{*}}^{\infty} \mathrm{h}_{\mathrm{i}}\left[\mathrm{v}_{\mathrm{i}}\left(\varphi_{\mathrm{i}}\right)\right] \mu_{\mathrm{i}}\left(\varphi_{\mathrm{i}}\right) \mathrm{d} \varphi_{\mathrm{i}}=\int_{\varphi_{\mathrm{i}}^{*}}^{\infty} \mathrm{M}_{\mathrm{i}}\left[\left(\mathrm{q}_{\mathrm{i}}\left(\varphi_{\mathrm{i}}\right) / \varphi_{\mathrm{i}}\right)+\mathrm{f}_{\mathrm{i}}\right] \mu_{\mathrm{i}}\left(\varphi_{\mathrm{i}}\right) \mathrm{d} \varphi_{\mathrm{i}} \tag{9}
\end{equation*}
$$

Likewise, the exporting firms producing output $q_{i x}$ with productivity $\varphi_{i}$ have a resource cost of:

$$
\begin{equation*}
\int_{\varphi_{i x}^{*}}^{\infty} \mathrm{h}_{\mathrm{i}}\left[\mathrm{v}_{\mathrm{ix}}\left(\varphi_{\mathrm{i}}\right)\right] \mu_{\mathrm{i}}\left(\varphi_{\mathrm{i}}\right) \mathrm{d} \varphi_{\mathrm{i}}=\int_{\varphi_{\mathrm{ix}}^{*}}^{\infty} \mathrm{M}_{\mathrm{i}}\left[\left(\mathrm{q}_{\mathrm{ix}}\left(\varphi_{\mathrm{i}}\right) / \varphi_{\mathrm{i}}\right)+\mathrm{f}_{\mathrm{ix}}\right] \mu_{\mathrm{i}}\left(\varphi_{\mathrm{i}}\right) \mathrm{d} \varphi_{\mathrm{i}} \tag{10}
\end{equation*}
$$

where $f_{i x}$ is the additional fixed cost for exporters. We assume that the function $h_{i}$ for export and domestic production is the same, so that the factor proportions in the two activities are equal.

### 2.1 Social Planner's Problem

Total GDP in the economy is obtained by summing revenue over the sectors:

$$
\begin{equation*}
\mathrm{R}=\sum_{\mathrm{i}}^{\mathrm{N}} \mathrm{R}_{\mathrm{id}}+\mathrm{R}_{\mathrm{ix}} \tag{11}
\end{equation*}
$$

Consider now a social planner's problem of maximizing GDP in (11), subject to the resource constraints for the economy, which are (9), (10) and:

$$
\begin{equation*}
\sum_{i=1}^{N}\left[\int_{\varphi_{i}^{*}}^{\infty} v_{i}\left(\varphi_{i}\right) \mu_{i}\left(\varphi_{i}\right) d \varphi_{i}+\int_{\varphi_{i x}^{*}}^{\infty} v_{i x}\left(\varphi_{i}\right) \mu_{i}\left(\varphi_{i}\right) d \varphi_{i}\right]=V-\sum_{i=1}^{N} M_{i e} v_{i e} \tag{12}
\end{equation*}
$$

The summation on the left of (12) is total resources used in production. On the right of (12), V is the vector of factor endowments for the economy, and from that we subtract the vector of fixed costs $v_{i e}$ from entry into each sector, times the mass of entering firms $\mathrm{M}_{\mathrm{ie}}$. The number of entering firms is given by (1), which we can substitute into (12), obtaining:

$$
\begin{equation*}
\sum_{i=1}^{N}\left[\int_{\varphi_{i}^{*}}^{\infty} v_{i}\left(\varphi_{i}\right) \mu_{i}\left(\varphi_{i}\right) d \varphi_{i}+\int_{\varphi_{i x}^{*}}^{\infty} v_{i x}\left(\varphi_{i}\right) \mu_{i}\left(\varphi_{i}\right) d \varphi_{i}\right]=V-\sum_{i=1}^{N} \frac{\delta M_{i}}{\left[1-G_{i}\left(\varphi_{i}^{*}\right)\right]} v_{i e} . \tag{12'}
\end{equation*}
$$

In maximizing GDP we will hold fixed the shift parameters $A_{i d}$ and $A_{i x}$, which is analogous to holding product prices constant in a conventional GDP function. Then we have:

## Proposition 1

Choose $q_{i}\left(\varphi_{i}\right), q_{i x}\left(\varphi_{i}\right), v_{i}\left(\varphi_{i}\right), v_{i x}\left(\varphi_{i}\right), \varphi_{i}^{*}, \varphi_{i x}^{*}$, and $M_{i}$ for $i=1, \ldots, N$, to maximize GDP in (11), subject to (9), (10) and (12'). Holding fixed the shift parameters $A_{d}=\left(A_{1 d}, \ldots, A_{N d}\right)$ and $\mathrm{A}_{\mathrm{x}}=\left(\mathrm{A}_{1 \mathrm{x}}, \ldots, \mathrm{A}_{\mathrm{Nx}}\right)$, the first-order conditions for an interior maximum are identical to the equilibrium conditions for the monopolistically competitive economy. Then GDP can be written as a function $R\left(\mathrm{~A}_{\mathrm{d}}, \mathrm{A}_{\mathrm{x}}, \mathrm{V}\right)$, and satisfies:
(a) $R\left(A_{d}, A_{x}, V\right)$ is homogeneous of degree one in $\left(A_{d}, A_{x}\right)$, and if $R$ is differentiable then,

$$
\begin{equation*}
\frac{\partial \ln \mathrm{R}}{\partial \ln \mathrm{~A}_{\mathrm{id}}}=\frac{\mathrm{R}_{\mathrm{id}}}{\mathrm{R}}, \text { and } \frac{\partial \ln \mathrm{R}}{\partial \ln \mathrm{~A}_{\mathrm{ix}}}=\frac{\mathrm{R}_{\mathrm{ix}}}{\mathrm{R}} ; \tag{13}
\end{equation*}
$$

(b) $R\left(A_{d}, A_{x}, V\right)$ is homogeneous of degree one in $V$, and $\partial R / \partial V=w$ which is the vector of factor prices.

The proof of Proposition 1 proceeds by setting up the Lagrangian and obtaining the firstorder conditions. Details are provided in the Appendix, and here we simply describe the results. The first-order condition with respect to the domestic and export quantities are that marginal revenue equal marginal cost:

$$
\begin{equation*}
\left(\frac{\sigma_{i}-1}{\sigma_{i}}\right) p_{i}\left(\varphi_{i}\right)=\frac{m_{i}}{\varphi_{i}}, \text { and, } \quad\left(\frac{\sigma_{i}-1}{\sigma_{i}}\right) p_{i x}\left(\varphi_{i}\right)=\frac{m_{i}}{\varphi_{i}} . \tag{14}
\end{equation*}
$$

where $m_{i}$ is the Lagrange multiplier for (9) and (10), representing marginal costs. Using (14), we can calculate profits from domestic and export sales as $r_{i}\left(\varphi_{i}\right)-\left(m_{i} / \varphi_{i}\right) q_{i}\left(\varphi_{i}\right)=r_{i}\left(\varphi_{i}\right) / \sigma_{i}$, and $\mathrm{r}_{\mathrm{ix}}\left(\varphi_{\mathrm{i}}\right)-\left(\mathrm{m}_{\mathrm{i}} / \varphi_{\mathrm{i}}\right) \mathrm{q}_{\mathrm{ix}}\left(\varphi_{\mathrm{i}}\right)=\mathrm{r}_{\mathrm{ix}}\left(\varphi_{\mathrm{i}}\right) / \sigma_{\mathrm{i}}$.

Next, the first-order conditions with respect to productivity cutoffs $\varphi_{\mathrm{i}}^{*}$, and $\varphi_{\mathrm{ix}}^{*}$, are:

$$
\begin{equation*}
\mathrm{r}_{\mathrm{i}}\left(\varphi_{\mathrm{i}}^{*}\right) / \sigma_{\mathrm{i}}=\mathrm{m}_{\mathrm{i}} \mathrm{f}_{\mathrm{i}}, \quad \text { and }, \quad \mathrm{r}_{\mathrm{ix}}\left(\varphi_{\mathrm{ix}}^{*}\right) / \sigma_{\mathrm{i}}=\mathrm{m}_{\mathrm{i}} \mathrm{f}_{\mathrm{ix}} . \tag{15}
\end{equation*}
$$

Thus, the profits earned by the marginal domestic producer and exporter should just cover fixed costs, which are the zero-cutoff-profit (ZCP) conditions of Melitz (2003). ${ }^{7}$ Substituting for domestic and export revenue from (3) and (5), we can confirm that exporters have higher productivity, $\varphi_{i x}^{*} \geq \varphi_{i}^{*}$, provided that: $\left(\tau_{i} \mathrm{P}_{\mathrm{i}}^{\mathrm{H}} / \mathrm{P}_{\mathrm{i}}^{\mathrm{F}}\right)^{\sigma_{i}-1} \mathrm{f}_{\mathrm{ix}} \geq \mathrm{f}_{\mathrm{i}}\left(\mathrm{E}_{\mathrm{i}}^{\mathrm{F}} / \mathrm{E}_{\mathrm{i}}^{\mathrm{H}}\right)$. This is the same as condition used by Melitz (2003, p. 1709) and Bernard, Redding and Schott (2005, p. 13), which we also assume holds.

Finally, the first-order condition with respect to $\mathrm{M}_{\mathrm{i}}$ is:

$$
\frac{\left[1-\mathrm{G}_{\mathrm{i}}\left(\varphi_{\mathrm{i}}^{*}\right)\right]}{\delta}\left[\int_{\varphi_{\mathrm{i}}^{*}}^{\infty} \frac{\mathrm{r}_{\mathrm{i}}\left(\varphi_{\mathrm{i}}\right)}{\sigma_{\mathrm{i}}} \mu_{\mathrm{i}}\left(\varphi_{\mathrm{i}}\right) \mathrm{d} \varphi_{\mathrm{i}}-\mathrm{m}_{\mathrm{i}} \mathrm{f}_{\mathrm{i}}+\int_{\varphi_{\mathrm{ix}}^{*}}^{\infty} \frac{\mathrm{r}_{\mathrm{ix}}\left(\varphi_{\mathrm{i}}\right)}{\sigma_{\mathrm{i}}} \mu_{\mathrm{i}}\left(\varphi_{\mathrm{i}}\right) \mathrm{d} \varphi_{\mathrm{i}}-\mathrm{m}_{\mathrm{i}} \mathrm{f}_{\mathrm{ix}}\right]=\mathrm{w}^{\prime} v_{\mathrm{ie}},
$$

[^5]where the term in brackets is the average profits earned by a successful entrant. This condition states that expected discounted profits equal the fixed costs of entry, which is the free-entry condition in Melitz (2003).

Proposition 1 shows that the optimal choices for quantities and cutoff-productivities by individual firms are identical to that for the social planner, and maximize GDP: there is no distortion on the production-side of the economy. ${ }^{8}$ Properties (a) and (b) further show that the GDP function satisfies the usual properties as in the competitive case, ${ }^{9}$ but estimating this function will be more difficult than in the competitive case for several reasons. First, the shift parameters are themselves endogenous, since they depend on CES prices indexes $P_{i}^{H}$ in (4) and $P_{i}^{F}$, which are endogenous. Second, the shift parameters are not directly observed, so we will need to develop some proxies for them. Third, we still need to determine the appropriate functional form for the GDP function. All of these issues will be addressed in the next section, using a Pareto distribution for firm productivities.

### 2.2 Functional Form for GDP

Following Helpman, Melitz and Yeaple (2004) and Chaney (2005), we assume the Pareto distribution for productivities, defined by: ${ }^{10}$

$$
\begin{equation*}
\mathrm{G}_{\mathrm{i}}\left(\varphi_{\mathrm{i}}\right)=1-\varphi_{\mathrm{i}}^{-\theta_{\mathrm{i}}}, \text { with } \theta_{\mathrm{i}}>\sigma_{\mathrm{i}}-1 \tag{16}
\end{equation*}
$$

The parameter $\theta_{\mathrm{i}}$ is a measure of dispersion of the Pareto distribution, with lower $\theta_{\mathrm{i}}$ having

[^6]more weight in the upper tail. Using this distribution, we calculate export relative to domestic variety in (8) as:
\[

$$
\begin{equation*}
\chi_{\mathrm{i}}=\left(\frac{1-\mathrm{G}\left(\varphi_{\mathrm{ix}}^{*}\right)}{1-\mathrm{G}\left(\varphi_{\mathrm{i}}^{*}\right)}\right)=\left(\frac{\varphi_{\mathrm{i}}^{*}}{\varphi_{\mathrm{ix}}^{*}}\right)^{\theta_{\mathrm{i}}} . \tag{17}
\end{equation*}
$$

\]

Relative export variety can be further simplified by using (3) and (5) to compute $\mathrm{r}_{\mathrm{i}}\left(\varphi_{\mathrm{i}}^{*}\right)$
and $r_{i x}\left(\varphi_{\mathrm{ix}}^{*}\right)$, together with the equilibrium conditions (14) and (15). Then we obtain:

$$
\frac{r_{i x}\left(\varphi_{i x}^{*}\right)}{r_{i}\left(\varphi_{i}^{*}\right)}=\left(\frac{\varphi_{i x}^{*} P_{i}^{F} / \tau_{i}}{\varphi_{i}^{*} P_{i}^{H}}\right)^{\sigma_{i}-1} \frac{\mathrm{E}_{\mathrm{i}}^{\mathrm{F}}}{\mathrm{E}_{\mathrm{i}}^{\mathrm{H}}}=\frac{\mathrm{f}_{\mathrm{ix}}}{\mathrm{f}_{\mathrm{i}}} .
$$

Raising this expression to the power $\left(1 / \sigma_{\mathrm{i}}\right)$, and re-arranging, we have:

$$
\begin{equation*}
\left(\frac{\mathrm{A}_{\mathrm{ix}}}{\mathrm{~A}_{\mathrm{id}}}\right)=\chi_{\mathrm{i}}^{\frac{\sigma_{\mathrm{i}}-1}{\sigma_{\mathrm{i}} \theta_{\mathrm{i}}}}\left(\frac{\mathrm{f}_{\mathrm{ix}}}{\mathrm{f}_{\mathrm{i}}}\right)^{1 / \sigma_{\mathrm{i}}}, \quad 0<\frac{\sigma_{\mathrm{i}}-1}{\sigma_{\mathrm{i}} \theta_{\mathrm{i}}}<1 . \tag{18}
\end{equation*}
$$

Thus, the ratio of the export/domestic shift parameters in demand equals relative export variety raised to a positive power, adjusted for a term involving fixed costs. This means that relative export variety can be used as a proxy for $\left(\mathrm{A}_{\mathrm{ix}} / \mathrm{A}_{\mathrm{id}}\right)$.

It is surprising that relative export variety is positively related to $\left(\mathrm{A}_{\mathrm{ix}} / \mathrm{A}_{\mathrm{id}}\right)$ in (18). We normally expect an increase in variety to lower the CES price index, and therefore lower the shift parameter, which depend on the CES price index. To understand the positive relation in (18) we use the ZCP condition in (15), illustrated in Figure 1 for the export market (a similar diagram holds for the domestic market). The marginal exporter with productivity $\varphi_{\mathrm{ix}}^{*}$ earns zero profits, shown by the tangency between the average costs curve $A C_{1}$ and the demand curve $D_{1}$, at $q_{1}$. Suppose now that the shift parameter $\mathrm{A}_{\mathrm{ix}}$ is higher, due to in increase in the CES export price index or a reduction in transport costs. Then demand shifts up to $\mathrm{D}_{2}$, and the exporter with
productivity $\varphi_{\mathrm{ix}}^{*}$ earns positive profits. A lower-productivity exporter, with average cost curve $A C_{2}$, would earn zero profits at $q_{2}$. So the increase in $A_{i x}$ leads to entry and expands export variety, which explains the positive relation between $\left(\mathrm{A}_{\mathrm{ix}} / \mathrm{A}_{\mathrm{id}}\right)$ and relative export variety.

Furthermore, the increase in $\mathrm{A}_{\mathrm{ix}}$ also leads to higher GDP, from Theorem 1. So increases in relative export variety are associated with rising GDP. One explanation for this result is that increases in $\mathrm{A}_{\mathrm{ix}}$ allow exporters to charge higher prices, so there is a terms of trade gain. A second explanation, stressed by Melitz (2003) and Bernard, Redding and Schott (2005), is that exporters are more productive than domestic firms. So even though an expansion in relative export variety necessary means the addition of less efficient exporters, those firms are still more productive than the marginal domestic firms, who are forced out of the market due to rising factor prices when exports increase. So expansions in relative export variety is associated with more-productive firms on average, and rising GDP.

The Pareto distribution further enables us to aggregate domestic and export sales in each sector. In particular, we shown in the Appendix that the Pareto distribution, along with condition (15), allows us to write export sales relative to domestic sales in each sector as:

$$
\begin{equation*}
\frac{\mathrm{R}_{\mathrm{ix}}}{\mathrm{R}_{\mathrm{id}}}=\chi_{\mathrm{i}}\left(\frac{\mathrm{f}_{\mathrm{ix}}}{\mathrm{f}_{\mathrm{i}}}\right) \tag{19}
\end{equation*}
$$

As one might expect, relative export variety is directly related to relative export sales. Then substituting from (18), the ratio of export to domestic sales becomes:

$$
\begin{equation*}
\frac{\mathrm{R}_{\mathrm{ix}}}{\mathrm{R}_{\mathrm{id}}}=\left(\frac{\mathrm{A}_{\mathrm{ix}}}{\mathrm{~A}_{\mathrm{id}}}\right)^{\frac{\sigma_{\mathrm{i}} \theta_{\mathrm{i}}}{\left(\sigma_{\mathrm{i}}-1\right)}}\left(\frac{\mathrm{f}_{\mathrm{ix}}}{\mathrm{f}_{\mathrm{i}}}\right)^{1-\frac{\theta_{\mathrm{i}}}{\left(\sigma_{\mathrm{i}}-1\right)}} \tag{20}
\end{equation*}
$$

Thus, the sales ratio is a constant-elasticity function of the relative export shift parameters. This implies that: first, the shift parameters $\left(\mathrm{A}_{\mathrm{ix}}, \mathrm{A}_{\mathrm{id}}\right)$ for sector i are weakly separable from all other
variables in the GDP function; and second, the appropriate aggregator for $\left(\mathrm{A}_{\mathrm{ix}}, \mathrm{A}_{\mathrm{id}}\right)$ is a CES function. These results are summarized by:

## Proposition 2

Assume that the distribution of firm productivity in Pareto, as in (16). Then the domestic and export parameters $\left(\mathrm{A}_{\mathrm{id}}, \mathrm{A}_{\mathrm{ix}}\right)$ can be aggregated into a CES function:

$$
\begin{equation*}
P_{i}=\psi_{i}\left(A_{i d}, A_{i x}\right) \equiv\left[A_{i d}^{\frac{\sigma_{i} \theta_{i}}{\left(\sigma_{i}-1\right)}}+A_{i x}^{\frac{\sigma_{i} \theta_{i}}{\left(\sigma_{i}-1\right)}}\left(f_{i x} / f_{i}\right)^{1-\frac{\theta_{i}}{\left(\sigma_{i}-1\right)}}\right]^{\frac{\left(\sigma_{i}-1\right)}{\sigma_{i} \theta_{i}}} \tag{21}
\end{equation*}
$$

It follows that GDP can be written as $\mathrm{R}\left(\mathrm{P}_{1}, \ldots, \mathrm{P}_{\mathrm{N}}, \mathrm{V}\right)$, and if R is differentiable then:

$$
\begin{equation*}
\frac{\partial \ln \mathrm{R}}{\partial \ln \mathrm{P}_{\mathrm{i}}}=\frac{\left(\mathrm{R}_{\mathrm{id}}+\mathrm{R}_{\mathrm{ix}}\right)}{\mathrm{R}} \tag{22}
\end{equation*}
$$

which is the share of sector i in GDP.

The terms $P_{i}=\psi_{i}\left(A_{i d}, A_{i x}\right)$ enter the GDP function like sectoral "prices," and the GDP function $R\left(P_{1}, \ldots, P_{N}, V\right)$ is still homogeneous of degree one in $\left(P_{1}, \ldots, P_{N}\right)$. Because we are aggregating the domestic and export prices, the elasticity of the GDP function with respect to $\mathrm{P}_{\mathrm{i}}$ is the share of sector i in GDP including both domestic and export sales. Part (b) of Proposition 1 still holds, so that $R(P, V)$ is homogeneous of degree one in $V$, with $\partial R / \partial V=w$.

Our finding that the Pareto distribution leads to a CES function between domestic and exported varieties is related to the results in Chaney (2005), who derives a gravity equation for country exports to different destination markets. For the case of a single export market, we can obtain his results by substituting for $\mathrm{A}_{\mathrm{id}}$ and $\mathrm{A}_{\mathrm{ix}}$ from (3') and (5') into (20), obtaining:

$$
\begin{equation*}
\frac{R_{i x}}{R_{i d}}=\chi_{i}\left(\frac{f_{i x}}{f_{i}}\right)=\left(\frac{P_{i}^{F} / \tau_{i}}{P_{i}^{H}}\right)^{\theta_{i}}\left(\frac{E_{i}^{F}}{E_{i}^{H}}\right)^{\frac{\theta_{i}}{\left(\sigma_{i}-1\right)}}\left(\frac{f_{i x}}{f_{i}}\right)^{1-\frac{\theta_{i}}{\left(\sigma_{i}-1\right)}} \tag{23}
\end{equation*}
$$

Thus, the export/domestic share is negatively related to the transport costs, with an exponent of $-\theta_{\mathrm{i}}$ in (23), just as in the gravity equation derived by Chaney. The fixed costs in (23) also have the same exponent as found by Chaney (2005).

## 3. Empirical Specification

### 3.1 Translog GDP Function

Following Harrigan (1997), we assume a translog functional form for GDP across sectors while using the CES function in (21) within each sector. Introducing a country superscript c and the time subscript $t$, let $P_{i t}^{c}=\psi_{i}\left(A_{i d t}^{c}, A_{i x t}^{c}\right)$ denote the value of that CES function, which are the sectoral "prices." Define the vector $\mathrm{P}_{\mathrm{t}}^{\mathrm{c}}=\left(\mathrm{P}_{1 \mathrm{t}}^{\mathrm{c}}, \ldots, \mathrm{P}_{\mathrm{N}+1, \mathrm{t}}^{\mathrm{c}}\right)$ to also include a price for non-traded sector $N+1$. Denoting the factor endowments by the vector $V_{t}^{c}=\left(v_{1 t}^{c}, \ldots, v_{K t}^{c}\right)$, the translog GDP function is:

$$
\begin{align*}
\ln \mathrm{R}_{\mathrm{t}}^{\mathrm{c}}\left(\mathrm{P}_{\mathrm{t}}^{\mathrm{c}}, \mathrm{~V}_{\mathrm{t}}^{\mathrm{c}}\right)= & \alpha_{0}^{\mathrm{c}}+\beta_{0 \mathrm{t}}+\sum_{\mathrm{i}=1}^{\mathrm{N}+1} \alpha_{\mathrm{i}} \ln \mathrm{P}_{\mathrm{it}}^{\mathrm{c}}+\sum_{\mathrm{k}=1}^{\mathrm{K}} \beta_{\mathrm{k}} \ln \mathrm{v}_{\mathrm{kt}}^{\mathrm{c}}+\frac{1}{2} \sum_{\mathrm{i}=1}^{\mathrm{N}+\ln +1} \sum_{\mathrm{j}=1}^{\mathrm{N}+1} \gamma_{\mathrm{ij}} \ln \mathrm{P}_{\mathrm{it}}^{\mathrm{c}} \ln \mathrm{P}_{\mathrm{jt}}^{\mathrm{c}}  \tag{24}\\
& +\frac{1}{2} \sum_{\mathrm{k}=1}^{\mathrm{K}} \sum_{\ell=1}^{\mathrm{K}} \delta_{\mathrm{k} \ell} \ln \mathrm{v}_{\mathrm{kt}}^{\mathrm{c}} \ln \mathrm{v}_{\ell \mathrm{t}}^{\mathrm{c}}+\sum_{\mathrm{i}=1}^{\mathrm{N}+1} \sum_{\mathrm{k}=1}^{\mathrm{K}} \phi_{\mathrm{ik}} \ln \mathrm{P}_{\mathrm{it}}^{\mathrm{c}} \ln \mathrm{v}_{\mathrm{kt}}^{\mathrm{c}} .
\end{align*}
$$

We allow this function to differ across countries based on the fixed-effects $\alpha_{0}^{\mathrm{c}}$, which reflect exogenous technology differences, and also allow for the year fixed-effects $\beta_{0 t}$, which are equal across countries. In treating all other parameters of the translog function as common across both countries and time, we are assuming that the distribution function $G_{i}\left(\varphi_{i}\right)$ and the fixed costs $f_{i}$ and $f_{i x}$ do not vary over these dimensions.

To satisfy homogeneity of degree one in prices and endowments, we test the restrictions:

$$
\begin{equation*}
\gamma_{\mathrm{ij}}=\gamma_{\mathrm{ji}}, \delta_{\mathrm{k} \ell}=\delta_{\ell \mathrm{k}}, \sum_{\mathrm{i}=1}^{\mathrm{N}+1} \alpha_{\mathrm{n}}=\sum_{\mathrm{k}=1}^{\mathrm{K}} \beta_{\mathrm{k}}=1, \sum_{\mathrm{i}=1}^{\mathrm{N}+1} \gamma_{\mathrm{ij}}=\sum_{\mathrm{i}=1}^{\mathrm{N}+1} \phi_{\mathrm{ik}}=\sum_{\mathrm{k}=1}^{\mathrm{K}} \delta_{\mathrm{k} \ell}=\sum_{\mathrm{k}=1}^{\mathrm{K}} \phi_{\mathrm{ik}}=0 . \tag{25}
\end{equation*}
$$

As shown in Proposition 2, the share of sector $i$ in GDP equals the derivative of $\ln R_{t}^{c}\left(P_{t}^{c}, V_{t}^{c}\right)$ with respect to $\ln P_{i t}^{c}$ :

$$
\begin{equation*}
\mathrm{s}_{\mathrm{it}}^{\mathrm{c}}=\alpha_{\mathrm{i}}+\sum_{\mathrm{j}=1}^{\mathrm{N}+1} \gamma_{\mathrm{ij}} \ln \mathrm{P}_{\mathrm{jt}}^{\mathrm{c}}+\sum_{\mathrm{k}=1}^{\mathrm{K}} \phi_{\mathrm{ik}} \ln \mathrm{v}_{\mathrm{kt}}^{\mathrm{c}}, \quad \mathrm{i}=1, \ldots, \mathrm{~N}+1 . \tag{26}
\end{equation*}
$$

Also, the share of factor $k$ in GDP equals the derivative of $\ln R_{t}^{c}\left(P_{t}^{c}, V_{t}^{c}\right)$ with respect to $\ln v_{k t}^{c}$ :

$$
\begin{equation*}
\mathrm{s}_{\mathrm{kt}}^{\mathrm{c}}=\beta_{\mathrm{k}}+\sum_{\ell=1}^{\mathrm{K}} \delta_{\mathrm{k} \ell} \ln \mathrm{v}_{\ell \mathrm{t}}^{\mathrm{c}}+\sum_{\mathrm{i}=1}^{\mathrm{N}+1} \phi_{\mathrm{kn}} \ln \mathrm{P}_{\mathrm{it}}^{\mathrm{c}}, \mathrm{k}=1, \ldots, \mathrm{~K} . \tag{27}
\end{equation*}
$$

### 3.2 CES Sectoral Aggregates

Key to the empirical work will be to measure the CES aggregates $P_{i t}^{c}=\psi_{i}\left(A_{i d t}^{c}, A_{i x t}^{c}\right)$ in each sector. To this end, we will difference the GDP and share equations with respect to a comparison country denoted by "a". The CES aggregate in each sector will also be differenced with respect to country " $a$ " in log form, which means we take the $\log$ of the ratio $P_{i t}^{c} / P_{i t}^{a}$.

To evaluate the ratio of CES functions, we can apply the index number formula due to Sato (1976) and Vartia (1976). ${ }^{11}$ Under our assumption that the fixed costs $\left(\mathrm{f}_{\mathrm{ix}} / \mathrm{f}_{\mathrm{i}}\right)$ appearing in (21) are the same across countries, the CES ratio in sector $i$ equals:

$$
\begin{equation*}
\frac{\Psi_{i}\left(A_{i d t}^{c}, A_{i x t}^{c}\right)}{\Psi_{i}\left(A_{i d t}^{a}, A_{i x t}^{a}\right)}=\left(\frac{A_{i d t}^{c}}{A_{i d t}^{a}}\right)^{1-W_{i t}^{c}}\left(\frac{A_{i x t}^{c}}{A_{i x t}^{a}}\right)^{W_{i t}^{c}}=\left(\frac{A_{i d t}^{c}}{A_{i d t}^{a}}\right)\left(\frac{A_{i x t}^{c} / A_{i d t}^{c}}{A_{i x t}^{\mathrm{a}} / A_{i d t}^{a}}\right)^{W_{i t}^{c}}=\frac{A_{i d t}^{c}}{A_{i d t}^{a}}\left(\frac{\chi_{i t}^{c}}{\chi_{i t}^{\mathrm{a}}}\right)^{\frac{\left(\sigma_{i}-1\right)}{\sigma_{i} \theta_{i}}} W_{i t}^{c} \tag{28}
\end{equation*}
$$

[^7]where $W_{i t}^{c}$ is the logarithmic mean of the export shares in countries a and c. ${ }^{12}$ The first equality in (28) follows directly from the Sato-Vartia formula, which allows us to evaluate the ratio of CES functions without knowledge of the fixed costs $\left(\mathrm{f}_{\mathrm{ix}} / \mathrm{f}_{\mathrm{i}}\right)$, but using the data on export shares instead. The second equality follows by algebra; and the third equality follows by using (18) to replace the export/domestic shift parameters with relative export variety.

To further simplify (28), we substitute the definition of the domestic shift parameters $A_{i d t}^{c}$ from (3'), to obtain:

$$
\begin{equation*}
\left(\frac{\mathrm{P}_{\mathrm{it}}^{\mathrm{c}}}{\mathrm{P}_{\mathrm{it}}^{\mathrm{a}}}\right)=\frac{\psi_{\mathrm{i}}\left(\mathrm{~A}_{\mathrm{idt}}^{\mathrm{c}}, \mathrm{~A}_{\mathrm{ixt}}^{\mathrm{c}}\right)}{\psi_{\mathrm{i}}\left(\mathrm{~A}_{\mathrm{idt}}^{\mathrm{a}}, \mathrm{~A}_{\mathrm{ixt}}^{\mathrm{a}}\right)}=\left(\frac{\mathrm{P}_{\mathrm{it}}^{\mathrm{Hc}}}{\mathrm{P}_{\mathrm{it}}^{\mathrm{Ha}}}\right)\left(\frac{\mathrm{E}_{\mathrm{it}}^{\mathrm{Hc}} / \mathrm{P}_{\mathrm{it}}^{\mathrm{Hc}}}{\mathrm{E}_{\mathrm{it}}^{\mathrm{Ha}} / \mathrm{P}_{\mathrm{it}}^{\mathrm{Ha}}}\right)^{\frac{1}{\sigma_{\mathrm{i}}}}\left(\frac{\chi_{\mathrm{it}}^{\mathrm{c}}}{\chi_{\mathrm{it}}^{\mathrm{a}}}\right)^{\frac{\left(\sigma_{\mathrm{i}}-1\right)}{\sigma_{i} \theta_{\mathrm{i}}} \mathrm{~W}_{\mathrm{it}}^{\mathrm{c}}} . \tag{28'}
\end{equation*}
$$

This equation shows that the ratio of sectoral "prices" depend on three terms: the first term on the right is the ratio of the domestic CES price indexes in each sector; the second term is real expenditure; and the third term is the ratio of relative export variety in countries c and a . Each of these is difficult to measure and so we rely on some assumptions.

First, we have no data to measure the sectoral CES price indexes $\mathrm{P}_{\mathrm{it}}^{\mathrm{Hc}}$ shown in (4), and will assume that they reflect country-level prices $\mathrm{P}_{\mathrm{t}}^{\mathrm{Hc}}$ plus a sectoral error term:

$$
\begin{equation*}
\ln \left(\frac{P_{i t}^{\mathrm{Hc}}}{\mathrm{P}_{\mathrm{it}}^{\mathrm{Ha}}}\right)=\ln \left(\frac{\mathrm{P}_{\mathrm{t}}^{\mathrm{Hc}}}{\mathrm{P}_{\mathrm{t}}^{\mathrm{Ha}}}\right)+\mathrm{u}_{1 \mathrm{it}}^{\mathrm{c}} . \tag{29}
\end{equation*}
$$

Second, we will suppose that higher real expenditure in a sector leads more firms to enter, as is shown by Melitz (2003, p. 1712). ${ }^{13}$ So domestic variety in each sector is assumed to equal:

[^8]13 Actually, Melitz (2003, p. 1712) shows that the mass of entering firms is related to industry revenue $R_{i}$.

$$
\begin{equation*}
\ln \left(\frac{\mathrm{M}_{\mathrm{it}}^{\mathrm{c}}}{\mathrm{M}_{\mathrm{it}}^{\mathrm{a}}}\right)=\ln \left(\frac{\mathrm{E}_{\mathrm{it}}^{\mathrm{Hc}} / \mathrm{P}_{\mathrm{it}}^{\mathrm{Hc}}}{\mathrm{E}_{\mathrm{it}}^{\mathrm{Ha}} / \mathrm{P}_{\mathrm{it}}^{\mathrm{Ha}}}\right)+\mathrm{u}_{2 \mathrm{it}}^{\mathrm{c}} . \tag{30}
\end{equation*}
$$

Substituting (29) and (30) into (28'), we obtain the sectoral "prices":

$$
\begin{equation*}
\ln \left(\frac{\mathrm{P}_{\mathrm{it}}^{\mathrm{c}}}{\mathrm{P}_{\mathrm{it}}^{\mathrm{a}}}\right)=\ln \left(\frac{\mathrm{P}_{\mathrm{t}}^{\mathrm{Hc}}}{\mathrm{P}_{\mathrm{t}}^{\mathrm{Ha}}}\right)+\frac{1}{\sigma_{i}} \ln \left(\frac{\mathrm{M}_{\mathrm{it}}^{\mathrm{c}}}{\mathrm{M}_{\mathrm{it}}^{\mathrm{a}}}\right)+\frac{\left(\sigma_{\mathrm{i}}-1\right)}{\sigma_{i} \theta_{\mathrm{i}}} W_{i t}^{\mathrm{c}} \ln \left(\frac{\chi_{\mathrm{it}}^{\mathrm{c}}}{\chi_{\mathrm{it}}^{\mathrm{a}}}\right)+\left(u_{1 i t}^{\mathrm{c}}+u_{2 i t}^{\mathrm{c}}\right) . \tag{31}
\end{equation*}
$$

As discussed in the next section, we will be measuring total export variety in country c relative to country a, which is $\left(\mathrm{M}_{\mathrm{it}}^{\mathrm{c}} \chi_{\mathrm{it}}^{\mathrm{c}} / \mathrm{M}_{\mathrm{it}}^{\mathrm{c}} \chi_{\mathrm{it}}^{\mathrm{a}}\right)$. We have no way of separating this total into the ratio of export relative to domestic variety in the two countries, which is $\left(\chi_{\mathrm{it}}^{\mathrm{c}} / \chi_{\mathrm{it}}^{\mathrm{a}}\right)$, and the ratio of domestic varieties, which is $\left(M_{i t}^{c} / M_{i t}^{a}\right)$. So when using (31) in the estimation, we will be combining these two terms into total export variety:

$$
\begin{equation*}
\ln \left(\frac{P_{i t}^{c}}{P_{i t}^{\mathrm{a}}}\right)=\ln \left(\frac{P_{t}^{\mathrm{Hc}}}{P_{t}^{\mathrm{Ha}}}\right)+\rho_{i} W_{i t}^{c} \ln \left(\frac{M_{i t}^{c} \chi_{i t}^{c}}{M_{i t}^{\mathrm{a}} \chi_{i t}^{\mathrm{a}}}\right)+\left(u_{1 i t}^{c}+u_{2 i t}^{c}+u_{3 i t}^{c}\right), \tag{32}
\end{equation*}
$$

where $u_{3 i t}^{c}$ is the error term introduced by using total export variety. Because we are combining the two terms $\left(\chi_{i t}^{c} / \chi_{i t}^{\mathrm{a}}\right)$ and $\left(\mathrm{M}_{\mathrm{it}}^{\mathrm{c}} / \mathrm{M}_{\mathrm{it}}^{\mathrm{a}}\right)$, we might expect $\rho_{\mathrm{i}}$ to lie in-between the coefficients on these terms, so that case $\rho_{\mathrm{i}}$ lies within the range $\left[\left(\sigma_{\mathrm{i}}-1\right) / \theta_{\mathrm{i}} \sigma_{\mathrm{i}}, 1 / \sigma_{\mathrm{i}}\right] .{ }^{14}$

Differencing the share equation in (26) with respect to country "a", and substituting for the sectoral "prices" from (32), we obtain:

$$
\begin{equation*}
s_{i t}^{c}=s_{i t}^{a}+\sum_{j=1}^{N} \rho_{i} \gamma_{i j} W_{i t} \ln \left(\frac{M_{j \mathrm{j}}^{\mathrm{c}} \chi_{\mathrm{jt}}^{\mathrm{c}}}{M_{j \mathrm{jt}}^{\mathrm{a}} \chi_{\mathrm{jt}}^{\mathrm{a}}}\right)+\gamma_{i \mathrm{~N}+1} \ln \left(\frac{P_{\mathrm{N}+1 \mathrm{t}}^{\mathrm{c}} / P_{\mathrm{t}}^{\mathrm{Hc}}}{P_{\mathrm{N}+1 \mathrm{t}}^{\mathrm{a}} / P_{\mathrm{t}}^{\mathrm{Ha}}}\right)+\sum_{\mathrm{k}=1}^{\mathrm{K}} \phi_{\mathrm{ik}} \ln \left(\frac{v_{\mathrm{kt}}^{\mathrm{c}}}{v_{\mathrm{kt}}^{\mathrm{a}}}\right)+\varepsilon_{\mathrm{it}}^{\mathrm{c}}, \tag{33}
\end{equation*}
$$

[^9]where the error term $\varepsilon_{i t}^{c}$ in (33) consists of the errors $\left(u_{1 i t}^{c}+u_{2 i t}^{c}+u_{3 i t}^{c}\right)$ summed across sectors. Notice that the country-level prices $P_{t}^{\mathrm{Hc}}$ deflate the non-traded prices in (33), but do not appear otherwise by using $\sum_{\mathrm{j}=1}^{\mathrm{N}+1} \gamma_{\mathrm{ij}}=0$.

One problem with the share equations (33) is that the parameters $\rho_{\mathrm{i}}$ cannot be separately identified from the translog parameters $\gamma_{\mathrm{ij}}$. To overcome this, we estimate the share equations jointly with a country-level productivity equation, obtained by differencing the GDP equation (24) with respect to country "a":

$$
\begin{equation*}
\ln \left(\frac{R_{t}^{c}\left(P_{t}^{c}, V_{t}^{c}\right)}{R_{t}^{a}\left(P_{t}^{a}, V_{t}^{a}\right)}\right)=\alpha_{0}^{c}+\beta_{0 t}+\sum_{i=1}^{N+1} \frac{1}{2}\left(s_{i t}^{c}+s_{i t}^{a}\right) \ln \left(\frac{P_{i t}^{c}}{P_{i t}^{a}}\right)+\sum_{k=1}^{M} \frac{1}{2}\left(s_{k t}^{c}+s_{k t}^{a}\right) \ln \left(\frac{v_{k t}^{c}}{v_{k t}^{a}}\right) . \tag{34}
\end{equation*}
$$

The right-hand side of (34) equals fixed effects, plus a share-weighted index of relative prices, plus a share-weighted index of relative endowments. These terms provide a decomposition of relative GDP into its price and factor-endowment components. ${ }^{15}$

We can simplify (34) by using the price indexes (32), and moving the factor endowments and non-traded prices to the left:

$$
\begin{align*}
\mathrm{TFP}_{\mathrm{t}}^{\mathrm{c}} & \equiv \ln \left(\frac{\mathrm{RGDP}_{\mathrm{t}}^{\mathrm{c}}}{\operatorname{RGDP}_{\mathrm{t}}^{\mathrm{a}}}\right)-\sum_{\mathrm{k}=1}^{\mathrm{K}} \frac{1}{2}\left(\mathrm{~s}_{\mathrm{kt}}^{\mathrm{c}}+\mathrm{s}_{\mathrm{kt}}^{\mathrm{a}}\right) \ln \left(\frac{\mathrm{v}_{\mathrm{kt}}^{\mathrm{c}}}{v_{\mathrm{kt}}^{\mathrm{a}}}\right)-\frac{1}{2}\left(\mathrm{~s}_{\mathrm{N}+1 \mathrm{t}}^{\mathrm{c}}+\mathrm{s}_{\mathrm{N}+1 \mathrm{t}}^{\mathrm{a}}\right) \ln \left(\frac{\mathrm{P}_{\mathrm{N}+1 \mathrm{t}}^{\mathrm{c}} / \mathrm{P}_{\mathrm{t}}^{\mathrm{Hc}}}{\mathrm{P}_{\mathrm{N}+1 \mathrm{t}}^{\mathrm{a}} / \mathrm{P}_{\mathrm{t}}^{\mathrm{Ha}}}\right) \\
& =\alpha_{0}^{\mathrm{c}}+\beta_{0 t}+\sum_{\mathrm{i}=1}^{\mathrm{N}} \frac{1}{2}\left(s_{\mathrm{it}}^{\mathrm{c}}+\mathrm{s}_{\mathrm{it}}^{\mathrm{a}}\right) \rho_{\mathrm{i}} \mathrm{~W}_{\mathrm{it}} \ln \left(\frac{\mathrm{M}_{\mathrm{it}}^{\mathrm{c}} \chi_{i t}^{\mathrm{c}}}{\mathrm{M}_{\mathrm{it}}^{\mathrm{a}} \chi_{\mathrm{it}}^{\mathrm{a}}}\right)+\varepsilon_{\mathrm{t}}^{\mathrm{c}} . \tag{35}
\end{align*}
$$

where real GDP is $\operatorname{RGDP}_{t}^{c} \equiv \mathrm{R}_{\mathrm{t}}^{\mathrm{c}}\left(\mathrm{P}_{\mathrm{t}}^{\mathrm{c}}, \mathrm{V}_{\mathrm{t}}^{\mathrm{c}}\right) / \mathrm{P}_{\mathrm{t}}^{\mathrm{Hc}}$. The left-hand side of (35) is interpreted as total factor productivity (TFP) differences between country c and country a, with an adjustment for

15 The decomposition in (34) is a special case of results in Diewert and Morrison (1986), which are summarized by Feenstra (2004, Appendix A, Theorem 5). We assume that $\alpha_{0}^{a}=\beta_{0 t}=0$ for country " $a$ ".
non-traded good prices. These TFP differences across countries are explained by differences in export variety on the right, plus an error term obtained from $\left(u_{1 i t}^{c}+u_{2 i t}^{c}+u_{3 i t}^{c}\right)$.

### 3.3 Measuring Export Variety

The measure of export variety we use is derived from a CES utility function by Feenstra (1994), and has been employed recently by Broda and Weinstein (2005) and by Hummels and Klenow (2005) who call it the "extensive margin." Rather than indexing prices by the continuous productivity $\varphi_{i}$ in sector i , we will instead indexes prices by the discrete variable $\mathrm{j} \in \mathrm{J}_{\mathrm{it}}^{\mathrm{c}}$. So $p_{i t}^{c}(j)$ is the export price for variety $j$ in sector $i$, year $t$ and country $c$, with quantity $q_{i t}^{c}(j)$.

Suppose that the set of exports from countries a and c differ, but have some product varieties in common. Denote this common set by $J \equiv\left(J_{i t}^{a} \cap J_{i t}^{c}\right) \neq \varnothing$. From Feenstra (1994), an inverse measure of export variety from country c is:

$$
\begin{equation*}
\lambda_{i t}^{c}(\mathrm{~J}) \equiv \frac{\sum_{\mathrm{j} \in \mathrm{~J}} \mathrm{p}_{\mathrm{it}}^{\mathrm{c}}(\mathrm{j}) q_{i t}^{\mathrm{c}}(\mathrm{j})}{\sum_{\mathrm{j} \in \mathrm{~J}_{\mathrm{it}}^{\mathrm{c}}} \mathrm{p}_{\mathrm{it}}^{\mathrm{c}}(\mathrm{j}) q_{\mathrm{it}}^{\mathrm{c}}(\mathrm{j})} \tag{36}
\end{equation*}
$$

Notice that $\lambda_{\mathrm{it}}^{\mathrm{c}}(\mathrm{J}) \leq 1$ in (36) due to the differing summations in the numerator and denominator. This term will be strictly less than one if there are goods in the set $\mathrm{J}_{\text {it }}^{\mathrm{c}}$ that are not found in the common set J . In other words, if country c is selling some goods in period t that are not sold by country a, this will make $\lambda_{\mathrm{it}}^{\mathrm{c}}(\mathrm{J})<1$, so it is an inverse measure of country c export variety.

The ratio $\left[\lambda_{\mathrm{it}}^{\mathrm{c}}(\mathrm{J}) / \lambda_{\mathrm{it}}^{\mathrm{a}}(\mathrm{J})\right]$ is an inverse measure of export variety from country c relative to country a. Taking the reciprocal, we use $\left[\lambda_{i t}^{\mathrm{a}}(\mathrm{J}) / \lambda_{\mathrm{it}}^{\mathrm{c}}(\mathrm{J})\right]$ in place of $\left(\mathrm{M}_{\mathrm{it}}^{\mathrm{c}} \chi_{\mathrm{it}}^{\mathrm{c}} / \mathrm{M}_{\mathrm{it}}^{\mathrm{a}} \chi_{\mathrm{it}}^{\mathrm{a}}\right)$ in equations (33) and (35). We shall measure the ratio $\left[\lambda_{i t}^{\mathrm{a}}(\mathrm{J}) / \lambda_{i t}^{\mathrm{c}}(\mathrm{J})\right]$ using exports of countries to
the United States. While it would be preferable to use their worldwide exports, our data for the U.S. are more disaggregate and allows for a finer measurement of "unique" products sold by one country and not another. Specifically, for 1980-1988 we will use the 7-digit Tariff Schedule of the U.S. Annotated (TSUSA) classification of U.S. imports, and for 1989 - 2000 we shall use the 10-digit Harmonized System (HS) classification.

To measure the ratio $\left[\lambda_{i t}^{a}(J) / \lambda_{i t}^{c}(J)\right]$, we need a consistent comparison country "a". For this purpose, we shall use the worldwide exports from all countries to the U.S. as the comparison. Furthermore, we take the union of all products sold in any year, and we average export sales of each product over years. Denote this comparison country by " $a$ ", so that the set $J_{i}^{a}=\bigcup_{c, t} J_{i t}^{c}$ is the total set of varieties imported by the U.S. in sector i over all years, and $p_{i}^{a}(j) q_{i}^{a}(j)$ is the average value of imports for product j (summed over all source countries and averaged across years). Then comparing country c to country a , it is immediate that the common set of goods exported is $J \equiv J_{i t}^{c} \cap J_{i}^{a}=J_{i t}^{c}$, or simply the set of goods exported by country c. Therefore, from (36) we have that $\lambda_{\mathrm{it}}^{\mathrm{c}}\left(\mathrm{J}_{\mathrm{it}}^{\mathrm{c}}\right)=1$, and export variety by country c is measured by:

$$
\begin{equation*}
\Lambda_{i t}^{c} \equiv \frac{\lambda_{i t}^{a}\left(J_{i t}^{c}\right)}{\lambda_{i t}^{c}\left(J_{i t}^{c}\right)}=\frac{\sum_{j \in J_{i t}^{c}} p_{i}^{a}(j) q_{i}^{a}(j)}{\sum_{j \in J_{i}^{a}} p_{i}^{a}(j) q_{i}^{a}(j)} \tag{37}
\end{equation*}
$$

Notice that the measure of export variety in (37) changes over time or across country only due to changes in the set of goods sold by that country, $\mathrm{J}_{\text {it }}^{\mathrm{c}}$, which appears in the numerator on the right. The denominator is constant across countries and time. Therefore, (37) is a measure of product variety of exports that is consistent across countries and over time. Broda and Weinstein (2005) and Hummels and Klenow (2005) each use a similar formula to (36) or (37), but with
different "comparison cases": Broda and Weinstein focus on the time-series growth in import varieties in the U.S., so the comparison is import variety in a base year; whereas Hummels and Klenow focus on cross-sectional variety in a given year, so the comparison is worldwide variety in that year. Each of these formulations are appropriate for the question being asked, and by taking the union of all imported products in the U.S. over years and source countries, we obtain a consistent comparison across both dimensions. ${ }^{16}$

Summary statistics for the measure of export variety in (37) are provided in Table 1. There is a strong correlations with real GDP in the exporting countries, shown in the third row. In the next rows we show export variety in each sector for 1980, 1988, 1989 and 2000. There is a discrete fall in export variety from 1988 to 1989, due to the changing classification of U.S. import statistics from the TSUSA to the HS classification. We will account for that discrete fall by including year fixed-effects in all our estimating equations. Taking the growth rate of export variety over 1980-1988 and 1989-2000, the average growth is about $10 \%$ per year, which means that export variety doubles every seven years and increases eight-fold over the two decades. That eight-fold average increase is shown in the final row, and is lower in sectors like agriculture, petroleum and plastics, and mining and metals, but much higher in textiles and garments and the electronics industry.

### 3.4 Other Data

Our data set covers 44 countries from 1980 to 2000, a total of 509 observations. The GDP and endowment data are obtained from World Development Indicators (World Bank, 2005). Real GDP is measured in constant 2000 U.S. dollars (converted at nominal exchange

[^10]rates that year), so we are using GDP deflators to measure $P_{t}^{H c}$ and $\operatorname{RGDP}_{t}^{c} \equiv R_{t}^{c}\left(P_{t}^{c}, V_{t}^{c}\right) / P_{t}^{H c}$. There are three primary factor endowments: labor, capital and agriculture land. Labor is defined as the number of persons in the labor force of each country. Capital is constructed from real investment using the perpetual inventory method. ${ }^{17}$ Endowments of the comparison country "a" are measured by the sum of endowments for all sample countries, $v_{k t}^{a}=\sum_{c=1}^{C} v_{k t}^{c}$. We aggregate goods into $\mathrm{N}=7$ sectors, as shown in Tables 1 and 2. The $8^{\text {th }}$ sector is the nontraded good, with price $P_{8 t}^{c} \equiv\left(P_{N+1 t}^{c} / P_{t}^{H c}\right)$ obtained by netting the prices of traded goods, both export and import, from the country GDP deflators. This procedure may introduce some error into the nontraded price, which we address by using instrumental variables. ${ }^{18}$ The value added of these sectors are available in a UNIDO data set, used to construct the value added share of each sector, $\mathrm{s}_{\mathrm{it}}^{\mathrm{c}}$.

In Table 2 we show the sectoral shares for each traded sector, which jointly account for $20 \%$ of GDP on average. Instruments used to address the as endogeneity of export variety, as well as measurement error in the nontraded prices, consist of U.S. tariffs with each partner country, free trade agreements and distance to the U.S. The U.S. tariffs vary by sector, countries and years, and are summarized in Table 2. The textiles and garments sector has the highest tariffs, and a correlation of -0.16 with export variety. In the final rows of Table 2 we also show the drop in tariffs from 1980-2000, which are modest in size: -5.7 percentage points in electronics, and less than that in all other sectors. The small cuts in U.S. tariffs means that this variable will not be able to account for the very large growth in export variety.

[^11]
### 3.5 Estimating Equations

Substituting for export variety $\Lambda_{\mathrm{it}}^{\mathrm{c}}$ and the relative nontraded price $\left(\mathrm{P}_{8 \mathrm{t}}^{\mathrm{c}} / \mathrm{P}_{\mathrm{t}}^{\mathrm{Hc}}\right)$ in (33), the share equations become:

$$
\begin{equation*}
s_{\text {it }}^{\mathrm{c}}=\mathrm{s}_{\mathrm{it}}^{\mathrm{a}}+\sum_{\mathrm{j}=1}^{7} \gamma_{\mathrm{ij}} \rho_{\mathrm{i}} \mathrm{~W}_{\mathrm{it}} \ln \left(\Lambda_{\mathrm{jt}}^{\mathrm{c}}\right)+\gamma_{\mathrm{i} 8} \ln \left(\frac{\mathrm{P}_{8 \mathrm{t}}^{\mathrm{c}} / \mathrm{P}_{\mathrm{t}}^{\mathrm{Hc}}}{\mathrm{P}_{8 \mathrm{t}}^{\mathrm{a}} / \mathrm{P}_{\mathrm{t}}^{\mathrm{Ha}}}\right)+\sum_{\mathrm{k}=1}^{3} \phi_{\mathrm{ik}} \ln \left(\frac{\mathrm{v}_{\mathrm{kt}}^{\mathrm{c}}}{\mathrm{v}_{\mathrm{kt}}^{\mathrm{a}}}\right)+\varepsilon_{\mathrm{it}}^{\mathrm{c}} . \tag{38}
\end{equation*}
$$

We allow for year fixed effects when estimating (38). Homogeneity of degree zero in prices and endowments is tested by $\sum_{\mathrm{j}=1}^{8} \gamma_{\mathrm{ij}}=0$ and $\sum_{\mathrm{k}=1}^{3} \phi_{\mathrm{ik}}=0$, respectively.

Testing homogeneity of the TFP equation is slightly more complicated, because with the shares of sectors and factors summing to unity in (35), it is automatically homogeneous of degree one in both. To test that TFP is homogeneous of degree one in prices, we rewrite the non-traded share in (35) as $-\frac{1}{2}\left(s_{N+1 t}^{c}+s_{N+1 t}^{a}\right)=\frac{1}{2} \sum_{i=1}^{7}\left(s_{i t}^{c}+s_{i t}^{a}\right)-1$. Keep $\frac{1}{2} \sum_{i=1}^{7}\left(s_{i t}^{c}+s_{i t}^{a}\right) \ln \left(\frac{P_{8 t}^{c} / P_{t}^{\text {Hc }}}{P_{8 t}^{a} / P_{t}^{\text {Ha }}}\right)$ on the left, but move $\ln \left(\frac{\mathrm{P}_{8 t}^{\mathrm{c}} / \mathrm{P}_{t}^{\mathrm{Hc}}}{\mathrm{P}_{8 \mathrm{t}}^{\mathrm{a}} / \mathrm{P}_{t}^{\mathrm{Ha}}}\right)$ to the right of (35), while introducing the coefficient $\eta_{1}$ this term.

Then $\eta_{1}=1$ tests that the TFP equation is homogeneous of degree one in prices.
We use a similar approach to test that TFP is homogeneous of degree one in endowments. In this case we do not have we do not have the separate capital and land shares, though we do have the labor share of GDP. ${ }^{19}$ So letting $\eta_{2}$ denote the average share of land, the weighted endowments appearing on the left of (35) can be written as:

$$
-\sum_{\mathrm{k}=1}^{\mathrm{K}} \frac{1}{2}\left(\mathrm{~s}_{\mathrm{kt}}^{\mathrm{c}}+\mathrm{s}_{\mathrm{kt}}^{\mathrm{a}}\right) \ln \left(\frac{\mathrm{v}_{\mathrm{kt}}^{\mathrm{c}}}{\mathrm{v}_{\mathrm{kt}}^{\mathrm{a}}}\right)=-\frac{1}{2}\left(\mathrm{~s}_{\mathrm{Lt}}^{\mathrm{c}}+\mathrm{s}_{\mathrm{Lt}}^{\mathrm{a}}\right) \ln \left(\frac{\ell_{\mathrm{t}}^{\mathrm{c}}}{\ell_{\mathrm{t}}^{\mathrm{a}}}\right)-\left[1-\frac{1}{2}\left(\mathrm{~s}_{\mathrm{Lt}}^{\mathrm{c}}+\mathrm{s}_{\mathrm{Lt}}^{\mathrm{a}}\right)-\eta_{2}\right] \ln \left(\frac{\mathrm{k}_{\mathrm{t}}^{\mathrm{c}}}{\mathrm{k}_{\mathrm{t}}^{\mathrm{a}}}\right)-\ln \left(\frac{\mathrm{T}_{\mathrm{t}}^{\mathrm{c}}}{T_{\mathrm{t}}^{\mathrm{a}}}\right),
$$

where we measure the labor/land and capital/land endowments as $\ln \ell_{\mathrm{t}}^{\mathrm{c}} \equiv \ln \left(\mathrm{L}_{\mathrm{t}}^{\mathrm{c}} / \mathrm{T}_{\mathrm{t}}^{\mathrm{c}}\right)$ and

[^12]$\ln \mathrm{k}_{\mathrm{t}}^{\mathrm{c}} \equiv \ln \left(\mathrm{K}_{\mathrm{t}}^{\mathrm{c}} / \mathrm{T}_{\mathrm{t}}^{\mathrm{c}}\right)$, respectively. Those two terms continue to appear on the left of (35), but we move $-\eta_{2} \ln \left(k_{t}^{c} / k_{t}^{a}\right)$ and $\ln \left(T_{t}^{c} / T_{t}^{a}\right)$ to the right. We introduce the coefficient $\eta_{3}$ on the latter term, where $\eta_{3}=1$ tests that the TFP equation is homogeneous of degree one in endowments.

To implement these homogeneity tests we define "adjusted" TFP as:

Adj. $\operatorname{TFP}_{t}^{c} \equiv \ln \left(\frac{\operatorname{RGDP}_{t}^{c}}{\operatorname{RGDP}_{t}^{a}}\right)-\frac{1}{2}\left(s_{L t}^{c}+s_{L t}^{a}\right) \ln \left(\frac{\ell_{t}^{c}}{\ell_{t}^{\mathrm{a}}}\right)-\left[1-\frac{1}{2}\left(s_{L t}^{c}+s_{L t}^{a}\right)\right] \ln \left(\frac{\mathrm{k}_{t}^{c}}{\mathrm{k}_{\mathrm{t}}^{\mathrm{a}}}\right)+\frac{1}{2} \sum_{i=1}^{7}\left(s_{i t}^{c}+\mathrm{s}_{\mathrm{it}}^{\mathrm{a}}\right) \ln \left(\frac{\mathrm{P}_{8 t}^{\mathrm{c}} / \mathrm{P}_{\mathrm{t}}^{\mathrm{Hc}}}{\mathrm{P}_{8 t}^{\mathrm{a}} / P_{t}^{H a}}\right)$

Then the TFP equation (35) is rewritten as:

$$
\begin{align*}
\text { Adj. } \mathrm{TFP}_{\mathrm{t}}^{\mathrm{c}}=\alpha_{0}^{\mathrm{c}} & +\beta_{0 \mathrm{t}}^{\mathrm{c}}+\eta_{1} \ln \left(\frac{\mathrm{P}_{8 \mathrm{t}}^{\mathrm{c}} / \mathrm{P}_{\mathrm{t}}^{\mathrm{Hc}}}{\mathrm{P}_{8 \mathrm{t}}^{\mathrm{a}} / \mathrm{P}_{\mathrm{t}}^{\mathrm{Ha}}}\right)-\eta_{2} \ln \left(\frac{\mathrm{k}_{\mathrm{t}}^{\mathrm{c}}}{\mathrm{k}_{\mathrm{t}}^{\mathrm{a}}}\right)+\eta_{3} \ln \left(\frac{\mathrm{~T}_{\mathrm{t}}^{\mathrm{c}}}{\mathrm{~T}_{\mathrm{t}}^{\mathrm{a}}}\right)  \tag{39}\\
& +\sum_{\mathrm{j}=1}^{7} \frac{1}{2}\left(\mathrm{~s}_{\mathrm{it}}^{\mathrm{c}}+\mathrm{s}_{\mathrm{it}}^{\mathrm{a}}\right) \rho_{\mathrm{i}} \mathrm{~W}_{\mathrm{it}} \ln \left(\Lambda_{\mathrm{jt}}^{\mathrm{c}}\right)+\varepsilon_{\mathrm{t}}^{\mathrm{c}} .
\end{align*}
$$

With the estimated parameters from (39), we can reconstruct the country TFP differences as:

$$
\begin{align*}
& \text { Estimated } \mathrm{TFP}_{t}^{\mathrm{c}} \equiv \operatorname{Adj} \cdot \mathrm{TFP}_{\mathrm{t}}^{\mathrm{c}}-\hat{\eta}_{1} \ln \left(\frac{\mathrm{P}_{8 \mathrm{t}}^{\mathrm{c}} / \mathrm{P}_{\mathrm{t}}^{\mathrm{Hc}}}{\mathrm{P}_{8 \mathrm{t}}^{\mathrm{a}} / \mathrm{P}_{\mathrm{t}}^{\mathrm{Ha}}}\right)+\hat{\eta}_{2} \ln \left(\frac{\mathrm{k}_{\mathrm{t}}^{\mathrm{c}}}{\mathrm{k}_{\mathrm{t}}^{\mathrm{a}}}\right)-\hat{\eta}_{3} \ln \left(\frac{\mathrm{~T}_{\mathrm{t}}^{\mathrm{c}}}{\mathrm{~T}_{\mathrm{t}}^{\mathrm{a}}}\right)  \tag{40}\\
& =\hat{\alpha}_{0}^{\mathrm{c}}+\hat{\beta}_{0 \mathrm{t}}+\sum_{\mathrm{j}=1}^{7} \frac{1}{2}\left(\mathrm{~s}_{\mathrm{it}}^{\mathrm{c}}+\mathrm{s}_{\mathrm{it}}^{\mathrm{a}}\right) \hat{\rho}_{\mathrm{i}} \mathrm{~W}_{\mathrm{it}} \ln \left(\Lambda_{\mathrm{jt}}^{\mathrm{c}}\right)+\hat{\varepsilon}_{\mathrm{t}}^{\mathrm{c}},
\end{align*}
$$

which shows how estimated TFP is related to export variety and an error term.
We need to use nonlinear system estimation to estimate equations (38) and (39). In the next section, we proceed by estimating a full nonlinear 3SLS estimation with instrumental variables. A series of specification tests are performed: for homogeneity in prices, $\sum_{j=1}^{8} \gamma_{i j}=0$
in (38) and $\eta_{1}=1$ in (39); homogeneity in endowments, $\sum_{\mathrm{k}=1}^{3} \phi_{\mathrm{ik}}=0$ in (38) and $\eta_{3}=1$ in (39);
symmetry, $\gamma_{\mathrm{ij}}=\gamma_{\mathrm{ji}}$ in (38); as well as the over-identifying restrictions of the instruments.

## 4. Estimation Results

Table 3 presents the result of the nonlinear system of share equations (38) with the country TFP equation (39), estimated using three stage least squares regressions (3SLS). For brevity we report only one set of results: with the symmetric and homogeneity constraints in prices and endowments imposed in the share equations, ${ }^{20}$ but testing for homogeneity in the TFP equations by allowing for $\eta_{1} \neq 1$ and $\eta_{3} \neq 1$. Columns (1) to (7) of the table show the estimated coefficients of each of the sectoral share equations, and the last column shows the estimated coefficients of the TFP equation.

In the top part of Table 3 in columns (1) to (7) we report $\gamma_{\mathrm{ij}}$, which are the partial price effects due to export variety changes of the industry in the rows on the share of industries in the columns. All the own-price effects $\gamma_{\mathrm{ij}}$ are positive and most are highly significant. ${ }^{21}$ In other words, the underlying supply curves of these industries are positively sloped. The bottom part of Table 3 in columns (1) to (7) presents the Rybczynksi effects of endowments on the share of each industry. Positive point estimates indicate industry share expansions due to the increases in that endowments. For example, an increase in the labor endowment relative to land benefits the electronics industry. These findings are broadly similar to those of Harrigan (1997).

The top half of column (8) in Table 3 presents the 3SLS estimates of $\rho_{\mathrm{i}}$ for each sector.

All the point estimates are positive, and are smaller than one. Since we expect $\rho_{\mathrm{i}}$ to lie within the range $\left[\left(\sigma_{i}-1\right) / \theta_{i} \sigma_{\mathrm{i}}, 1 / \sigma_{\mathrm{i}}\right]$, these estimates imply that the elasticities of substitution exceed

[^13]unity and that the restriction $\theta_{i}>\sigma_{i}-1$ in (16) holds. The industry with the highest value of $\rho_{i}$ is electronics, so that increases in export variety contribute the most to country productivity, whereas the industry with the lowest value of $\rho_{\mathrm{i}}$ is agriculture, so export variety contributes little to productivity. While we cannot separately identify the elasticity of substitution from the Pareto parameter $\theta_{\mathrm{i}}$, one interpretation of these findings is that agriculture has a high value of $\sigma_{\mathrm{i}}$, or a high value of $\theta_{i}$ as compared to $\left(\sigma_{i}-1\right) / \sigma_{i}$. High $\sigma_{i}$ means homogeneous products, whereas high $\theta_{\mathrm{i}}$ means there is little dispersion in firm productivities, both of which seem appropriate for agriculture. The low value of $\rho_{\mathrm{i}}$ for electronics can be explained by a low value of $\sigma_{\mathrm{i}}$, or a low value of $\theta_{\mathrm{i}}$ as compared to $\left(\sigma_{\mathrm{i}}-1\right) / \sigma_{\mathrm{i}}$. This means either heterogeneous products or a wide dispersion of productivities, which again seems reasonable.

The coefficient of capital-land ratio in the lower part of column (8) in Table 3, which has the interpretation of the negative share of land in GDP is not statistically significant. This suggests that most of the factor returns in GDP are in fact due to labor and capital earnings. The coefficient on the relative land size (shown in the labor-land row) is not statistically different from one, which implies that homogeneity in factor endowments is not rejected. However, the coefficient $\eta_{1}$ on the price of nontraded goods is significantly less than one, which violates the homogeneity constraint on prices in the TFP equation.

Instruments used in Table 2 consisting of U.S. tariffs by industry, source country and year (bilateral industry tariffs for textiles and basic metals, the two industries that have the highest tariffs), NAFTA dummy, distance and its squares between exporting countries and US (in kilometers), and relative endowments. ${ }^{22}$ Given that the above nonlinear 3SLS estimation

[^14]involves minimizing the criterion function, the minimized value provides a test statistic for hypothesis testing. The difference between the values of the criterion functions of the restricted and unrestricted models is asymptotically chi-squared distributed with degree of freedom equal to the number of restrictions. According to Davidson and MacKinnon (1993, p. 665), it is important that the same estimate of variance-covariance matrix be used for both the restricted and unrestricted estimations, in order to ensure that the test statistic is positive. We us the variance-covariance matrix of the unrestricted model.

Table 4 presents the test statistics and the associated p-values of the hypothesis tests. First, we test the homogeneity constraints on prices and endowments in the share equations, along with the homogeneity constraint in endowments in the TFP equation. As shown in the first row of Table 4, these homogeneity constraints are not rejected. If we also test the homogeneity constraint on prices in the TFP equation (i.e. $\eta_{1}=1$ ), that constraint is easily rejected, possibly due to measurement errors in nontraded good prices. So that constraint is not imposed.

Next, the twenty-one symmetry constraints on the cross-price effects are tested on the whole system, which are not rejected. Third, we test for the 16 over-identifying restrictions due to the extra instruments, which are not rejected. Finally, the overall specification of the system is tested by jointly testing all these 44 constraints. This is done by comparing the value of criterion function of the restricted model to a just-identified model with no symmetry constraints and no extra instruments. The whole set of restriction are again not rejected, which supports the symmetry and homogeneity constraints and the validity of instruments. In the next section we explore the instruments further by reporting their regressions with export variety.

### 4.1 Effects of Tariffs and Distance on Export Variety

Table 5 presents least squares (LS) estimation linking export variety to all instruments and exogenous variables of the nonlinear 3SLS system presented in Table 3. This is similar but not identical to the first-stage estimation of the nonlinear system, which involves regressing the derivatives of each equation with respect to the parameters of the system on all the instruments and exogenous variables. ${ }^{23}$ In comparison, the regressions we present in Table 5 just uses the export variety index $\ln \Lambda_{\mathrm{it}}^{\mathrm{c}}$ as a dependent variable, which allows us to see the relationship between export variety and the tariff and distance variables.

The top part of Table 5 shows the effects of the U.S. tariff on the export variety of the industry in the columns. We expect industry export variety to decrease with the own tariff of the industry, while there may exist some positive effects due to reallocation of resources among industries when there is a tariff increase in other industries. All industry export variety indexes are negatively correlated with own tariffs except for the textiles \& garments and the electronics industries. For textiles \& garments, MFA quotas are known to be more restrictive and binding than tariffs, which may explain the insignificant effect of tariffs on export variety. For the electronics industry, it could be the case that non-tariff barriers, transport costs and skilled labor endowments are more important in explaining expansion in export variety than tariffs. For the rest of the industries, the own tariff effects are all negative and statistically significant.

A one percentage point increase in U.S. tariffs on petroleum and plastics lowers export variety of the industry by $9.9 \%$, at the highest, and a similar increase in the wood and paper tariff lowers export variety of the industry by $3 \%$, at the lowest. While these semi-elasticities show that tariffs have a statistically significant impact on product variety in most industries, the

[^15]economic magnitude of this effect is very modest. From the last rows of Table 2, we know that the observed drop in U.S. tariffs over 1980-2000 are quite small. Using these tariff reductions and the semi-elasticities in Table 5, we can calculate that the drop in U.S. tariffs has increased export variety by only $11 \%$ over the two decades. Recalling that average export variety increased by eight times over 1980-2000 (see Table 1), we conclude that fall in U.S. tariffs explains only a very small part of export variety growth.

The next section of Table 5 shows the marginal effects of NAFTA on export variety. Given that we already control for tariffs, these variables capture the effect of the reduction in non-tariff barriers due to the signing of such agreements on export variety. NAFTA is shown to have significant positive effects on the variety of agriculture and basic metals industries, and has a negative and significant effect on the export of petroleum and plastics industry.

The third section of Table 5 relates distance (in $\log$ of kilometers) and its squares to the export variety of the industries. Overall, the further a country is from the U.S., the less variety is exported. Such negative effects are particularly significant for textiles and garments industry, wood and paper industry and machinery and transport equipment industry. However, the effects are not linear since the coefficients on distance squares are most positive, which signal that that marginal effect of each addition kilometer diminishes with the overall distance between the two countries. Other than tariffs, distance and trade agreement dummies, we have also included all the right-hand side exogenous variables in Table 4 in the regressions. These variables are: a full set of year fixed effects, labor-land ratio, capital-land ratio, non-traded goods prices, and land area. The vast majority of the increase in export variety over time is explained by the year fixed effects, while the other variables explain its variation over countries.

### 4.3 Productivity Decomposition

To gain additional insight into the links between export variety and country productivity, we performed panel regressions of estimated productivity on export variety (constructed from the estimates in Table 3). Using (40), we relate the estimated country TFP to that portion due to export variety, $\sum_{\mathrm{i}=1}^{7} \frac{1}{2}\left(\mathrm{~s}_{\mathrm{it}}^{\mathrm{c}}+\mathrm{s}_{\mathrm{it}}^{\mathrm{a}}\right) \hat{\rho}_{\mathrm{i}} \mathrm{W}_{\mathrm{it}} \ln \left(\Lambda_{\mathrm{it}}^{\mathrm{c}}\right)$. Figure 2 plots the scatter graph of country TFP against industry export variety. Both variables are averaged over time so this scatter plot is equivalent to a "between" regression. It is evident that export variety has significant explanatory power for the variation of the country productivity differences: $\mathrm{R}^{2}=0.33$ for this univariate regression. The problem with this "between" regression, however, is that it omits country fixedeffects, which are included in the results from panel regressions reported in Table 6.

In the total sample, export variety can explain only $2.4 \%$ of the variation in country TFP, but $40.4 \%$ of within-country TFP variation. Thus, export variety is strongly correlated with the variation in country TFP over time, but explains only a small fraction of the variation in TFP across countries. This finding continues to hold if we investigate only the OECD countries (using the same parameters estimates as in Table 3). In that case, export variety explains $10.6 \%$ of the overall variation in country TFP, and $60.8 \%$ of within-country TFP variation.

To further illustrate the effects of export variety on country productivity, according to (40), a $1 \%$ increase in the export variety of each industry would increase country productivity by $\frac{1}{2}\left(s_{i t}^{c}+s_{i t}^{a}\right) \hat{\rho}_{i} W_{i t}$ percent. Thus, we can compute that at the sample mean, a doubling of export varieties of all industries could lead to $3.4 \%$ increase in country productivity. This effect is significant both statistically and economically. It implies that the eight-fold expansion of export variety over 1980-2000 explains more than a $10 \%$ increase in exporters' productivity. This is an estimate of the endogenous portion of productivity gains that is consistent with the monopolistic
competition model. As noted above, however, the variety increase itself is not well-explained by tariff cuts or other variables that change over time; instead, the increase in export variety is predicted by the time fixed effects in Table 5. In this sense, our empirical work does not give a full account of the mechanism of increased export activity and resulting productivity growth in the monopolistic competition model.

The tight time series linkage between export variety and productivity can be seen from Figures 3 and 4. Figure 3 compares Canada to the sample mean in terms of productivity, and average export variety, from 1980 to 2000. The relative export variety index is measured on the vertical left-hand scale, while relative country TFP index is measured on the right-hand scale. It is clear that these two series move together closely. In the years just after the Canada-U.S. free trade agreement in 1989, Canada has a boost in its export variety to the U.S. and in its TFP, but afterwards experienced a decline in both indexes relative to other countries. Figure 4 compares Japan to South Korea. Similar to the previous figure, average export variety is measured on the left-hand scale, while the productivity of Japan relative to Korea is measured on the right-hand scale. The movements of the two lines suggest that over the twenty year period, South Korea is catching up in terms of export variety as well as country productivity.

## 5. Conclusions

Current research in international trade has stressed that productivity is endogenous through the self-selection of exporters. The mechanism stressed by Melitz (2003) is that exporters are more productive on average than domestic firms, so an increase in export activity is associated with rising productivity. In this paper we have attempted to estimate the relation between export variety and productivity using a GDP function across countries and over time. In using the translog GDP function we are following Harrigan (1997), who hypothesized that export
prices would differ across countries due to total factor productivity in exports. We have used a CES measure of export variety, which enters like a sectoral "price" into the GDP function and sectoral share equations. We have treated export variety as an endogenous variable, and as instruments use those suggested by Melitz (2003): tariffs, trade agreements and distance.

The measure of export variety we use is constructed to be consistent across countries and over time. It shows an average $10 \%$ increase in export variety to the United States, which therefore increased by eight times over 1980-2000. Only a very small amount of that increase is explained by observed cuts in U.S. tariffs, however, and the majority is accounted for by time fixed-effects. Each doubling of export variety leads to a $3.4 \%$ increase in country productivity, so the eight-fold expansion of export variety over 1980-2000 explains more than a $10 \%$ increase in exporters' productivity. This is an estimate of the endogenous portion of productivity gains, though the variety increase itself is not well-explained by tariff cuts. In the time series we are able to explain a substantial portion of productivity gains across countries, but we cannot account for the enormous cross-country differences in productivity. In the total sample, export variety explains $40.4 \%$ of within-country variation in productivity, but only $2.4 \%$ of overall country productivity variation. Restricting attention to the OECD countries, export variety explains $60.8 \%$ of the within-country variation in productivity, but only $10.6 \%$ for the overall country variation. We conclude that the monopolistic competition model with endogenous productivity differences is quite effective at accounting for the time-series variation within countries, but not the large absolute differences in productivity between them.

## Appendix

## Proof of Proposition 1:

Introduce $m_{i}$ as the Lagrange multiplier on constraint (9), $m_{i x}$ as the Lagrange multiplier on (10), and the vector w as the multipliers on (12'). Then the Lagrangian can be written as:

$$
\begin{aligned}
& L=\sum_{i=1}^{N}\left\{\int_{\varphi_{i}^{*}}^{\infty} M_{i} A_{i d} q_{i}\left(\varphi_{i}\right)^{\frac{\sigma_{i}-1}{\sigma_{i}}} \mu_{i}\left(\varphi_{i}\right) d \varphi_{i}+\int_{\varphi_{i x}^{*}}^{\infty} M_{i} A_{i x} q_{i x}\left(\varphi_{i}\right)^{\frac{\sigma_{i}-1}{\sigma_{i}}} \mu_{i}\left(\varphi_{i}\right) d \varphi_{i}\right\} \\
& +\mathrm{m}_{\mathrm{i}} \sum_{\mathrm{i}=1}^{\mathrm{N}}\left\{\int_{\varphi_{\mathrm{i}}^{*}}^{\infty} \mathrm{h}_{\mathrm{i}}\left[\mathrm{v}_{\mathrm{i}}\left(\varphi_{\mathrm{i}}\right)\right] \mu\left(\varphi_{\mathrm{i}}\right) \mathrm{d} \varphi_{\mathrm{i}}-\int_{\varphi_{\mathrm{i}}^{*}}^{\infty} \mathrm{M}_{\mathrm{i}}\left[\left(\mathrm{q}_{\mathrm{i}} / \varphi_{\mathrm{i}}\right)+\mathrm{f}_{\mathrm{i}}\right] \mu\left(\varphi_{\mathrm{i}}\right) \mathrm{d} \varphi_{\mathrm{i}}\right\} \\
& +\mathrm{m}_{\mathrm{ix}} \sum_{\mathrm{i}=1}^{\mathrm{N}}\left\{\int_{\varphi_{\mathrm{ix}}^{*}}^{\infty} \mathrm{h}_{\mathrm{i}}\left[\mathrm{v}_{\mathrm{ix}}\left(\varphi_{\mathrm{i}}\right)\right] \mu\left(\varphi_{\mathrm{i}}\right) \mathrm{d} \varphi_{\mathrm{i}}-\int_{\varphi_{\mathrm{ix}}^{*}}^{\infty} \mathrm{M}_{\mathrm{i}}\left[\left(\mathrm{q}_{\mathrm{ix}} / \varphi_{\mathrm{i}}\right)+\mathrm{f}_{\mathrm{ix}}\right] \mu\left(\varphi_{\mathrm{i}}\right) \mathrm{d} \varphi_{\mathrm{i}}\right\} \\
& +w^{\prime}\left\{V-\sum_{i=1}^{N} \frac{\delta M_{i}}{\left[1-G_{i}\left(\varphi_{i}^{*}\right)\right]} v_{i e}-\sum_{i=1}^{N}\left[\int_{\varphi_{i}^{*}}^{\infty} v_{i}\left(\varphi_{i}\right) \mu_{i}\left(\varphi_{i}\right) d \varphi_{i}+\int_{\varphi_{i x}^{*}}^{\infty} v_{i x}\left(\varphi_{i}\right) \mu_{i}\left(\varphi_{i}\right) d \varphi_{i}\right]\right\} \\
& =\sum_{i=1}^{N} \int_{\varphi_{i}^{*}}^{\infty} M_{i}\left\{\left[A_{i d} q_{i}\left(\varphi_{i}\right)^{\frac{\sigma_{i}-1}{\sigma_{i}}}-m_{i}\left(\frac{q_{i}\left(\varphi_{i}\right)}{\varphi_{i}}+f_{i}\right)\right\} \mu_{i}\left(\varphi_{i}\right) d \varphi_{i}\right. \\
& +\sum_{i=1}^{N} \int_{\varphi_{i x}^{*}}^{\infty} M_{i}\left\{\left[A_{i x} q_{i x}\left(\varphi_{i}\right)^{\frac{\sigma_{i}-1}{\sigma_{i}}}-m_{i x}\left(\frac{q_{i x}\left(\varphi_{i}\right)}{\varphi_{i}}+f_{i x}\right)\right\} \mu_{i}\left(\varphi_{i}\right) d \varphi_{i}\right. \\
& +\sum_{i=1}^{N}\left[\int_{\varphi_{i}^{*}}^{\infty} m_{i} h_{i}\left(v_{i}\left(\varphi_{i}\right)\right) \mu_{i}\left(\varphi_{i}\right) d \varphi_{i}-\int_{\varphi_{i}^{*}}^{\infty} w^{\prime} v_{i}\left(\varphi_{i}\right) \mu_{i}\left(\varphi_{i}\right) d \varphi_{i}\right] \\
& +\sum_{i=1}^{N}\left[\int_{\varphi_{i x}^{*}}^{\infty} m_{i x} h_{i x}\left(v_{i x}\left(\varphi_{i}\right)\right) \mu_{i}\left(\varphi_{i}\right) d \varphi_{i}-\int_{\varphi_{i x}^{*}}^{\infty} w^{\prime} v_{i x}\left(\varphi_{i}\right) \mu_{i}\left(\varphi_{i}\right) d \varphi_{i}\right]+w^{\prime}\left[V-\sum_{i=1}^{N} \frac{\delta M_{i} v_{i e}}{\left[1-G_{i}\left(\varphi_{i}^{*}\right)\right]}\right]
\end{aligned}
$$

where (A1) follows by grouping terms within the integrals.

By inspection of the definition of the Lagrangian, the maximized value of GDP is a function $R\left(A_{d}, A_{x}, V\right)$. The first-order condition with respect to the domestic quantity $q_{i}\left(\varphi_{i}\right)$ is obtained by differentiating the terms within the first integral in (A1), and yields:

$$
\begin{equation*}
\left(\frac{\sigma_{i}-1}{\sigma_{i}}\right) A_{i d} q_{i}\left(\varphi_{i}\right)^{-\frac{1}{\sigma_{i}}}=\frac{m_{i}}{\varphi_{i}} . \tag{A2}
\end{equation*}
$$

The expression $A_{i d} q_{i}\left(\varphi_{i}\right)^{-\frac{1}{\sigma_{i}}}$ equals price $p_{i}\left(\varphi_{i}\right)$, as can be seen from the demand function (3), so this first-order condition is just marginal revenue equals marginal cost:

$$
\begin{equation*}
\left(\frac{\sigma_{i}-1}{\sigma_{i}}\right) \mathrm{p}_{\mathrm{i}}\left(\varphi_{\mathrm{i}}\right)=\frac{\mathrm{m}_{\mathrm{i}}}{\varphi_{\mathrm{i}}} \tag{A3}
\end{equation*}
$$

Likewise, maximization of the Lagrangian with respect to the export quantity $\mathrm{q}_{\mathrm{ix}}\left(\varphi_{\mathrm{i}}\right)$, in the second integral of (A1), also yields marginal revenue equal to marginal cost, for exports:

$$
\begin{equation*}
\left(\frac{\sigma_{\mathrm{i}}-1}{\sigma_{\mathrm{i}}}\right) \mathrm{p}_{\mathrm{ix}}\left(\varphi_{\mathrm{i}}\right)=\frac{\mathrm{m}_{\mathrm{ix}}}{\varphi_{\mathrm{i}}} . \tag{A4}
\end{equation*}
$$

Using (A3) and (A4), we can calculate that the profits from domestic and export sales are,

$$
\begin{equation*}
r_{i}\left(\varphi_{i}\right)-\left(\frac{m_{i}}{\varphi_{i}}\right) q_{i}\left(\varphi_{i}\right)=\frac{r_{i}\left(\varphi_{i}\right)}{\sigma_{i}} \text {, and } r_{i x}\left(\varphi_{i}\right)-\left(\frac{m_{i x}}{\varphi_{i}}\right) q_{i x}\left(\varphi_{i}\right)=\frac{r_{i x}\left(\varphi_{i}\right)}{\sigma_{i}} . \tag{A5}
\end{equation*}
$$

Maximization with respect to $\mathrm{v}_{\mathrm{i}}\left(\varphi_{\mathrm{i}}\right)$ and $\mathrm{v}_{\mathrm{ix}}\left(\varphi_{\mathrm{i}}\right)$ in the third and forth integrals yields,

$$
\begin{equation*}
\mathrm{m}_{\mathrm{i}} \frac{\partial \mathrm{~h}_{\mathrm{i}}\left(\mathrm{v}_{\mathrm{i}}\left(\varphi_{\mathrm{i}}\right)\right)}{\partial \mathrm{v}_{\mathrm{i}}\left(\varphi_{\mathrm{i}}\right)}=\mathrm{w}, \tag{A6}
\end{equation*}
$$

and,

$$
\begin{equation*}
\mathrm{m}_{\mathrm{ix}} \frac{\partial \mathrm{~h}_{\mathrm{i}}\left(\mathrm{v}_{\mathrm{ix}}\left(\varphi_{\mathrm{i}}\right)\right)}{\partial \mathrm{v}_{\mathrm{ix}}\left(\varphi_{\mathrm{i}}\right)}=\mathrm{w} . \tag{A7}
\end{equation*}
$$

Therefore: $\quad \frac{\partial \mathrm{h}_{\mathrm{i}}\left(\mathrm{v}_{\mathrm{i}}\left(\varphi_{\mathrm{i}}\right)\right)}{\partial \mathrm{v}_{\mathrm{ik}}\left(\varphi_{\mathrm{i}}\right)} / \frac{\partial \mathrm{h}_{\mathrm{i}}\left(\mathrm{v}_{\mathrm{i}}\left(\varphi_{\mathrm{i}}\right)\right)}{\partial \mathrm{v}_{\mathrm{i} \ell}\left(\varphi_{\mathrm{i}}\right)}=\mathrm{w}_{\mathrm{k}} / \mathrm{w}_{\ell}=\frac{\partial \mathrm{h}_{\mathrm{i}}\left(\mathrm{v}_{\mathrm{ix}}\left(\varphi_{\mathrm{i}}\right)\right)}{\partial \mathrm{v}_{\mathrm{ixk}}\left(\varphi_{\mathrm{i}}\right)} / \frac{\partial \mathrm{h}_{\mathrm{i}}\left(\mathrm{v}_{\mathrm{ix}}\left(\varphi_{\mathrm{i}}\right)\right)}{\partial \mathrm{v}_{\mathrm{ix} \ell}\left(\varphi_{\mathrm{i}}\right)}$.

Since the function $h_{i}$ is assumed to be strictly quasi-concave, it follows from (A8) that the ratio of demand for factors k and $\ell$ are identical in domestic and export use. Therefore, the values of $v_{i}$ and $v_{i x}$ are multiples of each other, $v_{i}=\lambda_{i} v_{i x}$. But since $h_{i}$ homogeneous of degree one, its first derivative is homogeneous of degree zero, so any solution $\lambda_{i} v_{i x}$ in (A6) yields exactly the same value for the derivatives $\partial \mathrm{h}_{\mathrm{i}}\left(\lambda_{\mathrm{i}} \mathrm{v}_{\mathrm{ix}}\right) / \partial \mathrm{v}_{\mathrm{i}}$ as does $\partial \mathrm{h}_{\mathrm{i}}\left(\mathrm{v}_{\mathrm{ix}}\right) / \partial \mathrm{v}_{\mathrm{ix}}$ in (A7). It follows that the equalities in (A6) and (A7) can hold if and only if $m_{i}=m_{i x}$, so the marginal costs of domestic production and exporting are equal. Furthermore, multiplying (A6) and (A7) by $v_{i}$ and $v_{i x}$, we immediately obtain $\left[\mathrm{m}_{\mathrm{i}} \mathrm{h}_{\mathrm{i}}\left(\mathrm{v}_{\mathrm{i}}\right)-\mathrm{w}^{\prime} \mathrm{v}_{\mathrm{i}}\right]=\left[\mathrm{m}_{\mathrm{i}} \mathrm{h}_{\mathrm{i}}\left(\mathrm{v}_{\mathrm{i}}\right)-\mathrm{w}^{\prime} \mathrm{v}_{\mathrm{i}}\right]=0$.

Substituting these relations into (A1), and using (A5), we can rewrite the Lagrangian as:

$$
\sum_{i=1}^{N}\left[\int_{\varphi_{i}^{*}}^{\infty} M_{i}\left(\frac{r_{i}\left(\varphi_{i}\right)}{\sigma_{i}}-m_{i} f_{i}\right) \mu_{i}\left(\varphi_{i}\right) d \varphi+\int_{\varphi_{i x}^{*}}^{\infty} M_{i}\left(\frac{r_{i x}\left(\varphi_{i}\right)}{\sigma_{i}}-m_{i} f_{i x}\right) \mu_{i}\left(\varphi_{i}\right) d \varphi\right]+w^{\prime}\left[V-\sum_{i=1}^{N} \frac{\delta M_{i} v_{i e}}{\left[1-G_{i}\left(\varphi_{i}^{*}\right)\right]}\right]
$$

Differentiating this Lagrangian with respect to the export cutoff productivity $\varphi_{i x}^{*}$, we obtain $\mathrm{r}_{\mathrm{ix}}\left(\varphi_{\mathrm{ix}}^{*}\right) / \sigma_{\mathrm{i}}=\mathrm{m}_{\mathrm{i}} \mathrm{f}_{\mathrm{ix}}$, which states that the profits earned by the marginal exporter should just cover fixed costs. This is an equilibrium condition in Melitz (2003). Differentiating the Lagrangian with respect to $\mathrm{M}_{\mathrm{i}}$, we obtain:

$$
\begin{equation*}
\frac{\left[1-\mathrm{G}_{\mathrm{i}}\left(\varphi_{\mathrm{i}}^{*}\right)\right]}{\delta}\left[\int_{\varphi_{\mathrm{i}}^{*}}^{\infty} \frac{\mathrm{r}_{\mathrm{i}}\left(\varphi_{\mathrm{i}}\right)}{\sigma_{\mathrm{i}}} \mu_{\mathrm{i}}\left(\varphi_{\mathrm{i}}\right) \mathrm{d} \varphi_{\mathrm{i}}-\mathrm{m}_{\mathrm{i}} \mathrm{f}_{\mathrm{i}}+\int_{\varphi_{\mathrm{ix}}^{*}}^{\infty} \frac{\mathrm{r}_{\mathrm{ix}}\left(\varphi_{\mathrm{i}}\right)}{\sigma_{\mathrm{i}}} \mu_{\mathrm{i}}\left(\varphi_{\mathrm{i}}\right) \mathrm{d} \varphi_{\mathrm{i}}-\mathrm{m}_{\mathrm{i}} \mathrm{f}_{\mathrm{ix}}\right]=\mathrm{w}^{\prime} \mathrm{v}_{\mathrm{ie}} \tag{A9}
\end{equation*}
$$

where the term in brackets is the average profits earned by a successful entrant. This condition states that expected discounted profits equal the fixed costs of entry, which is the free-entry condition in Melitz (2003). This condition ensures that that total revenue earned equals factor payments in the economy, $\mathrm{R}=\mathrm{w}^{\prime} \mathrm{V}$.

Finally, differentiating with respect to the domestic cutoff productivity $\varphi_{i}^{*}$, we need to recognize that the marginal distributions $\mu_{\mathrm{i}}\left(\varphi_{\mathrm{i}}\right)$ are divided by $\left[1-\mathrm{G}_{\mathrm{i}}\left(\varphi_{\mathrm{i}}^{*}\right)\right]$. Taking into account the derivative of this term with respect to $\varphi_{i}^{*}$, and then using the free-entry condition (A9), we obtain $\mathrm{r}_{\mathrm{i}}\left(\varphi_{\mathrm{i}}^{*}\right) / \sigma_{\mathrm{i}}=\mathrm{m}_{\mathrm{i}} \mathrm{f}_{\mathrm{i}}$, so the profits earned by the marginal domestic producer with productivity $\varphi_{i}^{*}$ just cover fixed costs.

Part (a) of the Proposition follows by differentiating (A1) with respect to $\mathrm{A}_{\mathrm{id}}$, obtaining:

$$
\frac{\partial \mathrm{L}}{\partial \mathrm{~A}_{\mathrm{id}}}=\frac{\partial \mathrm{R}}{\partial \mathrm{~A}_{\mathrm{id}}}=\int_{\varphi_{\mathrm{i}}^{*}}^{\infty} \mathrm{M}_{\mathrm{i}} \mathrm{q}_{\mathrm{i}}\left(\varphi_{\mathrm{i}}\right)^{\frac{\sigma_{\mathrm{i}}-1}{\sigma_{\mathrm{i}}}} \mu_{\mathrm{i}}\left(\varphi_{\mathrm{i}}\right) \mathrm{d} \varphi_{\mathrm{i}}=\mathrm{R}_{\mathrm{i}} / \mathrm{A}_{\mathrm{id}}
$$

It follows that $\partial \ln \mathrm{R} / \partial \ln \mathrm{A}_{\mathrm{id}}=\left(\mathrm{R}_{\mathrm{i}} / \mathrm{R}\right)$, and a similar condition holds for exports. Multiplying all $\mathrm{A}_{\mathrm{id}}$ and $\mathrm{A}_{\mathrm{ix}}$ in (A1)-(A4) by $\lambda>0$ will increase prices $\mathrm{p}_{\mathrm{i}}$ and $\mathrm{p}_{\mathrm{ix}}$, wages w and marginal costs $\mathrm{m}_{\mathrm{i}}=\mathrm{m}_{\mathrm{ix}}$ by that amount, with no change in any quantities. It follows that objective function is multiplied by $\lambda>0$, so $R\left(A_{d}, A_{x}, V\right)$ is homogeneous of degree one in $\left(A_{d}, A_{x}\right)$.

The result that $\partial \mathrm{R} / \partial \mathrm{V}=\mathrm{w}$ in part (b) is a property of Lagrange multipliers. Multiplying the endowments $V$ in (A1)-(A7) by $\lambda>0$ will increase all quantities by that amount, with no change in the prices, wages or marginal costs. It follows that objective function is multiplied by $\lambda>0$, so $R\left(A_{d}, A_{x}, V\right)$ is homogeneous of degree one in $V$. QED

## Proof of Proposition 2:

From (14) we see that prices are inversely proportional to productivities $\varphi_{i}$, so from (3) and (5) the revenue earned by firms of various productivities satisfies $\mathrm{r}\left(\varphi_{\mathrm{i}}^{\prime \prime}\right) / \mathrm{r}\left(\varphi_{\mathrm{i}}^{\prime}\right)=\left(\varphi_{\mathrm{i}}^{\prime \prime} / \varphi_{\mathrm{i}}^{\prime}\right)^{\sigma_{\mathrm{i}}-1}$. For example, compared to the cut-off productivity $\varphi_{i}^{*}$, we have $r_{i}\left(\varphi_{i}\right)=\left(\varphi_{i} / \varphi_{i}^{*}\right)^{\sigma_{i}-1} r_{i}\left(\varphi_{i}^{*}\right)$. Using
this relation and (3') to evaluate $r_{i}\left(\varphi_{i}^{*}\right)$, total revenue earned from domestic sales in sector $i$ is:

$$
\begin{equation*}
R_{i d}=\int_{\varphi_{i}^{*}}^{\infty} M_{i} r_{i}\left(\varphi_{i}\right) \mu_{i}(\varphi) d \varphi=M_{i} A_{i d}\left[\frac{\widetilde{\varphi}_{i}\left(\varphi_{i}^{*}\right)}{\varphi_{i}^{*}}\right]^{\sigma_{i}-1} q_{i}\left(\varphi_{i}^{*}\right)^{\frac{\sigma_{i}-1}{\sigma_{i}}}, \tag{A10}
\end{equation*}
$$

where $\tilde{\varphi}_{\mathrm{i}}\left(\varphi_{\mathrm{i}}^{*}\right)$ is the average productivity across firms, defined as in Melitz (2003) by:

$$
\begin{equation*}
\widetilde{\varphi}_{\mathrm{i}}\left(\varphi_{\mathrm{i}}^{*}\right) \equiv\left(\int_{\varphi_{\mathrm{i}}^{*}}^{\infty} \varphi_{\mathrm{i}}^{\sigma_{\mathrm{i}}-1} \mu_{\mathrm{i}}(\varphi) \mathrm{d} \varphi\right)^{\frac{1}{\left(\sigma_{\mathrm{i}}-1\right)}} \tag{A11}
\end{equation*}
$$

Equation (A10) shows that the domestic sales of home firms is equal to the sales from a mass $\mathrm{M}_{\mathrm{i}}$ of representative firms, all with productivity $\widetilde{\varphi}_{i}\left(\varphi_{i}^{*}\right) / \varphi_{i}^{*}$.

On the export side, it follows using the same steps as above that revenue equals $\mathrm{r}_{\mathrm{ix}}\left(\varphi_{\mathrm{ix}}\right)=\left(\varphi_{\mathrm{ix}} / \varphi_{\mathrm{ix}}^{*}\right)^{\sigma_{\mathrm{i}}-1} \mathrm{r}_{\mathrm{ix}}\left(\varphi_{\mathrm{ix}}^{*}\right)$. Then using (5') to evaluate $\mathrm{r}_{\mathrm{ix}}\left(\varphi_{\mathrm{ix}}^{*}\right)$, total revenue earned from export sales in sector i is:

$$
\begin{equation*}
\mathrm{R}_{\mathrm{ix}}=\int_{\varphi_{\mathrm{ix}}^{*}}^{\infty} \mathrm{M}_{\mathrm{i}} \mathrm{r}_{\mathrm{ix}}\left(\varphi_{\mathrm{i}}\right) \mu_{\mathrm{i}}(\varphi) \mathrm{d} \varphi=\chi_{\mathrm{i}} \mathrm{M}_{\mathrm{i}} \mathrm{~A}_{\mathrm{ix}}\left[\frac{\widetilde{\varphi}_{\mathrm{ix}}\left(\varphi_{\mathrm{ix}}^{*}\right)}{\varphi_{\mathrm{ix}}^{*}}\right]^{\sigma_{\mathrm{i}}-1} \mathrm{q}_{\mathrm{ix}}\left(\varphi_{\mathrm{ix}}^{*}\right)^{\frac{\sigma_{\mathrm{i}}-1}{\sigma_{\mathrm{i}}}}, \tag{A12}
\end{equation*}
$$

where $\tilde{\varphi}_{\mathrm{ix}}\left(\varphi_{\mathrm{ix}}^{*}\right)$ is the average productivity across exporting firms, defined analogously to (A11) but with the cutoff productivity $\varphi_{\mathrm{ix}}^{*}$ :

$$
\begin{equation*}
\tilde{\varphi}_{\mathrm{ix}}\left(\varphi_{\mathrm{ix}}^{*}\right) \equiv\left(\int_{\varphi_{\mathrm{ix}}^{*}}^{\infty} \varphi_{\mathrm{i}}^{\sigma_{i}-1} \frac{\mathrm{~g}_{\mathrm{i}}\left(\varphi_{\mathrm{i}}\right)}{\left[1-\mathrm{G}_{\mathrm{i}}\left(\varphi_{\mathrm{ix}}^{*}\right)\right]} \mathrm{d} \varphi_{\mathrm{i}}\right)^{\frac{1}{\left(\sigma_{i}-1\right)}} \tag{A13}
\end{equation*}
$$

We can think of the terms $\left[\widetilde{\varphi}_{i}\left(\varphi_{i}^{*}\right) / \varphi_{i}^{*}\right]$ or $\left[\widetilde{\varphi}_{i x}\left(\varphi_{i x}^{*}\right) / \varphi_{i x}^{*}\right]$ as a measure of the skewness of productivity. Calculating the average productivities using the Pareto distribution, we obtain:

$$
\begin{equation*}
\left[\frac{\widetilde{\varphi}_{\mathrm{i}}\left(\varphi_{\mathrm{i}}^{*}\right)}{\varphi_{\mathrm{i}}^{*}}\right]^{\sigma_{\mathrm{i}}-1}=\left[\frac{\widetilde{\varphi}_{\mathrm{ix}}\left(\varphi_{\mathrm{ix}}^{*}\right)}{\varphi_{\mathrm{ix}}^{*}}\right]^{\sigma_{\mathrm{i}}-1}=\left(\frac{\theta_{\mathrm{i}}}{\theta_{\mathrm{i}}-\sigma_{\mathrm{i}}+1}\right), \quad \theta_{\mathrm{i}}>\sigma_{\mathrm{i}}-1 \tag{A14}
\end{equation*}
$$

Thus, the skewness of the Pareto distribution is independent of the cutoff productivity.
Substituting (A14) into (A10) and (A12), dividing these and using (15), we obtain (19) in the text, from which (20) is obtained. These show that the export relative to domestic share in each sector is independent of the shift parameters in all other sectors and of factor endowments. Therefore, the parameters $\left(\mathrm{A}_{\mathrm{id}}, \mathrm{A}_{\mathrm{ix}}\right)$ in the GDP function are weakly separable from all other shift parameters and from the endowments. It follows that GDP can be written as a function $R\left[\psi_{1}\left(A_{1 d}, A_{1 x}\right), \ldots, \psi_{N}\left(A_{N d}, A_{N x}\right), V\right]$, for some linearly homogeneous functions $\psi_{i}, i=1, \ldots, N$.

Furthermore, (20) proves that the $\psi_{\mathrm{i}}$ are CES functions, $\psi_{\mathrm{i}}\left(\mathrm{A}_{\mathrm{id}}, \mathrm{A}_{\mathrm{ix}}\right)=\left(\mathrm{A}_{\mathrm{id}}^{\alpha}+\beta \mathrm{A}_{\mathrm{ix}}^{\alpha}\right)^{1 / \alpha}$, for some parameters $\alpha$ and $\beta$. Then using Proposition 1(a) to calculate (20), we obtain:

$$
\begin{equation*}
\frac{\mathrm{R}_{\mathrm{ix}}}{\mathrm{R}_{\mathrm{id}}}=\frac{\left(\frac{\partial \ln \mathrm{R}}{\partial \ln \psi_{\mathrm{i}}}\right)\left(\frac{\partial \ln \psi_{\mathrm{i}}}{\partial \ln \mathrm{~A}_{\mathrm{ix}}}\right)}{\left(\frac{\partial \ln \mathrm{R}}{\partial \ln \psi_{\mathrm{i}}}\right)\left(\frac{\partial \ln \psi_{\mathrm{i}}}{\partial \ln \mathrm{~A}_{\mathrm{id}}}\right)}=\beta\left(\frac{\mathrm{A}_{\mathrm{ix}}}{\mathrm{~A}_{\mathrm{id}}}\right)^{\alpha}=\left(\frac{\mathrm{A}_{\mathrm{ix}}}{\mathrm{~A}_{\mathrm{id}}}\right)^{\frac{\sigma_{\mathrm{i}} \theta_{\mathrm{i}}}{\left(\sigma_{\mathrm{i}}-1\right)}}\left(\frac{\mathrm{f}_{\mathrm{ix}}}{\mathrm{f}_{\mathrm{i}}}\right)^{1-\frac{\theta_{\mathrm{i}}}{\left(\sigma_{\mathrm{i}}-1\right)}} . \tag{A15}
\end{equation*}
$$

Therefore, $\alpha=\sigma_{i} \theta_{\mathrm{i}} /\left(\sigma_{\mathrm{i}}-1\right)$ and $\beta=\left(\mathrm{f}_{\mathrm{ix}} / \mathrm{f}_{\mathrm{i}}\right)^{1-\frac{\theta_{\mathrm{i}}}{\left(\sigma_{\mathrm{i}}-1\right)}}$, from which (21) is obtained.
To obtain (22), we use (13) and $\mathrm{R}\left[\psi_{1}\left(\mathrm{~A}_{1 \mathrm{~d}}, \mathrm{~A}_{1 \mathrm{x}}\right), \ldots, \psi_{\mathrm{N}}\left(\mathrm{A}_{\mathrm{Nd}}, \mathrm{A}_{\mathrm{Nx}}\right), \mathrm{V}\right]$ to compute:

$$
\begin{equation*}
\frac{\partial \ln \mathrm{R}}{\partial \ln \mathrm{~A}_{\mathrm{id}}}+\frac{\partial \ln \mathrm{R}}{\partial \ln \mathrm{~A}_{\mathrm{ix}}}=\frac{\mathrm{R}_{\mathrm{id}}+\mathrm{R}_{\mathrm{ix}}}{\mathrm{R}}=\left(\frac{\partial \ln \mathrm{R}}{\partial \ln \psi_{\mathrm{i}}}\right)\left[\left(\frac{\partial \ln \psi_{\mathrm{i}}}{\partial \ln \mathrm{~A}_{\mathrm{id}}}\right)+\left(\frac{\partial \ln \psi_{\mathrm{i}}}{\partial \ln \mathrm{~A}_{\mathrm{ix}}}\right)\right] . \tag{A16}
\end{equation*}
$$

The final term in brackets equals unity for the CES function, so that (22) follows. QED

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Table 1: Summary Statistics for Export Variety

|  | Industry <br> Average | Agriculture | Textiles \& Garments | Wood \& Paper | Petroleum \& Plastics | Mining \& Metals | Machinery \& Transport | Electronics |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Export Variety (percent) |  |  |  |  |  |  |  |  |
| Mean | 31.2 | 25.5 | 43.5 | 35.3 | 30.9 | 23.4 | 24.7 | 32.1 |
| Stan.Dev. | 19.7 | 14.9 | 25.9 | 22.8 | 26.4 | 19.1 | 23.3 | 23.6 |
| Correlation with GDP | 0.46 | 0.30 | 0.35 | 0.39 | 0.29 | 0.56 | 0.54 | 0.35 |
| 1980 | 18.1 | 23.6 | 14.8 | 25.7 | 30.2 | 19.8 | 10.8 | 11.6 |
| 1988 | 43.3 | 36.6 | 57.0 | 57.4 | 40.3 | 28.6 | 29.8 | 56.6 |
| 1989 | 19.9 | 17.4 | 29.6 | 20.7 | 22.9 | 18.1 | 19.2 | 15.4 |
| 2000 | 58.9 | 29.2 | 82.3 | 58.5 | 55.4 | 35.7 | 49.2 | 64.6 |
| Annual Growth, 1980-1988 | 10.9 | 5.5 | 16.8 | 10.0 | 3.6 | 4.6 | 12.6 | 19.8 |
| Annual Growth, 1989-2000 | 9.9 | 4.7 | 9.3 | 9.5 | 8.0 | 6.2 | 8.5 | 13.0 |
| $\begin{aligned} & \hline \text { Variety } 2000 \\ & \hline \text { Variety } 1980 \\ & \hline \end{aligned}$ | 7.9 | 2.7 | 12.2 | 7.0 | 3.4 | 3.0 | 7.8 | 24.1 |

## Notes:

1. Correlations with exporter real GDP is computed across years and countries.
2. Export variety falls from 1988 to 1989 due to the change in classification of U.S. imports, from the TSUSA classification to the Harmonized System.
3. Annual growth is computed as the difference in $\log$ varieties, divided by the number of years in the interval.
4. $($ Variety $2000 /$ Variety 1980$)=\exp [($ annual growth, $1980-1988) \times 8.5+($ annual growth, 1989-2000 $) \times 11.5]$. This calculation attributes average growth in export variety for the 1988-1989 year, when growth is not observed.

Table 2: Summary Statistics for Traded Sectors

|  | Industry Average | Agriculture | Textiles \& Garments | Wood \& Paper | Petroleum <br> \& Plastics | Mining \& Metals | Machinery <br> \& Transport | Electronics |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Value Added Share in GDP (percent) |  |  |  |  |  |  |  |  |
| Mean | 2.8 | 4.0 | 2.2 | 2.2 | 3.6 | 2.2 | 2.6 | 3.0 |
| Stan. Dev. | 0.8 | 1.8 | 1.5 | 1.1 | 1.8 | 1.1 | 1.7 | 2.1 |
| U.S. Tariffs (percent) |  |  |  |  |  |  |  |  |
| Mean | 3.4 | 2.6 | 11.6 | 2.1 | 2.1 | 3.3 | 2.5 | 3.1 |
| Stan. Dev. | 3.5 | 3.3 | 7.5 | 5.0 | 3.4 | 5.5 | 3.2 | 5.0 |
| Correlation with Variety | -0.25 | -0.12 | -0.16 | -0.13 | -0.20 | -0.06 | -0.10 | -0.20 |
| 1980 | 4.0 | 3.5 | 14.3 | 3.8 | 2.5 | 4.5 | 3.7 | 6.2 |
| 2000 | 2.1 | 1.4 | 10.3 | 0.7 | 1.0 | 1.7 | 1.0 | 0.5 |
| Difference | -1.9 | -2.1 | -4.0 | -3.1 | -1.5 | -2.7 | -2.7 | -5.7 |

## Notes:

1. Correlations between U.S. tariffs and export variety in that sector are computed across years and countries.

Table 3: Dependent Variables - Industry Shares in Columns (1) to (7), and Adjusted TFP in Column (8) Estimation method: Three Stage Least Squares Regressions
Total system observations: 4072


Note: For (1) to (7), each coefficient of the log of relative export variety in the row industry is the partial price effect of that industry on
the share of the column industry. These are the point estimates of gamma's. Own price effects are in bold.
For (8), each coefficient of the log of relative export variety in the row industry is the point estimate of $1 /(1$-sigma)*theta of that industry.
${ }^{1}$ Relative land area for (8).
*, $*^{*}$, and ${ }^{* * *}$ indicate significance at $90 \%, 95 \%$, and $99 \%$ confidence levels respectively, and White-robust standard errors are in parentheses. Instruments: effective tariffs, NAFTA dummy, distance, and distance squares, relative land, labor and capital endowments.

Table 4: Hypothesis Testing

| Null Hypotheses | Degree of Freedom | Test Statistics | P-values |
| :--- | :---: | :---: | :---: |
| Homogeneity | 7 | 3.339 | 0.852 |
| Symmetry | 21 | 10.469 | 0.972 |
| Over Identifying Restrictions | 16 | 9.080 | 0.910 |
| Overall Specification | 44 | 52.620 | 0.175 |

Table 5: Dependent Variables - Export Variety Index
Estimation method: Ordinary Least Squares

|  | Independent Variables: | Eq (1) <br> Agriculture | Eq(2) <br> Textiles \& Garments | $\mathrm{Eq}(3)$ <br> Wood \& Paper | Eq(4) <br> Petroleum \& Plastics | Eq(5) <br>  <br> Basic Metals | Eq(6) <br> Machinery \& Transports | $\mathrm{Eq}(7)$ Electronics |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Agriculture | $\begin{aligned} & \hline-6.625 * * * \\ & (0.963) \end{aligned}$ | $\begin{aligned} & \hline \hline 3.537 * * * \\ & (0.842) \end{aligned}$ | $\begin{aligned} & \hline \hline-0.395 \\ & (0.914) \end{aligned}$ | $\begin{aligned} & \hline \hline-8.164^{* * *} \\ & (2.083) \end{aligned}$ | $\begin{aligned} & \hline \hline-0.801 \\ & (1.133) \end{aligned}$ | $\begin{aligned} & \hline \hline 0.267 \\ & (1.237) \end{aligned}$ | $\begin{aligned} & \hline \hline-1.028 \\ & (0.980) \end{aligned}$ |
|  | Textiles \& Garments | $\begin{aligned} & 0.850 \\ & (0.596) \end{aligned}$ | $\begin{aligned} & \mathbf{0 . 2 3 1} \\ & (0.581) \end{aligned}$ | $\begin{aligned} & -0.386 \\ & (0.591) \end{aligned}$ | $\begin{aligned} & -2.877 * * \\ & (1.137) \end{aligned}$ | $\begin{aligned} & 0.190 \\ & (0.642) \end{aligned}$ | $\begin{aligned} & -1.597 * * \\ & (0.637) \end{aligned}$ | $\begin{aligned} & -1.999 * * * \\ & (0.536) \end{aligned}$ |
| 范 | Wood \& Paper | $\begin{aligned} & 1.775 \\ & (1.290) \end{aligned}$ | $\begin{aligned} & -0.250 \\ & (1.163) \end{aligned}$ | $\begin{aligned} & \mathbf{- 3 . 0 5 3 * * * *} \\ & (1.029) \end{aligned}$ | $\begin{aligned} & -3.048 \\ & (2.715) \end{aligned}$ | $\begin{aligned} & -3.136 * * \\ & (1.595) \end{aligned}$ | $\begin{aligned} & -2.206 \\ & (1.481) \end{aligned}$ | $\begin{aligned} & -2.589 * * \\ & (1.154) \end{aligned}$ |
| 烒 |  <br> Plastics | $\left\lvert\, \begin{aligned} & -7.842 * * * \\ & (1.603) \end{aligned}\right.$ | $\begin{aligned} & 2.993 * * * \\ & (1.024) \end{aligned}$ | $\begin{aligned} & -0.101 \\ & (1.072) \end{aligned}$ | $\begin{aligned} & \mathbf{- 9 . 8 6 2} \text { *** } \\ & (2.382) \end{aligned}$ | $\begin{aligned} & -0.217 \\ & (1.383) \end{aligned}$ | $\begin{aligned} & 1.976 \\ & (1.365) \end{aligned}$ | $\begin{aligned} & 1.897 \\ & (1.331) \end{aligned}$ |
| $\underset{0}{ \pm}$ | Mining \& Basic Metals | $\begin{aligned} & -1.465 \\ & (1.276) \end{aligned}$ | $\begin{aligned} & 1.615 \\ & (1.038) \end{aligned}$ | $\begin{aligned} & 0.522 \\ & (1.015) \end{aligned}$ | $\begin{aligned} & -1.754 \\ & (2.115) \end{aligned}$ | $\begin{aligned} & \text {-3.256**** } \\ & (1.243) \end{aligned}$ | $\begin{aligned} & 4.173 * * * \\ & (1.448) \end{aligned}$ | $\begin{aligned} & 3.239 * * * \\ & (1.195) \end{aligned}$ |
| $\bullet$ | Machinery \& Transports | $\begin{array}{\|l} -1.803 \\ (1.479) \end{array}$ | $\begin{aligned} & 0.760 \\ & (1.206) \end{aligned}$ | $\begin{aligned} & -2.571^{*} \\ & (1.362) \end{aligned}$ | $\begin{aligned} & -13.940^{* * *} \\ & (3.264) \end{aligned}$ | $\begin{aligned} & -4.120^{*} \\ & (2.351) \end{aligned}$ | $\begin{aligned} & \mathbf{- 8 . 7 4 8 * * *} \\ & (1.922) \end{aligned}$ | $\begin{aligned} & -7.672 * * * \\ & (1.451) \end{aligned}$ |
|  | Electronics | $\begin{aligned} & -0.089 \\ & (1.482) \end{aligned}$ | $\begin{aligned} & -4.894 * * * \\ & (1.692) \end{aligned}$ | $\begin{aligned} & 1.031 \\ & (1.403) \end{aligned}$ | $\begin{aligned} & 13.652 * * * \\ & (3.052) \end{aligned}$ | $\begin{aligned} & 3.996 * * \\ & (1.911) \end{aligned}$ | $\begin{aligned} & 4.922 * * * \\ & (1.780) \end{aligned}$ | $\begin{aligned} & \mathbf{3 . 0 3 7} * * \\ & (1.452) \end{aligned}$ |
|  | North America Free Trade Agreement | $\begin{aligned} & 0.271 * * * \\ & (0.090) \end{aligned}$ | $\begin{aligned} & 0.039 \\ & (0.069) \end{aligned}$ | $\begin{aligned} & 0.049 \\ & (0.076) \end{aligned}$ | $\begin{aligned} & -0.357 * * \\ & (0.141) \end{aligned}$ | $\begin{aligned} & 0.160 * * \\ & (0.074) \end{aligned}$ | $\begin{aligned} & 0.188 \\ & (0.136) \end{aligned}$ | $\begin{aligned} & 0.011 \\ & (0.103) \end{aligned}$ |
|  | Log of Distance | $\begin{aligned} & -0.152 \\ & (0.356) \end{aligned}$ | $\begin{aligned} & -1.447 * * * \\ & (0.294) \end{aligned}$ | $\begin{aligned} & -1.222 * * * \\ & (0.258) \end{aligned}$ | $\begin{aligned} & -0.010 \\ & (0.711) \end{aligned}$ | $\begin{aligned} & -0.302 \\ & (0.344) \end{aligned}$ | $\begin{aligned} & -3.012 * * * \\ & (0.359) \end{aligned}$ | $\begin{aligned} & -2.768 \\ & (0.319) \end{aligned}$ |
|  | $(\text { Log of Distance })^{2}$ | $\left(\begin{array}{l} -0.003 \\ (0.022) \end{array}\right.$ | $\begin{aligned} & 0.080 * * * \\ & (0.018) \end{aligned}$ | $\begin{aligned} & 0.067 * * * \\ & (0.016) \end{aligned}$ | $\begin{aligned} & -0.020 \\ & (0.044) \end{aligned}$ | $\begin{aligned} & 0.011 \\ & (0.021) \end{aligned}$ | $\begin{aligned} & 0.175 * * * \\ & (0.022) \end{aligned}$ | $\begin{aligned} & 0.167 \\ & (0.020) \end{aligned}$ |
|  | Endowment controls | Yes | Yes | Yes | Yes | Yes | Yes | Yes |
|  | Year Fixed Effects | Yes | Yes | Yes | Yes | Yes | Yes | Yes |
|  | R-squared | 0.5711 | 0.7110 | 0.7241 | 0.6191 | 0.8023 | 0.8352 | 0.7847 |

Note: All figures in bold are the own partial effects of effective tariffs. White robust standard errors are in parentheses.
Effective tariffs are the ratios of duties paid over industry exports.
Endowment controls included are the right-hand side variables of Table 1, which are log of relative labor-land ratio, capital-land ratio,
relative land area and nontraded good prices
*, **, and ${ }^{* * *}$ indicate significance at $90 \%, 95 \%$, and $99 \%$ confidence levels respectively.

## Table 6: Productivity Decompositions

|  | Overall Variation (in \%) |  | Within-Country Variation (in \%) |  |
| :--- | :---: | :---: | :---: | :---: |
|  | Full Sample | OECD | Full Sample | OECD |
| Variance of Estimated Country TFP | $0.368(100)$ | $0.168(100)$ | $0.047(100)$ | $0.038(100)$ |
| Explained by Country Fixed Effects | $0.259(70.5)$ | $0.079(47.3)$ |  |  |
| Explained by Average Variety | $0.009(2.4)$ | $0.018(10.6)$ | $0.019(40.4)$ | $0.023(60.8)$ |

Source: Authors calculation based on regression results of Table 3.


Figure 1: Equilibria with lower productivity at $\mathbf{q}_{\mathbf{2}}$


Figure 2: Average Country Productivity versus Export Variety


Figure 3: Canada compared to Sample Mean


Figure 4: Japan Compared to South Korea


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[^1]:    ${ }^{1}$ See, for example, from the work of Bowen, Leamer and Sveikauskas (1987), Trefler (1993, 1995), and Davis and Weinstein (2001) on the Heckscher-Ohlin model
    ${ }^{2}$ Explanations for the aggregate productivity differences across countries include geography/climate (Sachs, 2001), or colonial institutions (Acemoglu, et al, 2001), or social capital (Jones and Hall, 1999).

[^2]:    ${ }^{3}$ With a Pareto distribution for firm productivities, we also show that relative export variety enters with an exponent related to the elasticity of substitution and the Pareto productivity parameter. This result is similar to the formulation of the gravity equation in Chaney (2005).
    4 Assuming that the distribution of firms productivities is the same across countries is consistent with Melitz (2003), Bernard, Redding and Schott (2004) and Chaney (2005), but this assumption is not made by Eaton and Kortum (2002).

[^3]:    5 We identify conditions below to ensure that $\varphi_{\mathrm{ix}}^{*} \geq \varphi_{\mathrm{i}}^{*}$, as required in the model.

[^4]:    ${ }^{6}$ Notice that we are measuring export price and quantity at the factory-gate, or f.o.b., before of any transport costs. The c.i.f. prices would instead be $\tau_{\mathrm{i}} \mathrm{p}_{\mathrm{ix}}$, and the quantity inclusive of the amount lost in transit would be $\mathrm{q}_{\mathrm{ix}} / \tau_{\mathrm{i}}$. It would be equivalent to use these c.i.f. prices and quantities, but we find it convenient to use the f.o.b. variables since they show how the transport costs $\tau_{\mathrm{i}}$ enter the export shift parameter in ( $5^{\prime}$ ).

[^5]:    ${ }^{7}$ Fixed costs are multiplied by marginal costs in (15) because we used the same function $h_{i}$ to measure the resources needed for fixed and marginal costs.

[^6]:    ${ }^{8}$ There is a distortion between the production and consumption-side of the economy, since prices for consumers do not equal the marginal cost of production. Dixit and Stiglitz (1977) discussed this distortion in a monopolistically competitive economy with homogeneous firms, and argued that by adding the constraint that firms do not earn negative profits, then the equilibrium is still a constrained first-best for consumers.
    ${ }^{9}$ In condition (a), the GDP function will not be differentiable when there are more sectors than factors. This is the same problem that arises in the competitive case, and to avoid it we assume more factors than sectors.
    ${ }^{10}$ The restriction $\theta_{\mathrm{i}}>\sigma_{\mathrm{i}}-1$ is needed to ensure that average productivities have finite value; see the Appendix.

[^7]:    ${ }^{11}$ See Feenstra (1994) for a description of Sato-Vartia formula.

[^8]:    12 Let $S_{i t}^{\mathrm{c}}$ denote export/(export + domestic) sales in sector i and country c. Then $\mathrm{W}_{\mathrm{it}}^{\mathrm{c}}$ is constructed as: $\left[\left(\mathrm{S}_{\mathrm{it}}^{\mathrm{c}}-\mathrm{S}_{\mathrm{it}}^{\mathrm{a}}\right) /\left(\ln \mathrm{S}_{\mathrm{it}}^{\mathrm{c}}-\ln \mathrm{S}_{\mathrm{it}}^{\mathrm{a}}\right)\right] /\left\{\left[\left(\mathrm{S}_{\mathrm{it}}^{\mathrm{c}}-\mathrm{S}_{\mathrm{it}}^{\mathrm{a}}\right) /\left(\ln \mathrm{S}_{\mathrm{it}}^{\mathrm{c}}-\ln \mathrm{S}_{\mathrm{it}}^{\mathrm{a}}\right)\right]+\left[\left(1-\mathrm{S}_{\mathrm{it}}^{\mathrm{c}}\right)-\left(1-\mathrm{S}_{\mathrm{it}}^{\mathrm{a}}\right)\right] /\left[\ln \left(1-\mathrm{S}_{\mathrm{it}}^{\mathrm{c}}\right)-\ln \left(1-\mathrm{S}_{\mathrm{it}}^{\mathrm{a}}\right)\right]\right\}$.

[^9]:    14 This range for $\rho_{\mathrm{i}}$ is not precise because the export share $\mathrm{W}_{\mathrm{it}}^{\mathrm{c}}$ multiplies $\ln \left(\chi_{\mathrm{it}}^{\mathrm{c}} / \chi_{\mathrm{it}}^{\mathrm{a}}\right)$ in (31), but does not multiply the term $\ln \left(\mathrm{M}_{\mathrm{it}}^{\mathrm{a}} / \mathrm{M}_{\mathrm{it}}^{\mathrm{a}}\right)$. Nevertheless, we still use the export share in (32).

[^10]:    16 We thank a referee for pointing this out that this consistency was needed. If we instead use the worldwide exports to the U.S. in each year as the comparison, then the measures of export variety obtained are higher than those reported in Table 1, with lower growth rates.

[^11]:    ${ }^{17}$ Real investment is obtained by deflating the gross domestic capital formation of countries with that item's GDP deflator. In addition, we construct the base year capital stock using an infinite sum series of investment prior to the first year, assuming that the growth rate of investment of the first five years are good proxy for investment prior to the first year.
    18 The nontraded goods price for the comparison country is a weighted average of the country nontraded goods price indexes.

[^12]:    19 We thank Ann Harrison for providing this data.

[^13]:    ${ }^{20}$ Homogeneity in endowments means that (38) can be written as depending on the labor-land ratio and the capitalland ratio. Since the share equations sum to unity across sectors, their errors sum to zero. So one equation should be omitted for estimation, and we omit the share equation for the nontraded sector.
    ${ }^{21}$ Due to convergence problem, the $\gamma_{\mathrm{jj}}$ coefficient of the petroleum \& plastics industry (sector 4 ), is estimated separately, by fixing all the rest of the parameters in the optimal values. We repeated the process a few rounds, and the estimation results are very stable, as presented in Table 2.

[^14]:    ${ }^{22}$ Transport costs are not included as instrument as they are potentially endogenous - countries that trade a lot with the U.S. may have lower transport costs as a result. We thank a referee for this comment.

[^15]:    ${ }^{23}$ For example, differentiating (39) with respect to $\rho_{i}$ we obtain $\frac{1}{2}\left(s_{i t}^{c}+s_{i t}^{a}\right) W_{i t}^{c} \ln \Lambda_{i t}^{c}$, which is the export variety index for country c and sector $i$, times the average export share of that industry.

