Econ. C103 McFadden

PROBLEM SET 7 (due Nov. 4)

1. The following game in strategic form is called the Battle of the Sexes. Mary likes opera and John likes football, and they also like to do things together. Find the Nash equilibria.

		Mary's Action	
		Opera	Football
John's Action	Opera	(2,3)	(0,0)
	Football	(1,1)	(3,2)

2. Suppose the payoffs in the Battle of the Sexes game are modified because John mildly prefers football to opera, but Mary really hates football. Find the Nash equilibria.

		Mary's Action	
		Opera	Football
John's Action	Opera	(2,3)	(0,0)
	Football	(1,2)	(3,1)

3. Two players have actions x and z that are both in the unit interval, and have payoffs f(x,z) and g(x,z) respectively that are continuous and have the properties that f is concave in x, given z, and z is concave in z, given x. Does a Nash equilibrium always exist? If so prove that it does.

4. Prove that if a game in strategic form has a Nash equilibrium with mixed strategies, then the expected payoff to a player from any pure strategy in the support of the mixed strategy is the same, and all probability mixtures of strategies in the support have the same expected payoff.

5. Two firms are located at the ends of a street of unit length, with customers uniformly distributed along the street. A consumer at location $x \in [0,1]$ will choose to buy Left, None, or Right depending on which of the indirect utility levels $\{1 - p - x, 0, 1 - r - (1-x)\}$ is the largest, where p is the price of the Left firm and r is the price of the Right firm. Find a Nash Equilibrium in the pricing actions (p,r) of the two firms, assuming that they each have zero costs and sufficient capacity to supply the entire market, and seek to maximize profits.

5. A two-person strategic game is *symmetric* if both players have the same action sets, and if f(x,z) is the payoff to the first player at actions (x,z), then f(z,x) is the payoff to the second player at (x,z). Show that if the action sets are compact and f is concave and continuous in its arguments, then a symmetric Nash Equilibrium exists. Give an example of a finite symmetric game in which only asymmetric Nash Equilibria exist.